Super $\left\{\begin{matrix}\text{string}\\ \text{membrane}\end{matrix}\right\}$ Theory

both have Hadron physics origin!

Strings

q — $\bar{q}$

flux tube

$\downarrow$ quantize

graviton

susy to remove tachyon

$\downarrow$

$D=10$ superstring

$\downarrow$

D-branes

Membranes

bag $\rightarrow$ $\langle F\tilde{F}\rangle \neq 0$

$F\tilde{F} \sim F_4 = dA_3$

effective 3-form coupling to 'bag' membrane

$\downarrow$ susy.

$D=11$ supermembrane

$\downarrow$ quantize.

$SU(\infty)$ gauge Q.M.

---

semi-class. M-theory

$\downarrow$

$N \rightarrow \infty$ $\qquad$ $\infty \rightarrow N$

M(atrix) model

That takes us to c. '97. The two streams have converged!

Current Themes

Domain Walls → Brane World
→ Holographic R.G. flow
→ Phases of $N=1$ $D=4$ theories

Non-commutative Geometry → $[x,x] \sim x$ ("fuzzy geometry")
dielectric branes

→ $[x,x] \sim \text{const}$
Open S.T.

"ॐ theory" $\not\equiv$ $\boxed{\text{OM theory}}$ (Bergshoeff et al. Gopakumar et al.)
(J. Coltrane et al.)

\+ many others :
- Cosmological constant rethink
- Potentials on soliton moduli spaces
- Field Th. branes
- Calibrations

⋮

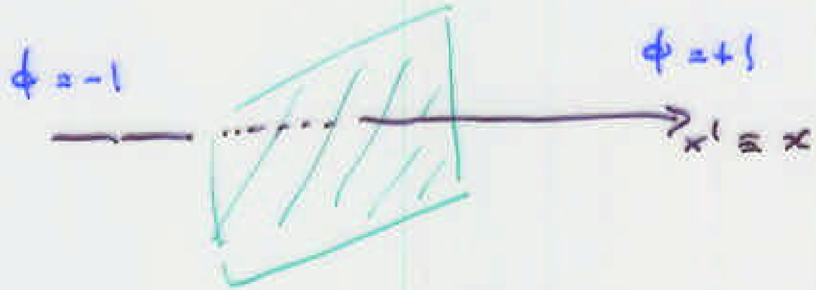
Wess-Zumino Domain Walls

$$\mathcal{L}_{WZ} = \frac{1}{2}|\partial\phi|^2 - \underbrace{\frac{1}{2}|W'(\phi)|^2}_{\text{potential, minima at critical pts. of holomorphic "superpotential" } W(\phi)} + \text{fermions}$$

complex scalar

e.g. $W(\phi) = \frac{1}{3}\phi^3 - \phi \Rightarrow W'(\phi) = \phi^2 - 1$

Expect <u>domain walls</u> between $\phi = \pm 1$ vacua



$$\text{Tension} = \frac{1}{2}\int_{-\infty}^{\infty} dx \left[\left|\frac{d\phi}{dx}\right|^2 + |W'(\phi)|^2 \right]$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \underbrace{\left|\frac{d\phi}{dx} \mp \overline{W'(\phi)}\right|^2}_{\geq 0} \pm \underbrace{\Big[\mathrm{Re}\, W\Big]_{-\infty}^{\infty}}_{\text{top. charge}}$$

Minimized by solns. of $\boxed{\frac{d\phi}{dx} = \pm \overline{W'(\phi)}}$

Susy fractions

A given bosonic field config. preserves some supersymmetry if

$$\boxed{\delta_\epsilon(\text{fermions}) = 0}$$

supersymmetry spinor parameter

For WZ domain walls this becomes

$$\underset{\sim}{n} \cdot \underset{\sim}{\gamma}\, \epsilon = \epsilon$$

This has 2 real solutions, so domain walls preserve $\frac{1}{2}$ susy

More generally, generic soln. of

$$\boxed{\frac{d\phi}{dz} = \overline{W'(\phi)}}$$ "Creek eq."

complex variable

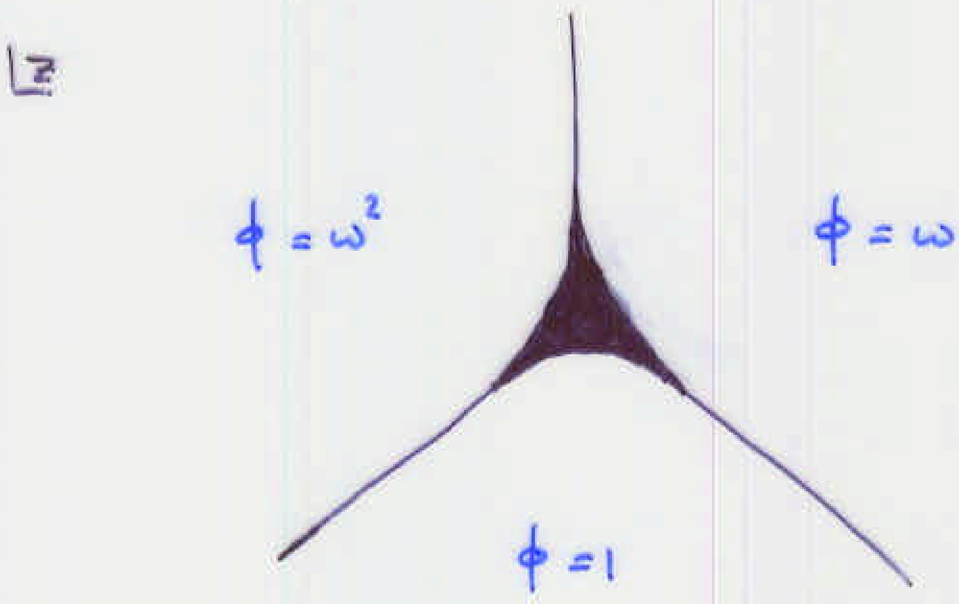
preserves $\frac{1}{4}$ susy

Domain Wall Junctions

Take $W(\phi) = \frac{1}{4}\phi^4 - \phi \quad \Rightarrow \quad W' = \phi^3 - 1$

Vacua at $\phi = \underbrace{(1, \omega, \omega^2)}_{\text{vacua permuted by } \mathbb{Z}_3} \qquad \omega = \sqrt[3]{-1}$

Expect $\mathbb{Z}_3$-symmetric domain wall junction in z-plane



Presenes ¼ susy (and minimizes energy) if it solves "creek" eq.

(Gibbons & PKT
Carroll, Hellerman & Trodden
Bazeia & ter Veldhuis)

But do non-singular solns. exist?

Numerical evidence : (Saffin) $\Rightarrow$ $\boxed{\text{YES}}$

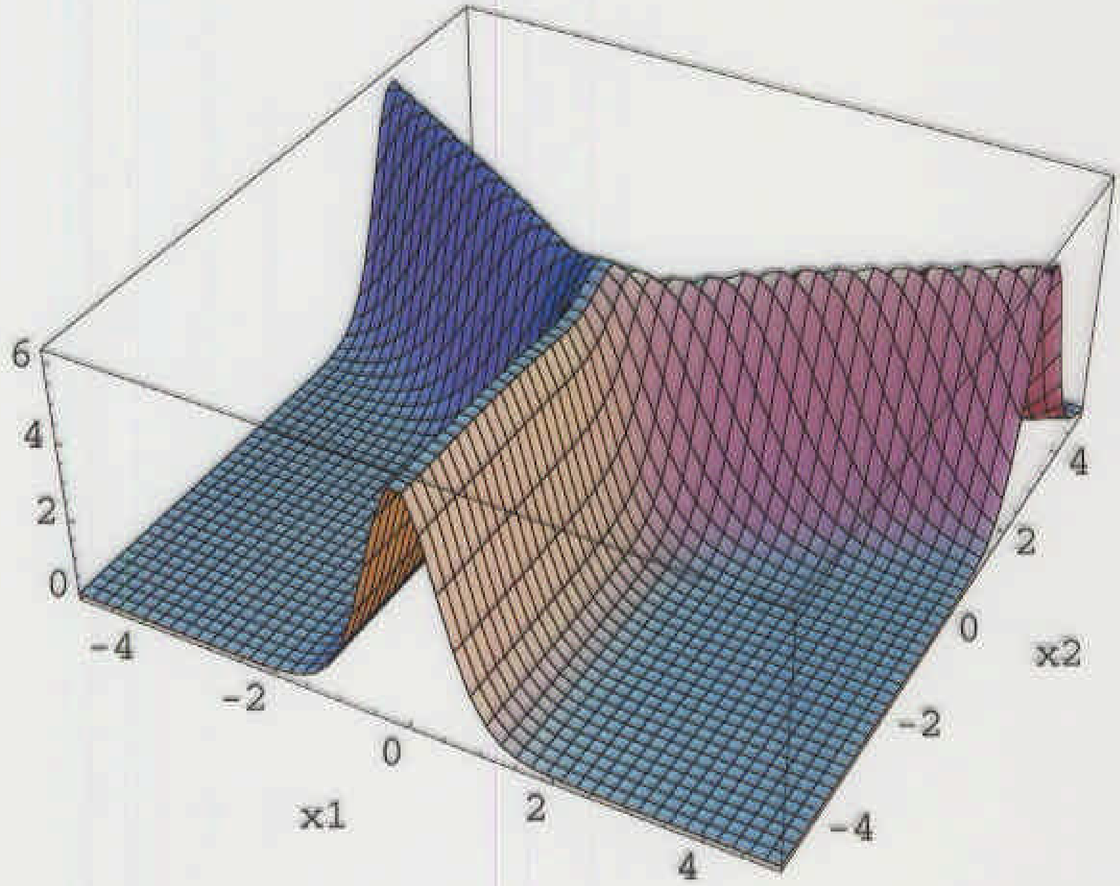
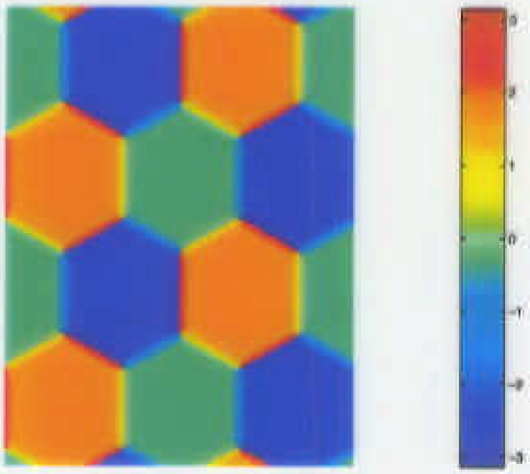


Figure 1: .

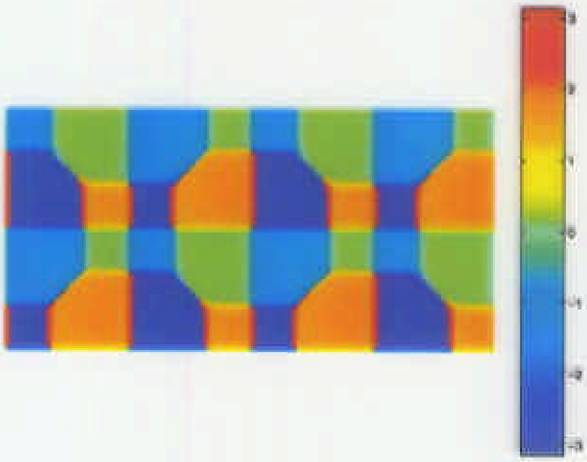
Domain Wall Junction
(P. Saffin)

Domain Wallpaper (P. Saffin)

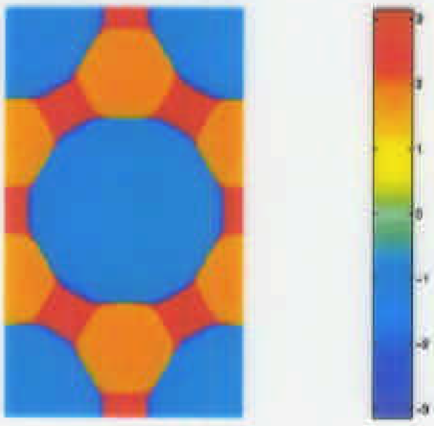
$W' = 1 - \phi^3$



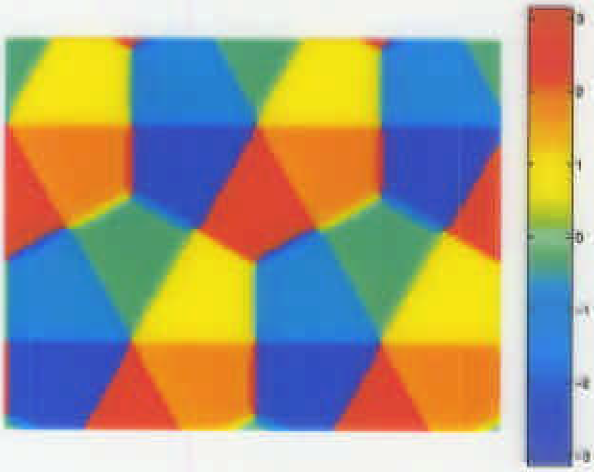
$W' = 1 - \phi^4$



$W' = 1 - \phi^6$



$W' = 1 - \phi^6$



The OINS model

(Oda, Ito, Naganuma & Sakai)

Generalize from one complex scalar to seven

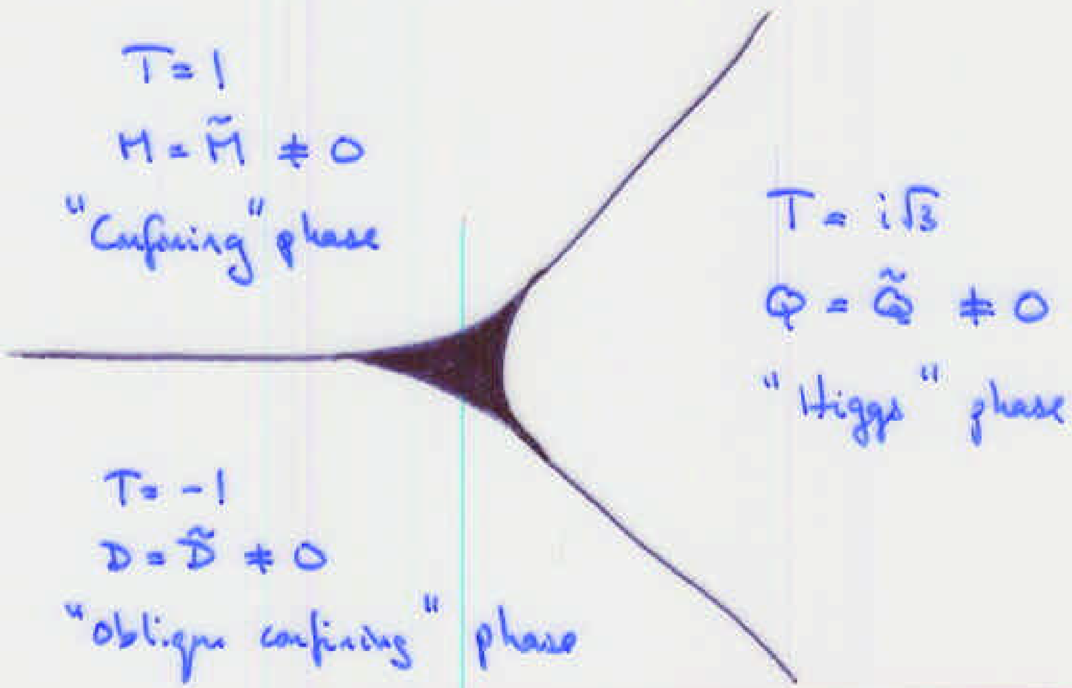
$$\left(T,\ Q,\tilde{Q},\ M,\tilde{M},\ D,\tilde{D}\right)$$

"quarks" "monopoles" "dyons"

Model provides effective description of some $N=1$ supersymmetric gauge theories.

$$W = (T-1)M\tilde{M} + (T+1)D\tilde{D} + (T-i\sqrt{3})Q\tilde{Q} - 2T$$

$\mathbb{Z}_3$-inv. superpotential



OINS found <u>exact</u> solution of "creek" eqn in this case.

A contradiction!

$$\boxed{\{Q_\alpha, Q_\beta\} = (C\gamma^\mu)_{\alpha\beta} P_\mu}$$

only N=1 D=4 susy algebra allowed by HKS theorem.

4 cpt. real spinor charge

Susy states are annihilated by some linear combination of Q's.

Fraction of susy preserved $= \frac{1}{4} \times \dim \ker\{Q, Q\}$

$$\neq 0 \Leftrightarrow \boxed{\det\{Q, Q\} = 0}$$

susy preservation condition

But $\det\{Q, Q\} = (P^2)^2$, so

either ① $P_\mu = 0$ vacuum

or ② $P_\mu \neq 0$ but $P^2 = 0$, massless particle
$\dim \ker\{Q, Q\} = 2$ ½ susy

> ½ susy domain walls & ¼ susy junctions forbidden by susy algebra!

Domain Wall charges

(de Azcárraga, Gauntlett, Izquierdo & PKT
Abraham & PKT
(Dvali & Shifman))

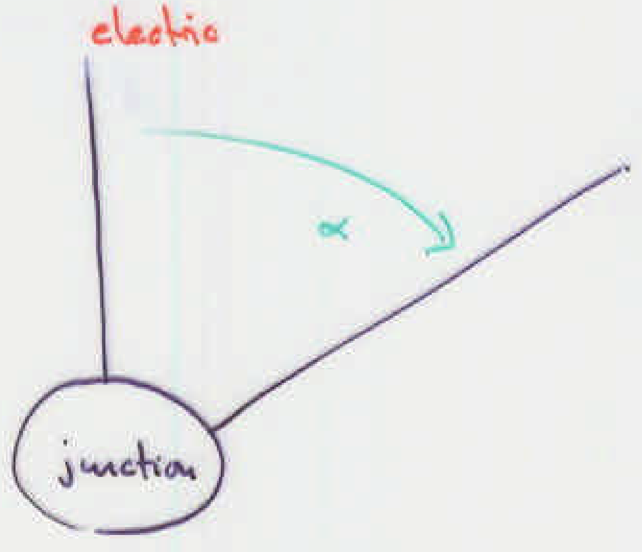
Domain walls carry 2-form charge

$$\boxed{\{Q_\alpha, Q_\beta\} = (C\gamma^\mu)_{\alpha\beta} P_\mu + (C\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu}}$$

$Z_{ab} \longleftrightarrow$ 'electric' wall

$\tilde{Z}_{ab} = \varepsilon_{abc} Z_{oc} \longleftrightarrow$ 'magnetic' wall

Algebra now allows both ½ susy walls and ¼ susy junctions



$$\frac{\text{electric}}{\text{magnetic}} = \cot\alpha$$

(Gibbons, Gauntlett, Wall & PKT)

M-theory

Applying same idea in $D=11$ we have

$$\boxed{\{Q, Q\} = (C\Gamma^{\mu})\, P_{\mu} + (C\Gamma^{\mu\nu})\, Z_{\mu\nu} + (C\Gamma^{\mu\nu\rho\sigma\lambda})\, Y_{\mu\nu\rho\sigma\lambda}}$$

32 cpt real spinor

$$= H + (\Gamma^{0a})\, \underset{\uparrow \atop \text{supergraviton}}{P_a} + (\Gamma^{0ab})\, \underset{\uparrow \atop \text{M2-brane}}{Z_{ab}} + (\Gamma^{0abcde})\, \underset{\uparrow \atop \text{M5-brane}}{Y_{abcde}}$$

$$+ (\Gamma^{a})\, \underset{\uparrow \atop \text{boundary}}{Z_{0a}} + (\Gamma^{abcd})\, \underset{\uparrow \atop \text{KK-monopole}}{Y_{0abcd}}$$

These are basic "ingredients" of M-theory

e.g. <u>Planar</u> M5-brane = <u>minimal surface</u> in $\mathbb{R}^{10}$

Planar $\Rightarrow$ $\frac{1}{2}$ susy

$$\boxed{\left.\begin{array}{l}\text{Non-planar} \\ \text{minimal surface}\end{array}\right\} \rightarrow \; < \tfrac{1}{2} \text{ susy}}$$

Minimal surfaces and MQCD

Minimal surfaces classified by Calibration Theory

Generic case is "Kähler": complex $2p$-surface Σ_{2p}

e.g.

$$MS = \mathbb{R}^3 \times (\underbrace{\Sigma_2 \text{ in } \mathbb{C}^6}_{\text{Kähler calibration}}) \longrightarrow \underbrace{1/8 \text{ susy}}_{\text{corresponds to } N=1 \text{ in } D=4}$$

effective 3-space

MS-brane fluctuations define strongly coupled $SU(n)$ gauge theory with n discrete vacua permuted by $\mathbb{Z}_n$ (MQCD) (Hori, Ooguri & Oz, Witten)

The domain walls & their junctions are given by the two exceptional calibrations

$$\boxed{\begin{array}{lllll} ① \text{"Associative"} & \Sigma_3 \text{ in } \mathbb{R}^7 & \rightarrow & 1/16 & \text{susy} \\ ② \text{"Cayley"} & \Sigma_4 \text{ in } \mathbb{R}^8 & \longrightarrow & 1/32 & \text{susy} \end{array}}$$

(Witten)

(Gibbons, Gauntlett & PKT)

Geometrical interpretation of previous results via M-theory!

Membranes & Matrices

Deformation of spherical membrane described by functions on 2-sphere

$$\underset{\text{deformation}}{f}(\underbrace{\sigma^1, \sigma^2}_{\text{membrane coords.}}) = \sum_{\text{harmonics}}$$

SU(2) irreps.

$$\text{"="} \quad \underset{\text{constant}}{\underset{\sim}{1}} + \underbrace{\underset{\sim}{3} + \cdots + \underset{\sim}{2n-1}}_{\text{decomposition of adjoint of } SU(N) \supset SU(2)} + \cdots \longrightarrow \infty$$

$$\text{Scalar on } S^2 \xrightarrow[\text{first } n \text{ harmonics}]{\text{truncate to}} n \times n \text{ hermitian matrix}$$

$$\underbrace{\{f, g\}_{PB}}_{\text{Poisson Bracket of functions on } S^2} \longrightarrow \underbrace{[f, g]}_{\text{commutator of } n \times n \text{ matrices}}$$

[J. Hoppe: Soryushin Kenkyu $\underline{18}$ (1989) pp 145–202]

Kyoto journal!

Supermembrane & M(atrix) model

Light-cone gauge Lagrangian for supermembrane in M-theory vacuum is (de Wit, Hoppe & Nicolai)

$$\mathcal{L} = \frac{1}{2}\int dt \int_{S^2} d^2\sigma \left\{ \left|\mathcal{D}_t \underset{\sim}{X}\right|^2 - \left|\{\underset{\sim}{X}, \underset{\sim}{X}\}_{PB}\right|^2 \right\} + \text{fermions}$$

connection for area-preserving diffeos.

$$= \frac{1}{2}\int dt \sum_{\text{harmonics}}$$

$$\xrightarrow{\text{truncate}} \frac{1}{2}\int dt \ \mathrm{tr}\left\{ \left|\mathcal{D}_t \underset{\sim}{X}\right|^2 - \left|[\underset{\sim}{X}, X]\right|^2 \right\} + \text{fermions}$$

trace over $n \times n$ matrices

$SU(n)$ connection

Ground State when $\mathcal{D}_t X = 0$ and

$$X = \begin{pmatrix} x_1 & & & 0 \\ & x_2 & & \\ & & \ddots & \\ 0 & & & x_n \end{pmatrix}$$

positions of n "partons"

i.e. <u>fuzzy supermembrane collapses to n pointlike constituents.</u>

Dielectric Branes (Myers, Emparan, ...)

Ground state matrices X need not commute in non-vacuum background, e.g.

$$F_4 = \underset{\text{constant}}{f}\, \varepsilon^{abc}\, dt \wedge dx^a \wedge dx^b \wedge dx^c$$

constant — 3 of transverse coords

yields

$$\boxed{[X^a, X^b] = f\, \varepsilon^{abc} X^c}$$

← Solved by any $n \times n$ rep of $SU(2)$

Lowest energy solution is irreducible $\Rightarrow$ n partons blown up to fuzzy sphere

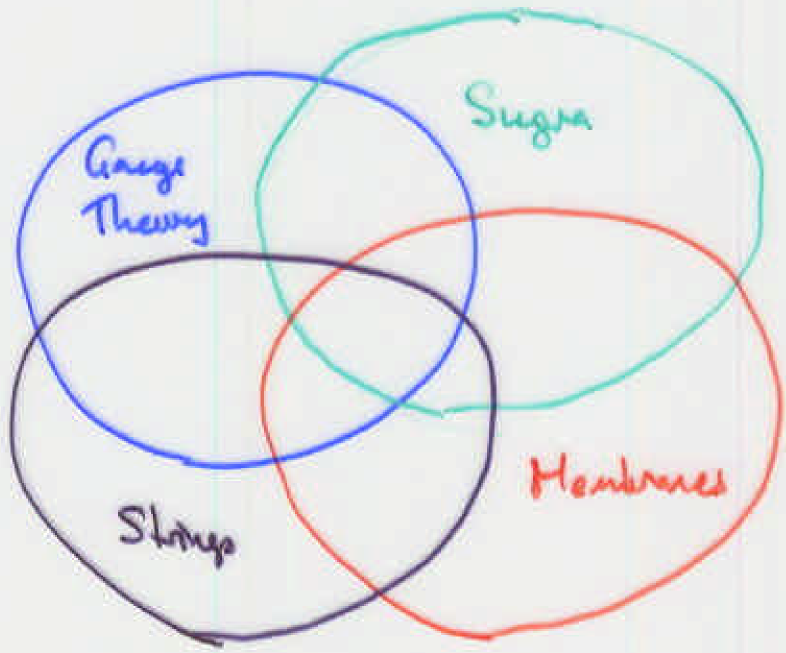
$n \to \infty \Rightarrow$ fuzzy sphere $\longrightarrow S^2$

Application D3-branes of brane world can be blown up to D5-branes wrapped on fuzzy sphere — resolves singularities of sugra duals of $N=1$ $D=4$ theories

(Polchinski & Strassler)

Summary

- Enormous progress since I was last in Japan (circa '81)
- Many different directions, but all are interrelated



May the next 20 yrs be as fruitful!