Lattice Field Theory

Richard Kenway
University of Edinburgh

overview

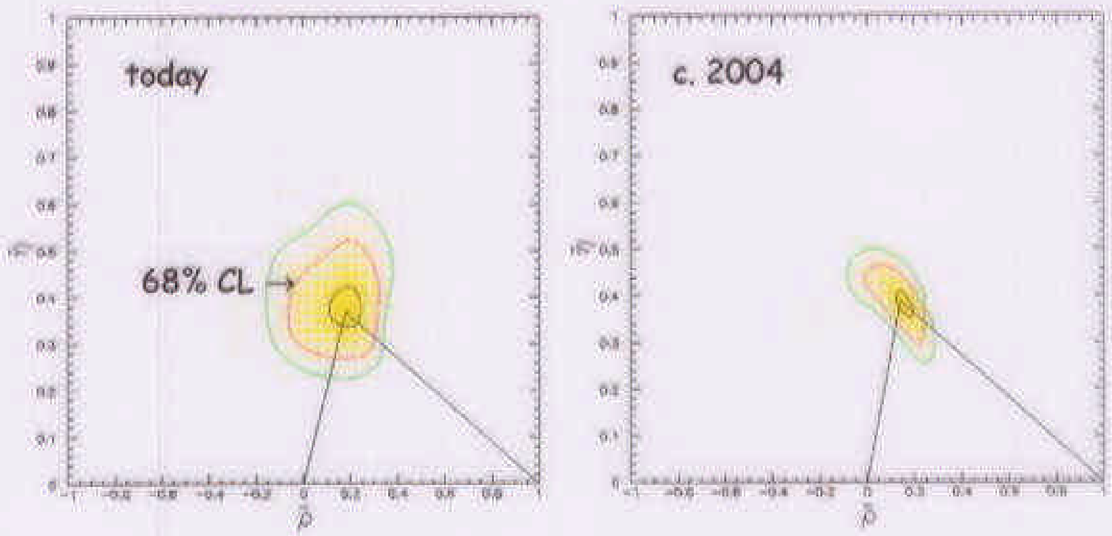
- after a few words about objectives, I will discuss:
  - theoretical progress
  - string breaking
  - hadron spectrum
  - quark masses
  - heavy-quark decays
  - structure functions
  - kaon physics
  - machines & prospects

- I will omit:
  - QCD thermodynamics
  - electroweak phase transition, Higgs, SUSY
  - topology and the confinement mechanism
  - gravity
  - some exploratory phenomenological applications
  - lots of theoretical details

for more see the Lattice 99 proceedings
Nucl. Phys. B (Proc.Suppl.) 83-84 (2000) MARCH 2000

objectives (1)

- to determine Standard Model parameters
  - due to confinement, the quark sector is not directly accessible by experiment
  - lattice QCD offers model-independent computation of hadronic masses and matrix elements
  - this is the main focus → search for new physics
  - a few years of running on a 10 Tflops machine would enable few-percent precision for α_s, quark masses, B parameters, decay constants, and semileptonic form factors (near zero recoil)
  - as a result, the allowed region of the unitarity triangle would shrink:

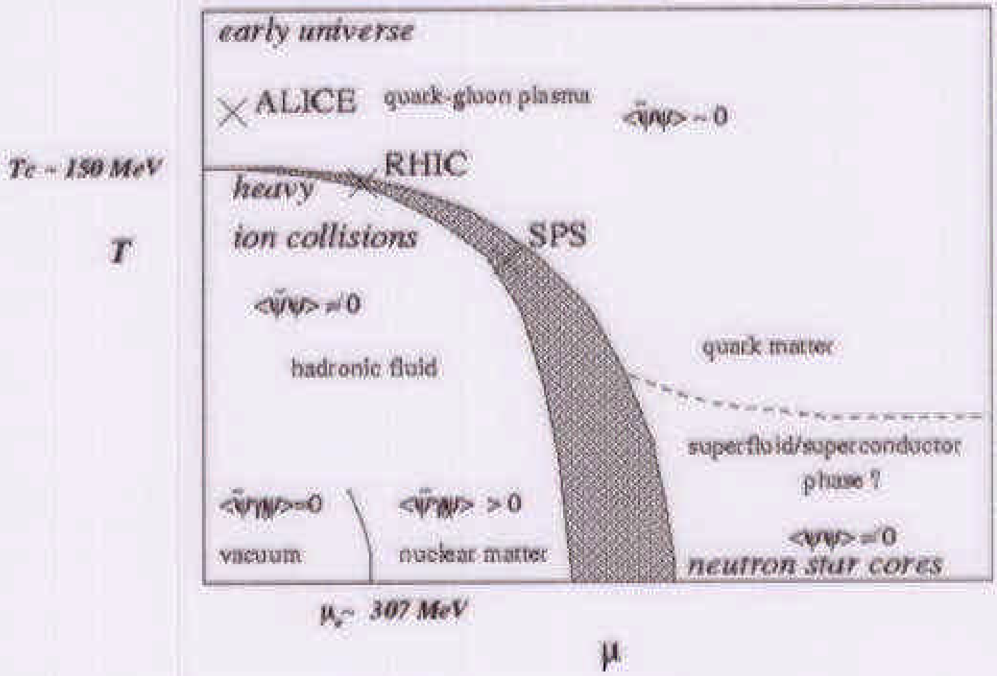


Ciuchini et al., hep-ph/9910236 and ECFA/99/200

  - the name of the game is to control systematics, particularly to:
    - quantify dynamical quark effects
    - reliably simulate/extrapolate to light and heavy quarks, and
    - validate QCD as the theory of strong interactions

objectives (2)

- to determine the phase structure of hadronic matter



  - sensitive to flavour content
    - T_c and the nature of the transition change with N_f
    - awaits simulations with realistic dynamical flavours
  - complex action for $\mu \neq 0$
    - Monte Carlo importance sampling fails
    - awaits an algorithmic breakthrough
  - short time extent of lattices hampers spectroscopy, which is needed for
    - leptonic decays of vector mesons, strangeness production, J/ψ suppression
    - anisotropic lattices will help
- to develop a general-purpose non-perturbative tool
  - progress in formulating lattice chiral symmetry has reawakened hopes of simulating chiral and SUSY theories

theoretical progress (1)

Lattice Chiral Symmetry

- major progress has been achieved following the rediscovery of the Ginsparg-Wilson relation
  - lattice Dirac operator is chosen to satisfy

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

$\{D, \gamma_5\} = 0$ in continuum

  - lattice version of chiral symmetry
  - separates chiral and continuum limits
  - forbids O(a) terms so fermion actions are improved

Kikukawa talk, PA-14b20

- three constructions:
  - overlap — Narayanan, Neuberger
  - domain wall — Kaplan
  - perfect action — Hasenfratz
- for vector theories (QCD) we can maintain global chiral symmetry at non-zero lattice spacing
  - should be possible to simulate u, d quarks with realistic masses
  - operators of different chiralities do not mix, eg simplifies kaon mixing and decay calculations
- abelian and non-abelian chiral gauge theories have been defined on the lattice
  - Standard Model can be defined non-perturbatively

Luescher
Neuberger

- prospect of lattice SUSY without fine tuning
  - lattice chiral symmetry forbids relevant SUSY-violating terms, eg for N=1 SYM

Kaplan et al,
hep-lat/0002030

theoretical progress (2)

Overlap

bare quark mass

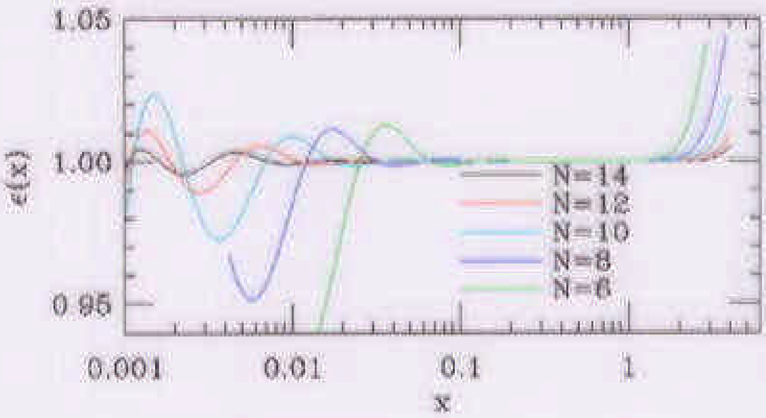
Wilson-Dirac matrix with large -ve mass

$$D_{\mathrm{ov}}(\mu)=\left(\frac{1+\overset{\downarrow}{\mu}}{2}\right)+\left(\frac{1-\mu}{2}\right)\gamma_5\,\mathrm{sgn}(\gamma_5\overset{\downarrow}{D_{\mathrm{W}}})$$

- $D_{ov}(0)$ obeys Ginsparg-Wilson relation
- matrix sign function by rational approximation

$$H_{\mathrm{W}} = \gamma_5 D_{\mathrm{W}}$$

$$\mathrm{sgn}(H_{\mathrm{W}}) \approx \varepsilon_N(H_{\mathrm{W}}) = \sum_{s=1}^{N}\frac{1}{c_s^2 H_{\mathrm{W}} + \dfrac{b_s^2}{H_{\mathrm{W}}}}$$

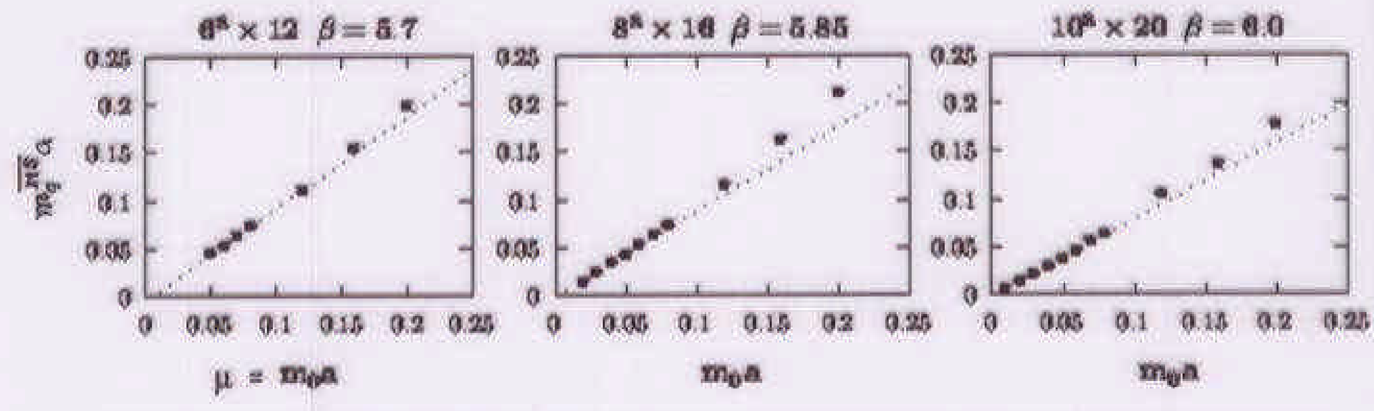


Edwards et al., hep-lat/0001013

- need to project out lowest 10-20 eigenvectors and treat their signs exactly

- no additive quark mass renormalisation

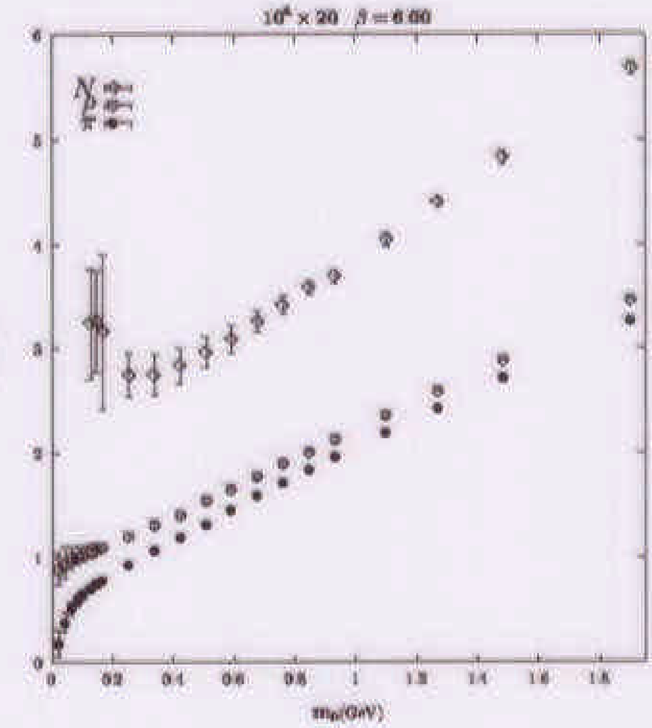
$$m_q^{\overline{\mathrm{MS}}} = Z_m^{-1}\mu\left(1+O(a^2)\right)$$



Liu et al., hep-lat/0006004

theoretical progress (3)

- quenched mass ratios scale at fixed quark mass
  - but in a small volume $(1\text{fm})^3$



finite size effects →

chiral logs with
$\delta = 0.2$ to 0.4 →

Liu et al., hep-lat/0006004

- computational cost for quenched QCD is comparable to simulating dynamical quarks

Hernandez et al., hep-lat/0001008

- introduce 2N auxiliary fields:

$$\bar{\psi}\,\mathrm{sgn}(H_W)\psi \approx \bar{\psi}\sum_{s=1}^{N}\frac{1}{c_s^2 H_W + \frac{b_s^2}{H_W}}\psi$$

$$\to \sum_s \left(\bar{\psi}_s\chi_s + \bar{\chi}_s\psi\right) + \bar{\chi}_s\left(c_s^2 H_W\right)\chi_s + b_s\left(\bar{\chi}_s\phi_s + \bar{\phi}_s\chi_s\right) - \bar{\phi}_s H_W \phi_s$$

  - 5 dimensional matrix, with condition number

$$\kappa \le \frac{1}{\mu}\kappa(H_W)$$

Narayanan & Neuberger, hep-lat/0005004

theoretical progress (4)

Domain Wall Quarks

- couple fermions to a mass defect in a 5th dimension

$$D_{\rm DW}(\mu) = \left(\frac{1+\mu}{2}\right) + \left(\frac{1-\mu}{2}\right)\gamma_5 \tanh\left(-\frac{L_s}{2}\log T\right)$$

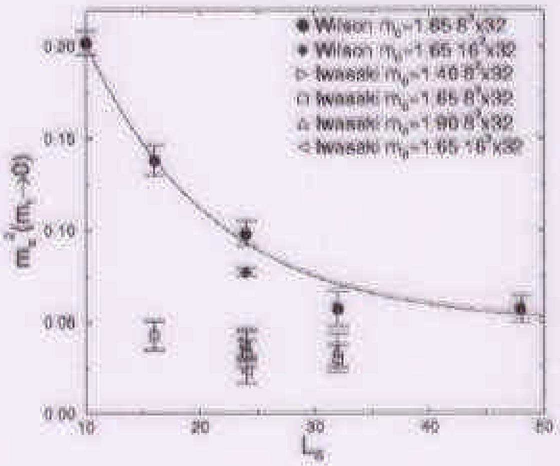
↑ transfer matrix in 5th dimension

- chiral modes of opposite chirality are trapped on 4-dimensional domain walls
- chiral symmetry breaking is exponentially suppressed by the size of the 5th dimension

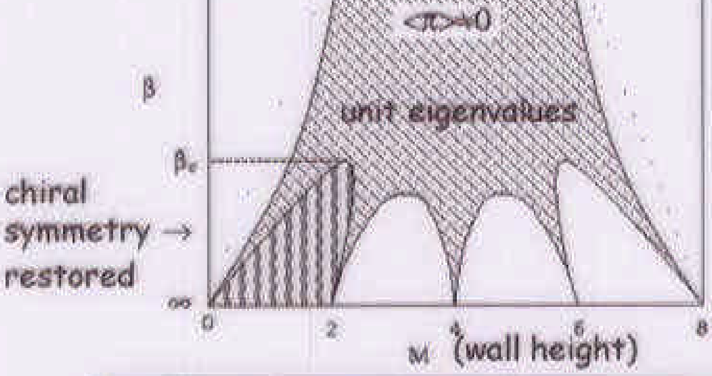
$$m_\pi^2 = C_0 + C_1(\mu + De^{-\alpha L_s})$$

$$C_0 \to 0 \text{ as } V \to \infty$$

- pion mass does not always vanish with quark mass due to near unit eigenvalues of T, which allow unsuppressed interactions between LH and RH fermions



Wu, hep-lat/9909117



- chiral symmetry is restored for RG improved action at weak coupling
- project out low eigenvectors and take their contribution with infinite L_s

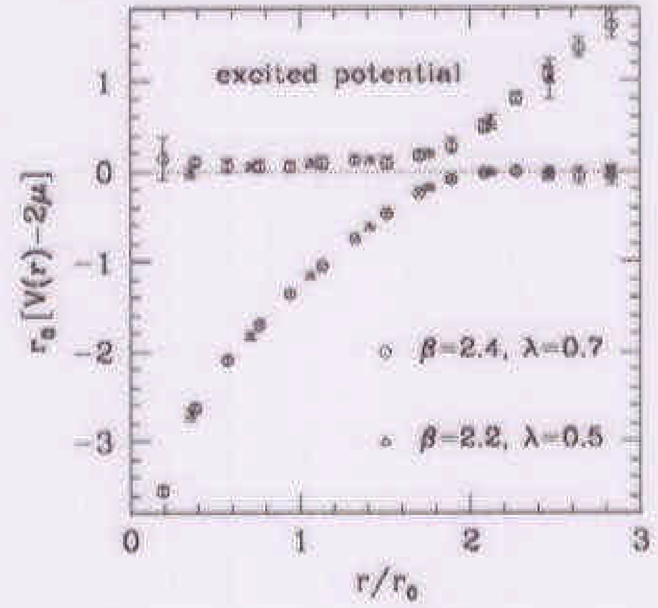
CP-PACS, hep-lat/0007014

Edwards & Heller, hep-lat/0005002

string breaking

- signature of dynamical quarks (at zero temperature)
- observed as level crossing in the confinement phase of SU(2) Higgs

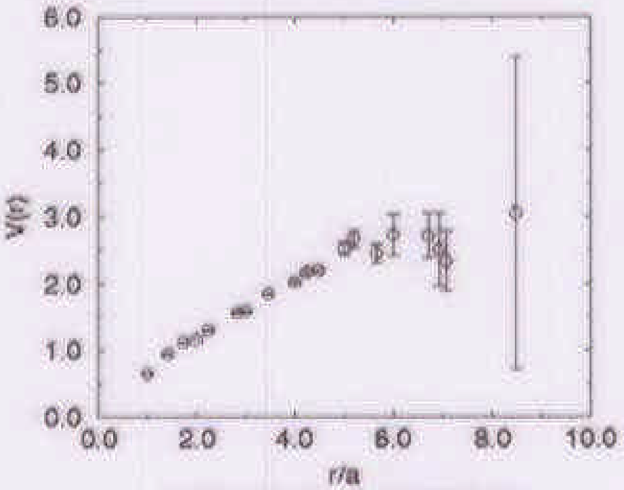
2 static-light "mesons" →



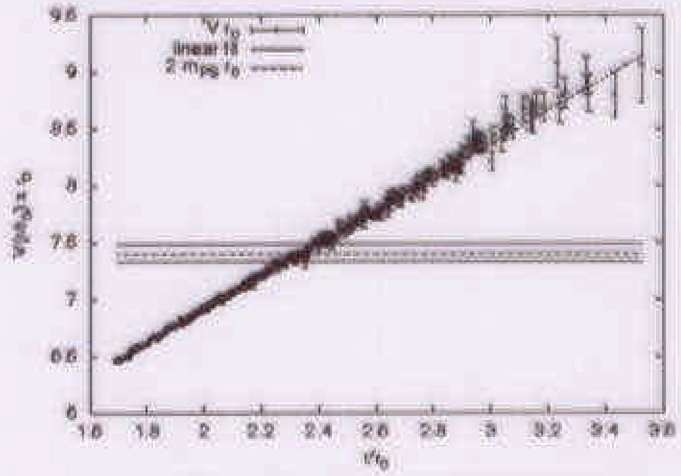
Knechtli, hep-lat/9909164

- not (yet convincingly) in QCD due to poor overlap of string states with 2 static-light mesons
  - costly … requires quark propagators at all sites
  - results, so far, for N_f = 2:
  - mixing matrix element is non-zero

UKQCD,
hep-lat/0001015



MILC, hep-lat/9909118

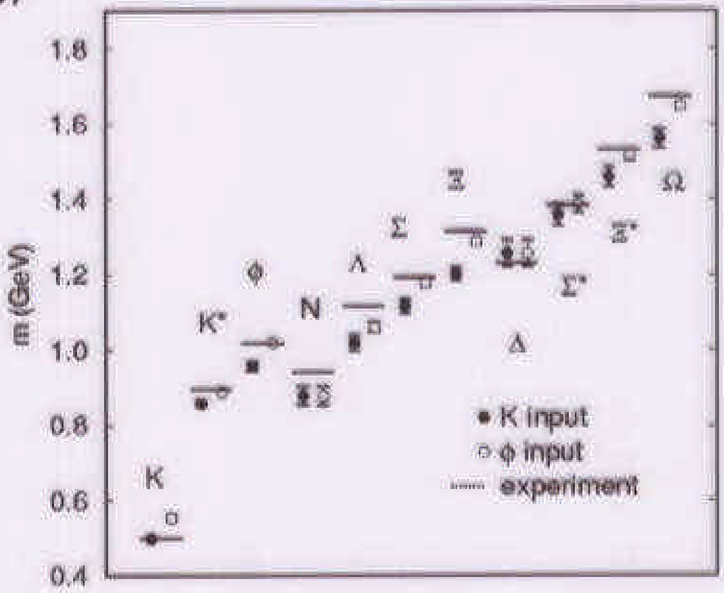


SESAM-TχL, hep-lat/0005018

hadron spectrum (1)

- quenched light hadrons
  - disagree with experiment...finally established in 1998!
  - but deviation is within 10% ... good news for phenomenology

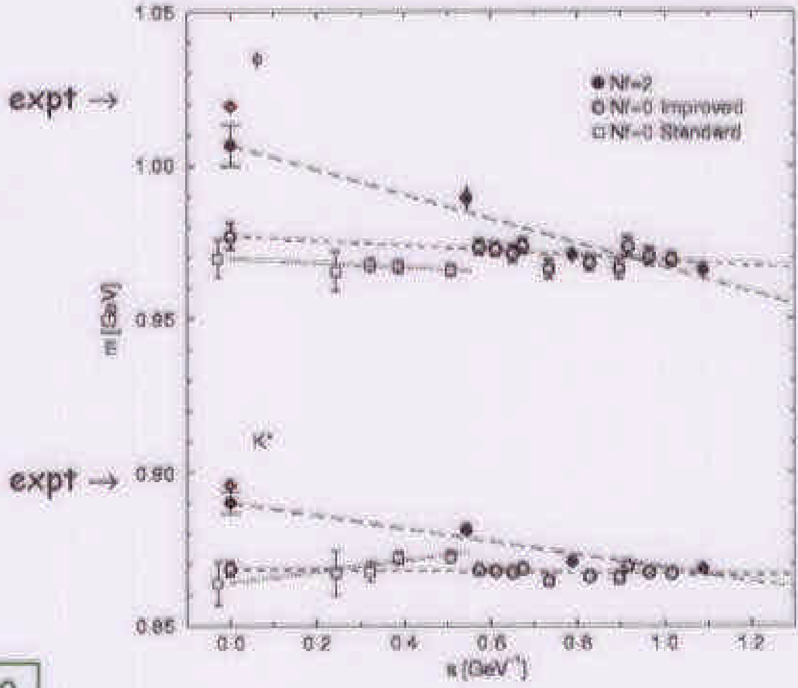
s quark mass cannot be defined consistently, eg from K and ϕ



CP-PACS, hep-lat/9904012

- dynamical u,d quarks, quenched s quark

strange meson spectrum is closer to experiment (m_s from M_K)



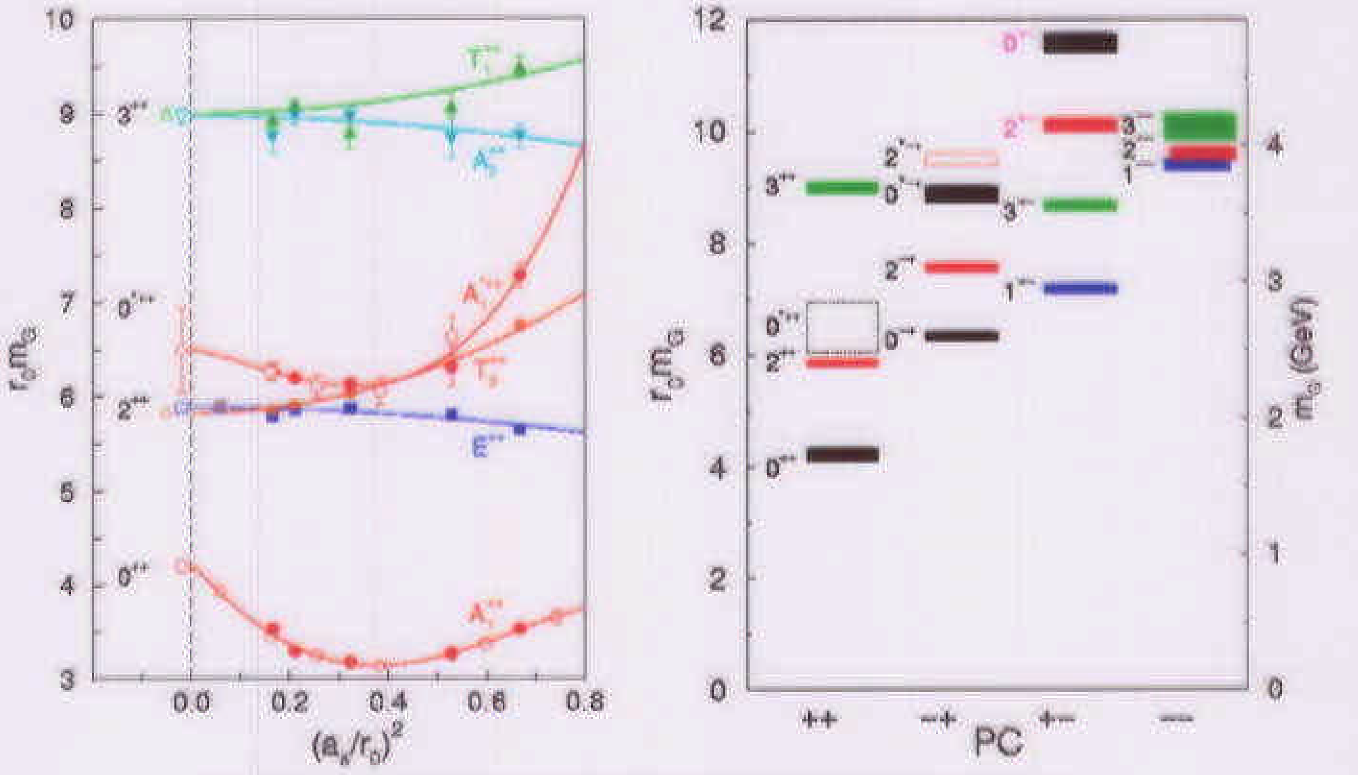
CP-PACS, hep-lat/9909050 and Lattice 00

Kanaya talk, PA-14a20

hadron spectrum (2)

- glueballs in quenched QCD
  - are well understood

Morningstar & Peardon, PRD60 (1999) 034509



- better scaling using the fixed-point action

Niedermayer et al., hep-lat/0007007

- mixing of the lightest scalar glueball with the lightest scalar quarkonium states:
  - $f_0(1710)$ is 74% glueball
  - $f_0(1500)$ is 98% quarkonium

Lee & Weingarten, hep-lat/9910008

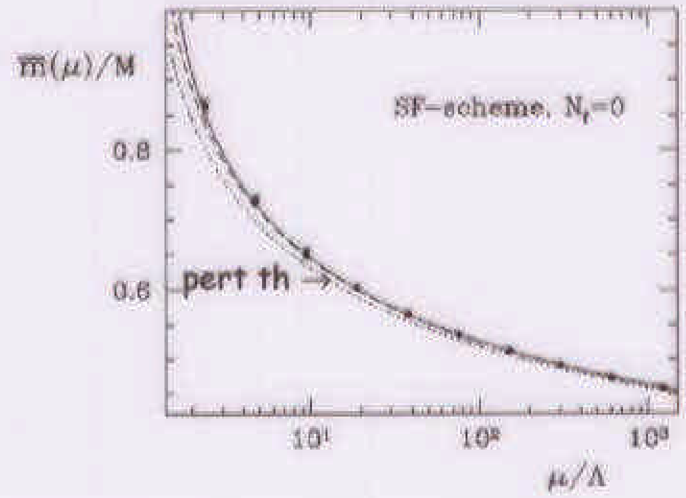
- flavour-singlet pseudoscalar mesons (η,η' mixing)
  - variance reduction techniques are producing a signal for disconnected diagrams
  - two sea-quark flavours (strange mass) + one quenched flavour (twice strange mass)
  - maximal mixing ($\theta = 45 \pm 2°$) of $(\bar{u}u + \bar{d}d)/\sqrt{2}$, $\bar{s}s$
  - need at least ten-times better statistics

UKQCD, hep-lat/0006020

quark masses (1)

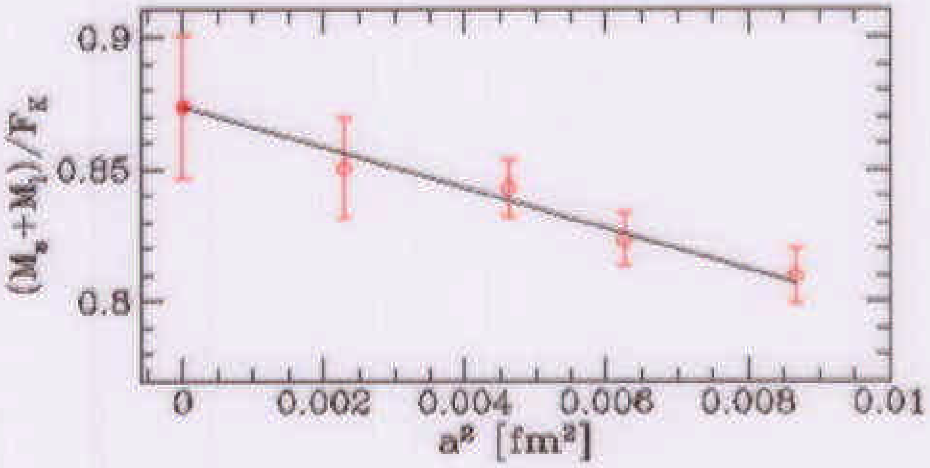
- encoded in hadron masses
- non-perturbative renormalisation
  - define intermediate scheme (eg SF) where scale dependence can be computed non-perturbatively
  - relate to perturbative scheme at high enough scale, eg via RG-invariant mass M (μ→∞)

Heitger talk, PA-14a10



Alpha NP B544 (1999) 669

- quenched light quark masses
  - different s quark masses from K and ϕ (20%)
  - sum of RG-invariant masses:

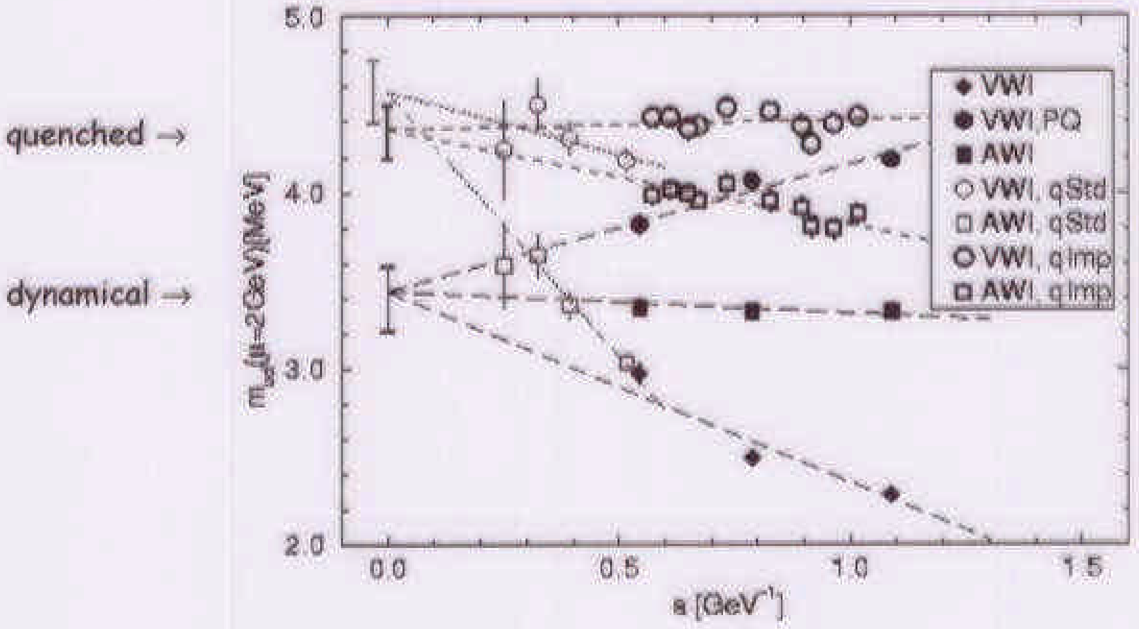


Alpha & UKQCD, hep-lat/9906013 hep-lat/9909098

$$m_s^{\overline{\rm MS}}(2\text{GeV}) = 97(4)\text{MeV} \ \ \text{(K mass input)}$$

quark masses (2)

- dynamical u,d quarks
  - improved action, fixed lattice size of $(2.5\text{fm})^3$
  - sea-quark masses corresponding to $m_{PS}/m_V \geq 0.6$



CP-PACS, hep-lat/0004010

  - from M_π/M_ρ with mean-field improved 1-loop matching
  - quark masses are roughly 25% less than in quenched QCD

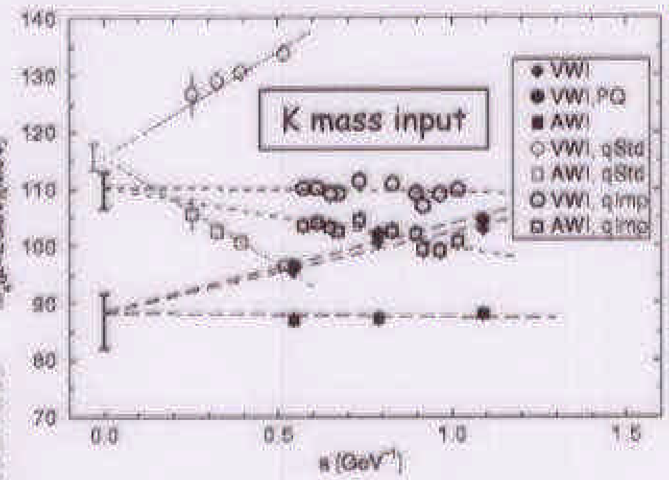
$$m_{ud}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 3.44^{+0.14}_{-0.22}\ \mathrm{MeV} \quad (N_f = 2)$$

$$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 4.36^{+0.14}_{-0.17}\ \mathrm{MeV} \quad (N_f = 0)$$

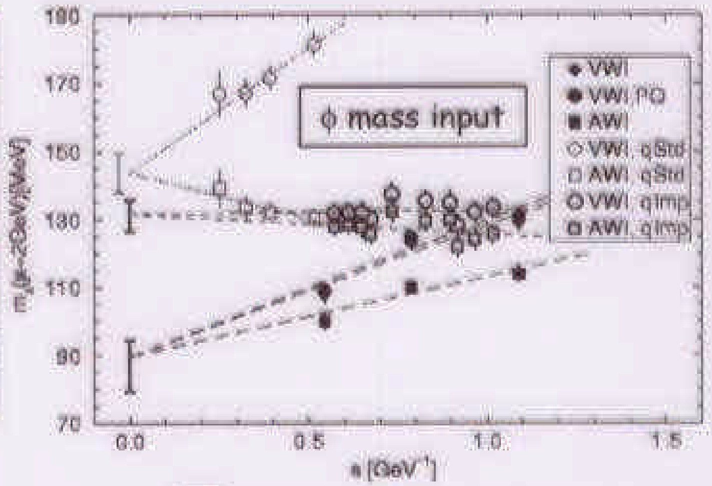
Kanaya talk, PA-14a20

quark masses (3)

- dynamical u,d quarks, quenched s quark
  - inconsistency in s quark mass disappears (within 10% error) compared to quenched QCD (20%)



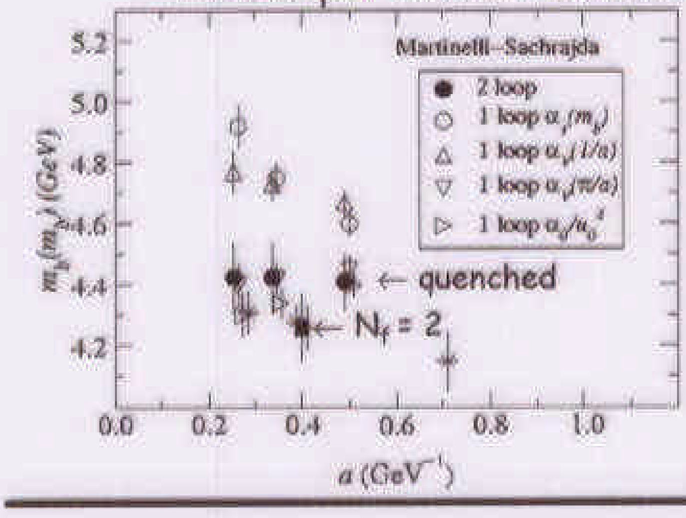
$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 88^{+4}_{-6}\ \mathrm{MeV}$



$m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 90^{+5}_{-11}\ \mathrm{MeV}$

CP-PACS, hep-lat/0004010

  - m_s/m_{ud} = 26(2), compared to 24.4(1.5) from chiral perturbation theory
  - smaller m_s suggests a larger value for ε'/ε
  - would a dynamical s quark reduce it further?
- dynamical u, d quarks, quenched b quark
  - from B_s binding energy at leading order in $1/m_b$ and NNLO perturbative matching



$$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}}) = 4.26(9)\,\mathrm{GeV}$$

Gimenez et al., hep-lat/0002007

Hashimoto, hep-lat/9909136

heavy-quark decays (1)

Leptonic Decays & Mixing

$\boxed{|V_{td}| \text{ and } |V_{ts}/V_{td}|}$

$$\Delta m_q = \frac{G_F^2}{6\pi^2} M_W^2 \eta_B S_0\left(m_t^2/M_W^2\right)\left|V_{tq}V_{tb}^*\right|^2 M_{B_q} f_{B_q}^2 \hat{B}_{B_q}$$

- quenched estimates for f_B have stabilised within large errors and irreducible scale uncertainty
  - only direct comparison with experiment is (!)

$N_f = 2$:
275(20) MeV
CP-PACS

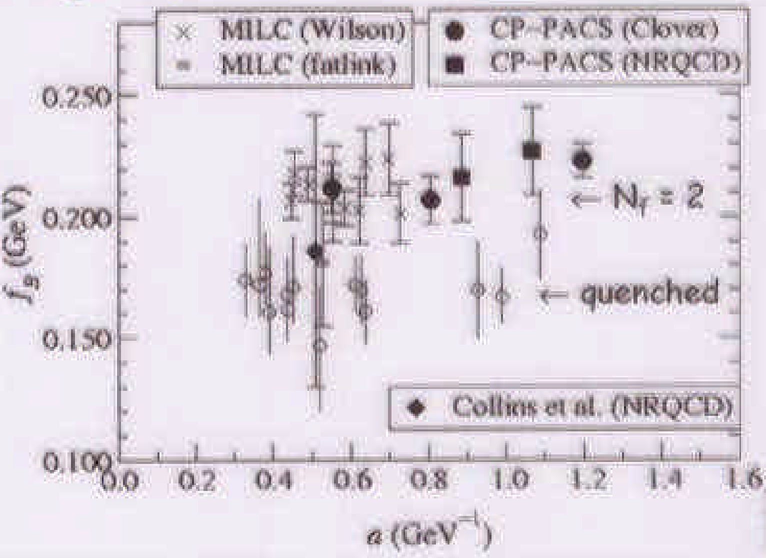
f_{D_s} = 241(30) MeV (lattice) UKQCD, hep-lat/0007020

= 241(32) MeV (expt world average) hep-ex/9903063

  - f_B involves extrapolation
- decay constants increase by up to 20% for two flavours of dynamical light quarks

	$N_f = 2$	$N_f = 0$
f_B (MeV)	210(30)	170(20)
f_{Bs} (MeV)	245(30)	195(20)
f_{Bs}/f_B	1.16(4)	1.15(4)

Hashimoto, hep-lat/9909136



Becirevic et al., hep-lat/0002025

- mixing (Δm) or CKM matrix element determination requires $f_B\sqrt{B_B}$ which may be computed directly

$$f_B\sqrt{\hat{B}_B} = 206(29)\ \text{MeV}$$

  - some systematics cancel in ratios

$$\frac{f_B}{f_{D_s}} = 0.74(5) \quad \text{and} \quad \xi = \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_B\sqrt{\hat{B}_B}} = 1.16(7)$$

heavy-quark decays (2)

Lifetimes

- B_s lifetime difference
  - from ratio of matrix elements and Δm_s
  - estimate Δm_s from Δm_d and ξ (until it is measured)
  - significant cancellation with $O(1/m_b)$ terms

$$\frac{\Delta\Gamma_{B_s}}{\Gamma_{B_s}} = 0.047(15)(16) \quad (< 0.31, \text{ experiment})$$

Becirevic et al., hep-ph/0006135

↓ unquenched f_{Bs}

$$= 0.107(26)(14)(17)$$

Hashimoto et al., hep-lat/0004022

  - NRQCD and relativistic actions give consistent results for ratio of matrix elements

- Λ_b lifetime remains a puzzle
  - to leading order in heavy-quark mass all b hadrons have the same lifetime
  - preliminary lattice calculations indicate that spectator effects are significant (6-10%)

$$\frac{\tau(\Lambda_b)}{\tau(B_0)} \approx 0.92 \quad (0.79(5), \text{ experiment})$$

UKQCD, hep-lat/9906031

Exclusive Semileptonic Decays

$\boxed{|V_{ub}| \text{ and } |V_{cb}|}$

- results are for quenched QCD
  - dynamical quark effects are beginning to be explored
  - assumed around 10%

Bernard et al., hep-lat/9909076

heavy-quark decays (3)

Heavy-to-Light Form Factors

$\boxed{B \to \pi \ell \nu}$

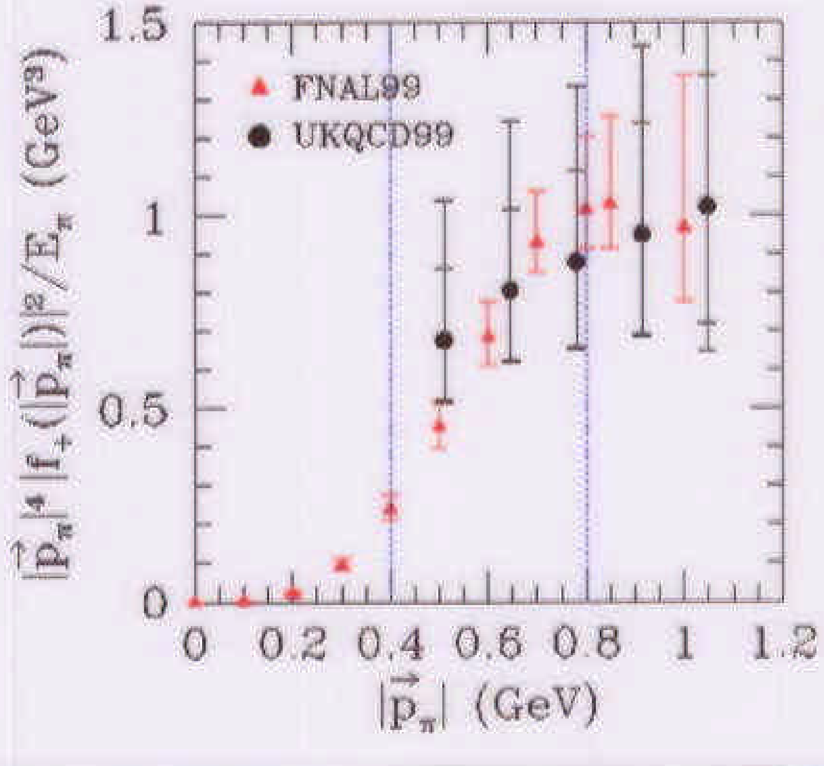
$$\langle \pi(p')|V^\mu|B(p)\rangle = \frac{M_B^2 - M_\pi^2}{q^2} q^\mu f_0(q^2) + \left(p^\mu + p'^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) f_+(q^2)$$

$$q = p - p', \quad V^\mu = \bar{b}\gamma^\mu u$$

- lattice QCD fixes normalisation (unlike HQET)
- kinematic range restricted to near zero recoil
  - experiment can measure corresponding differential rates
  - so model-independent determinations of CKM matrix elements will be possible
- differential decay rate

optimal range
← for matching →
to experiment

$\frac{\text{experiment}}{|V_{ub}|^2}$



FNAL, hep-lat/9910010
UKQCD, hep-lat/9911011

heavy-quark decays (4)

- model-dependent extrapolation is needed to obtain the full kinematic range
    - decay rate & branching ratio (using CLEO's $|V_{ub}|$):

$$\frac{\Gamma}{|V_{ub}|^2} = 9^{+3+2}_{-2-2}\,\mathrm{ps}^{-1}$$

$$\Gamma/\Gamma_{tot} = \left(1.5^{+0.5+0.3}_{-0.3-0.3} \pm 0.6\right)\times 10^{-4} \quad \text{UKQCD}$$

$$\Gamma/\Gamma_{tot} = \left(1.8 \pm 0.4 \pm 0.3 \pm 0.2\right)\times 10^{-4} \quad \text{CLEO}$$

UKQCD, hep-lat/9911011

- residue of B* pole in f_+ is related to $B^*B\pi$ coupling

$$g = 0.33(4)$$

JLQCD, hep-lat/9911036

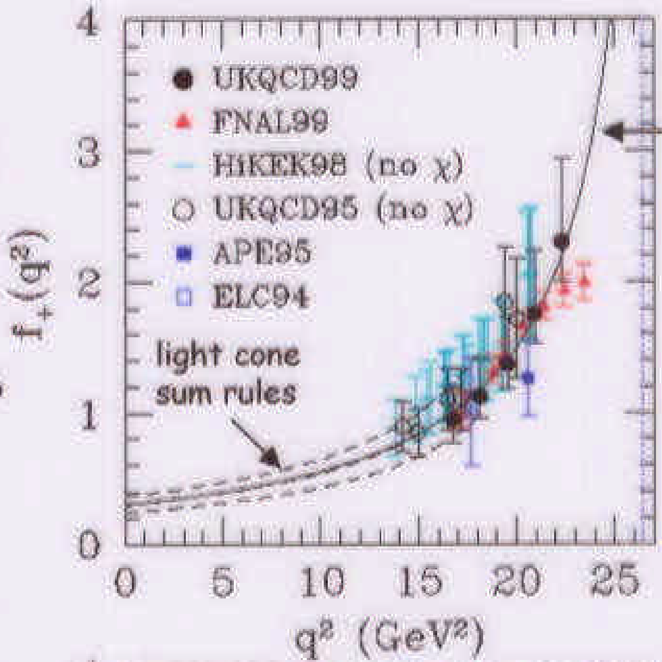
- soft pion theorem seems to be satisfied if care is taken with the chiral extrapolation

$$f_0\left(q^2_{\max}\right) = \frac{f_B}{f_\pi}$$

UKQCD, hep-lat/9909100
MILC, hep-lat/9909076

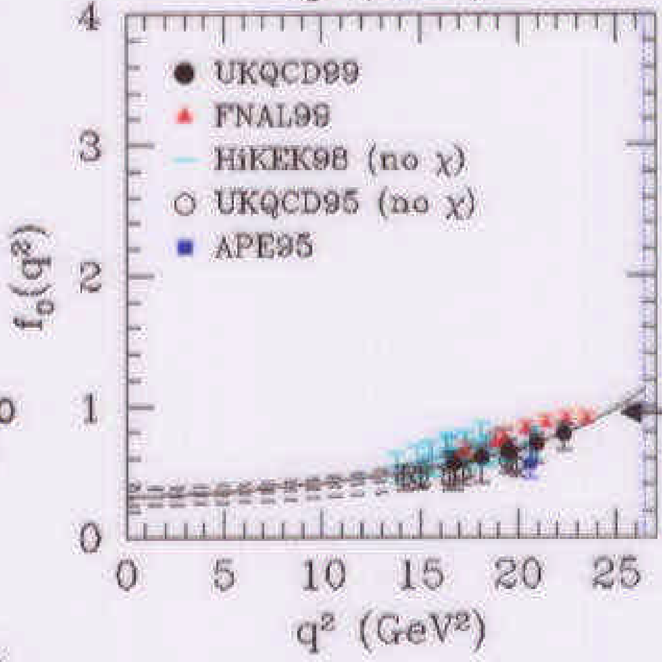
- this is still controversial

JLQCD, hep-lat/9911036



dipole fit to UKQCD99

Lellouch, hep-lat/9912353

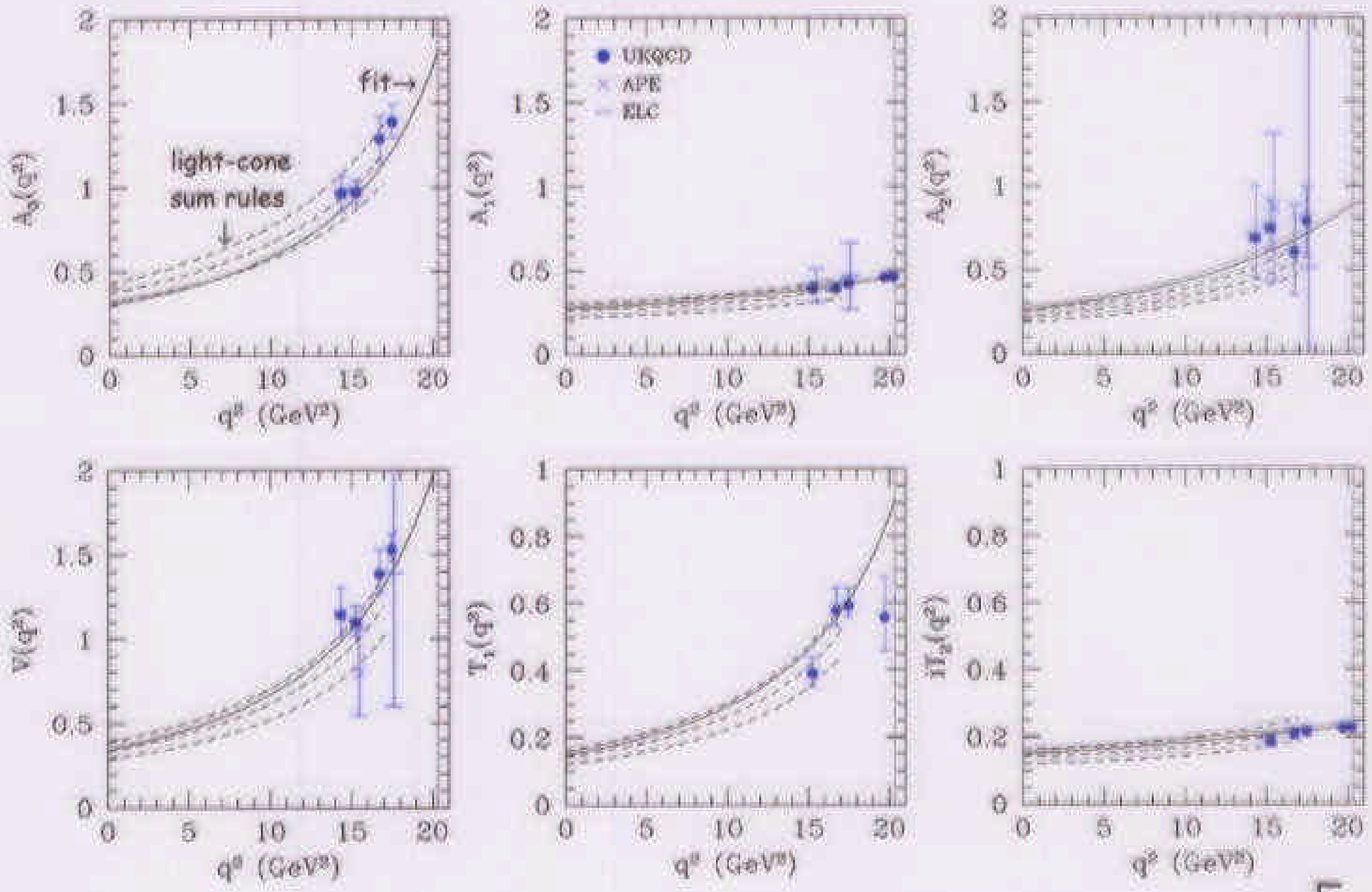


pole fit with $f_0(0) = f_+(0)$

heavy-quark decays (5)

$\boxed{B \to \rho\ell\nu \text{ and } B \to K^*\gamma}$

- new results soon from UKQCD
- existing lattice results may be parametrised using model form factors
  - eg 2-parameter fit to B→ρ form factors



UKQCD, PLB 416 (1998) 392

- $T_1(0)$ for K* final state determines the rate:

$$\frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} = 16^{+3}_{-4}\% \quad (13(4)\%, \text{CLEO})$$

Lellouch, hep-lat/9912353

heavy-quark decays (6)

Heavy-to-Heavy Form Factors $\boxed{B \to D^{(*)}\ell\nu}$

- smaller recoil so lattice covers full kinematic range
- HQET determines normalisation at zero recoil
  - $|V_{cb}|^2$ from extrapolating differential decay rate to zero recoil
  - lattice quantifies deviations from HQET at physical masses (need few percent accuracy)
- HQET for lattice QCD

  Kronfeld talk, PA-14b30

  - determine power corrections to F(ω=1)
  - many statistical and systematic errors cancel in ratio of matrix elements, eg

$$F_{D^*}(1) = 0.935(22)\binom{+8}{-11}(8)(20)$$

FNAL, hep-lat/9906376, 9910026

- recoil dependence and the Isgur-Wise function

UKQCD, hep-lat/9909126

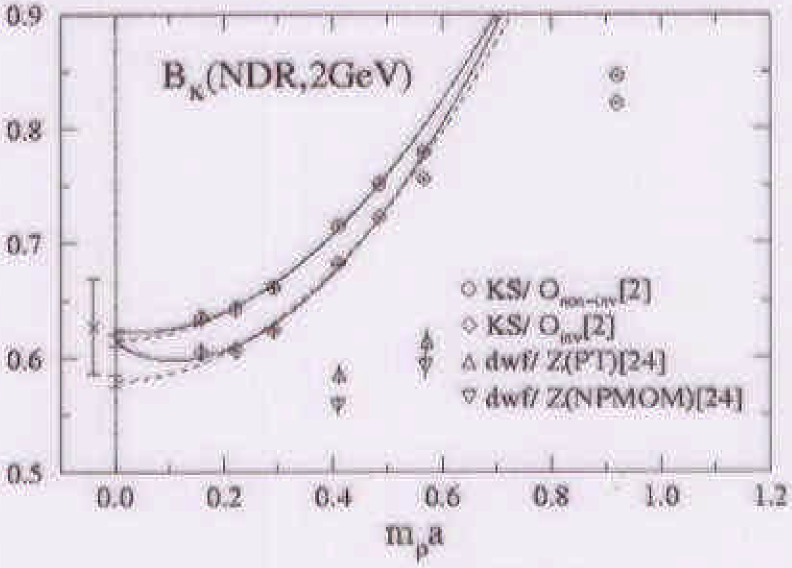
  - ξ(ω) is independent of heavy-quark mass around charm for 1 ≤ ω ≤ 1.2, and scales for 6.0 ≤ β ≤ 6.2

kaon physics (1)

B_K

$$B_K = \frac{3}{8}\frac{\left\langle \overline{K}^0 \left| \bar{s}\gamma_\mu(1-\gamma_5)d \cdot \bar{s}\gamma_\mu(1-\gamma_5)d \right| K^0 \right\rangle}{\left\langle \overline{K}^0 \left| \bar{s}\gamma_\mu\gamma_5 d \right| 0 \right\rangle \left\langle 0 \left| \bar{s}\gamma_\mu\gamma_5 d \right| K^0 \right\rangle}$$

- best determined weak matrix element in quenched QCD
  - staggered quarks, because mixing due to chiral symmetry breaking is non-trivial for Wilson quarks



Kuramashi, hep-lat/9910032

$$B_K(2\,\mathrm{GeV}) = 0.628(42)$$

  - the error is mostly from perturbative matching
  - need non-perturbative renormalisation to remove $O(\alpha^2)$ uncertainty
- dynamical quark effects raise B_K by around 5% at fixed lattice spacing

Testa talk, PA-14a30

kaon physics (2)

Non-Leptonic Decays

$K \rightarrow \pi\pi$

- no general method for dealing with multihadron final states (Maiani-Testa no-go theorem)
- renewed optimism for lattice calculations:
    - lattice chiral symmetry + large measured value for ε'/ε suggest cancellations between matrix elements can be controlled
    - use chiral perturbation theory + matrix elements which can be computed on the lattice, eg K → vacuum, π, or $\pi\pi$ at unphysical momenta
    - tune the volume so that one of the (discrete) energy levels of the two pions equals the kaon mass and relate the transition matrix element to the decay rate in infinite volume

Golterman et al., hep-lat/0006029

Lellouch et al., hep-lat/0003023

$B \rightarrow MM$

- impending flood of experimental data
- chiral perturbation theory no longer helps
- $B\rightarrow\pi\pi$ factorisation proved for $m_b >> \Lambda_{QCD}$:

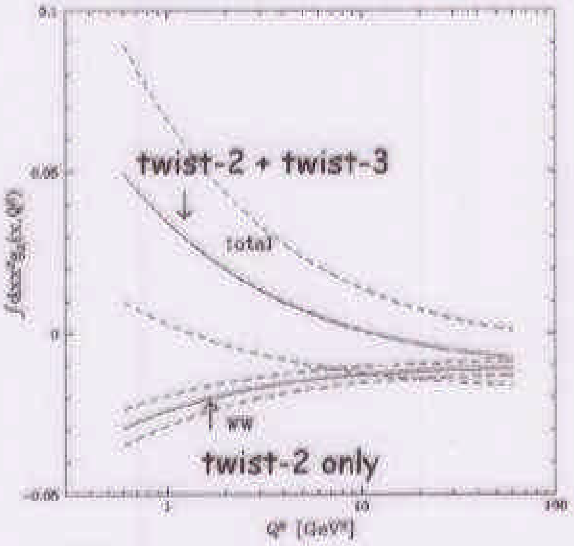
$$\langle \pi\pi | Q | B \rangle = \langle \pi | \underset{\substack{\uparrow \\ \text{form factor}}}{j_1} | B \rangle \langle \pi | \underset{\substack{\uparrow \\ \text{decay constant}}}{j_2} | 0 \rangle \left[1 + \sum r_n \alpha_s^n + O\left(\Lambda_{QCD}/m_b\right) \right]$$

Beneke et al., hep-ph/0006124

structure functions (1)

- lattice provides normalisation for parton densities
  - tests QCD and validity of perturbation theory
  - helps where experimental information is scarce, eg gluon distribution for x > 0.4
  - helps disentangle power corrections
- dynamical quark effects are crucial
  - results so far are quenched, but this will change soon

Jansen talk, PA-14b10



lowest moment of nucleon spin-dependent structure function g_2
→ strong mixing

QCDSF, hep-ph/9909253

- OPE relates moments of structure functions to hadronic matrix elements of local operators

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x,Q^2) = C_n^{(2)}\left(Q^2/\mu^2\,, g(\mu)\right) A_n^{(2)}(\mu) + O\left(1/Q^2\right)$$

Wilson coeffs (pert th)

hadronic matrix elements (lattice)

  - product $C^{(2)}A^{(2)}$ must be independent of μ
  - renormalisation is a major source of systematic error

$$A(\mu) = Z_{\mathrm{INT}}^{\overline{\mathrm{MS}}}(\mu) Z_{\mathrm{latt}}^{\mathrm{INT}}(\mu a) A^{\mathrm{latt}}(a)$$

needs a window: $a \ll \frac{1}{\mu} \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \ll L$

structure functions (2)

- INT = MOM normalises matrix element between quark states of momentum p ($p^2 = \mu^2$) in Landau gauge to the tree-level result

QCDSF, hep-lat/ 9807044

- INT = SF uses a step scaling function to relate the renormalised matrix element at a low (lattice) scale to a high scale where perturbative matching is valid → RG invariant matrix element ($\mu \to \infty$)

APETOV, hep-lat/ 9901016, 9903012

Jansen: quenched pion $\langle x \rangle$(2.4 GeV) = 0.30(3)

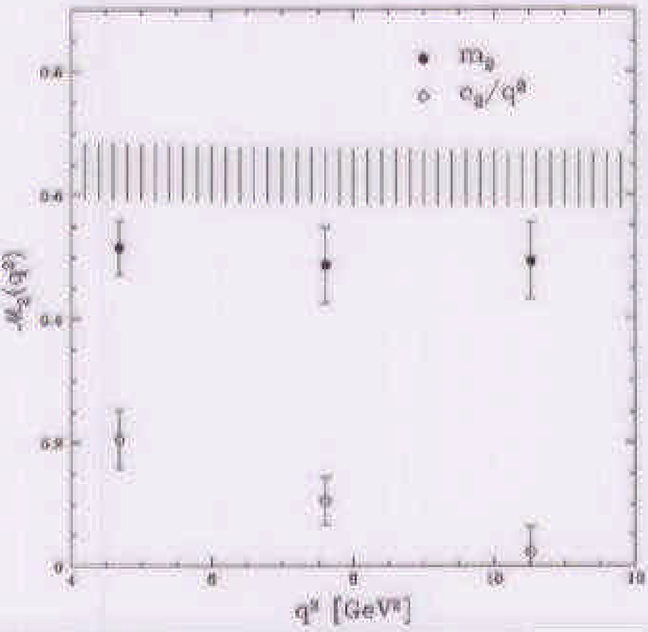
expt = 0.23(2)

- alternatively, compute $\langle h|J_\mu J_\nu|h\rangle$ directly on the lattice
  - determine Wilson coefficients non-perturbatively from matrix elements between quark states, by inverting

$$\langle p_k | J_\mu(q) J_\nu(-q) | p_k \rangle = \sum_{m,n} C^{(m)}_{\mu\nu,\mu_1 \cdots \mu_n}(a,q) \langle p_k | O^{(m)}_{\mu_1 \cdots \mu_n}(a) | p_k \rangle$$

  - avoids mixing and renormalon ambiguities
  - 62 operators and 70 momenta to extract *C*'s
  - reconstruct nucleon $\langle h|J_\mu J_\nu|h\rangle$ from nucleon matrix elements and *C*'s

QCDSF, hep-ph/9906320



lowest non-trivial moment of unpolarised structure function

$$M_2(q^2) = m_2 + c_2/q^2$$

→ large power corrections and strong mixing between twist-2 and twist-4 operators

machines & prospects (1)

Machines Today

CP-PACS: Tsukuba

- 300 Gflops Hitachi SR2201
- since 1996
- $73/Mflops

QCDSP: Columbia & Brookhaven

- 120 & 180 Gflops, custom-built using 32-bit DSP
- since 1998
- $10/Mflops

APEmille: Rome, Zeuthen, Swansea

- 70 Gflops (140 Gflops from Sep), 32-bit fully customised
- from 2000
- $5/Mflops

2003

- two projects are targeting 10 Tflops (64 bit):
  - QCDOC (Columbia, UKQCD) PowerPC node + 4D mesh
  - apeNEXT (INFN, DESY) custom node + 3D mesh
  - $1/Mflops
- beowulf (Alpha/Pentium + Myrinet) offers the prospect of cost-effective smaller systems

machines & prospects (2)

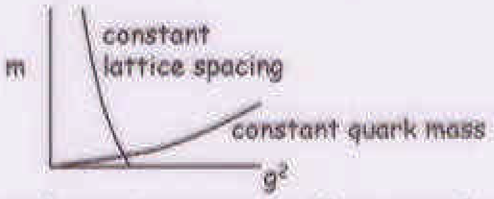
ECFA Report (Dec 99)

- "the future research program using lattice simulations is a very rich one, investigating problems of central importance to the development of our understanding of particle physics"
- "to remain competitive, the community will require access to a number of 10 Tflops machines by 2003"
- "it is unlikely to be possible to procure a 10 Tflops machine commercially at a reasonable price by 2003"

ECFA/99/200

Computational Challenge

- to achieve comparable precision to quenched QCD in simulations of 2 dynamical flavours with masses around 15 MeV will require 15-150 Tflops years
  - two-parameter space:



  - scaling of existing algorithms is not well enough understood
  - nothing is known about simulations with light enough quarks for $\rho\to\pi\pi$
- we need better algorithms and/or 100 Tflops machines

Sachrajda, hep-lat/9911016

machines & prospects (3)

Conclusions & Prospects

- the range of phenomenological applications continues to expand
  - key developments have been improved actions and non-perturbative renormalisation
  - there are good prospects for model-independent phenomenology
- lattice QCD continues to drive cost-effective high-performance computing technology
  - there are no technological limits in sight for QCD
- the primary objective is to extend the range of quark masses which can be simulated reliably
  - will need >100 Tflops machines (c. 2005) or an algorithmic breakthrough
- we are witnessing the "Second Lattice Field Theory Revolution" - the discovery of lattice chiral symmetry
  - will greatly expand the reach of non-perturbative methods