

Flavour Dynamics -- Central Mysteries of the Standard Model

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Notre Dame du Lac

3 mysteries

- \exists **family structure** !?

quarks \leftrightarrow leptons

- family **replication** !?

why 3 families? Is 3 fundamental?

- **pattern of masses** & mass related quantities \leftarrow **CKM**

- appears highly **nonaccidental**

- neutrinos massless?

2 strategies for obtaining answers

Strategy (A)

- have enough data!
- solve remaining fundamental challenge: **bring gravity into quantum world!**
 - ⇒ family structure as **side effect**

“Le Penseur”

Strategy (B)

nature might have a few more surprises up her sleeves -- need more hints from nature!

“David”

this talk: **Strategy (B)**



2021



New Landmarks

- 💣 Direct ~~CP~~ established in K decays!
- 💣 On the brink of observing ~~CP~~ in B decays -- a first outside $\Delta S \neq 0$!
- 💣 Reaching fertile ground for finding New Physics in $D^0 - \bar{D}^0$ and ~~CP~~

on the theory side:

- ☹ learning lessons of humility
- 😊 increasing sophistication of theoretical technologies
- 😊 pushing back new frontiers

New Challenges (to Theory)

- ⇒ regain **theoretical** control over ϵ'/ϵ !
- ⇒ develop **reliable quantitative** predictions for CP asymmetries in B decays!
- ⇒ refine those predictions into **precise** ones!
- ⇒ establish **theoretical** control over $D^0 - \bar{D}^0$ oscillations and CP violation!
- ⇒ develop **comprehensive** strategies to **distinguish** between different New Physics scenarios!

topics

○ Charged Current Couplings of Quarks

- the **unreasonable** success of CKM
- extracting CKM parameters

○ Nonleptonic Decays

- H_b & H_c lifetimes as **validation** studies
- exclusive NL decays $B \rightarrow M_1 M_2$
- quark-hadron duality

○ CP Violation

- ϵ'/ϵ
- $\Delta B \neq 0$

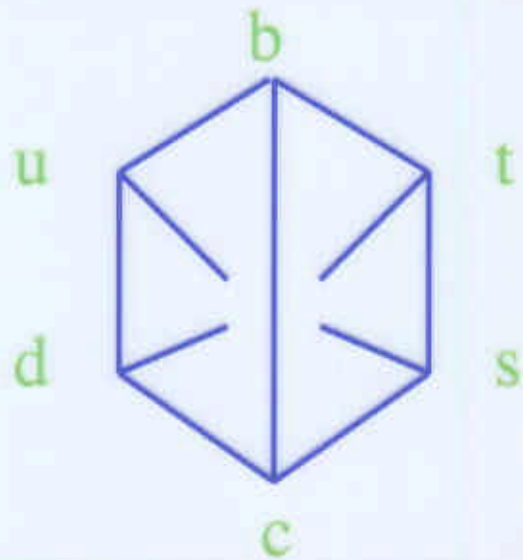
○ The Search for New Physics

- ◆ comments on **theoretical** uncertainties
- future CKM trigonometry
- “exotica”: $K_{\mu 3}$, EDM's, $\Delta C \neq 0$
- “Textures”

1:50,000 Scale Topographic Map/地形图

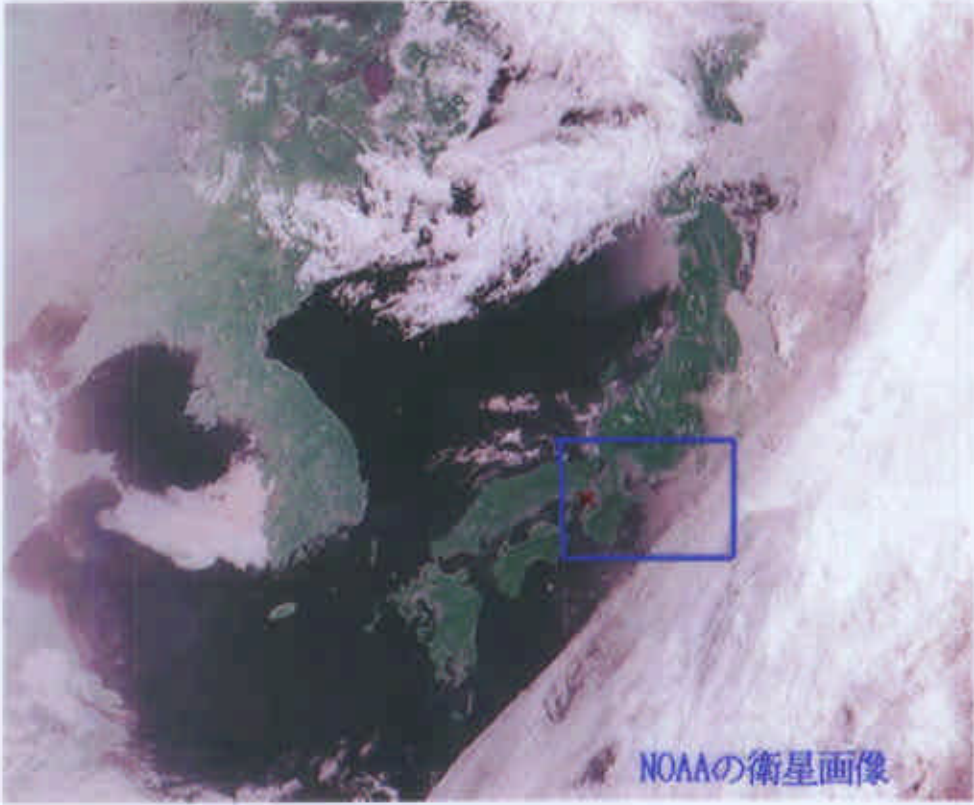


I. The Charged Current Couplings of Quarks



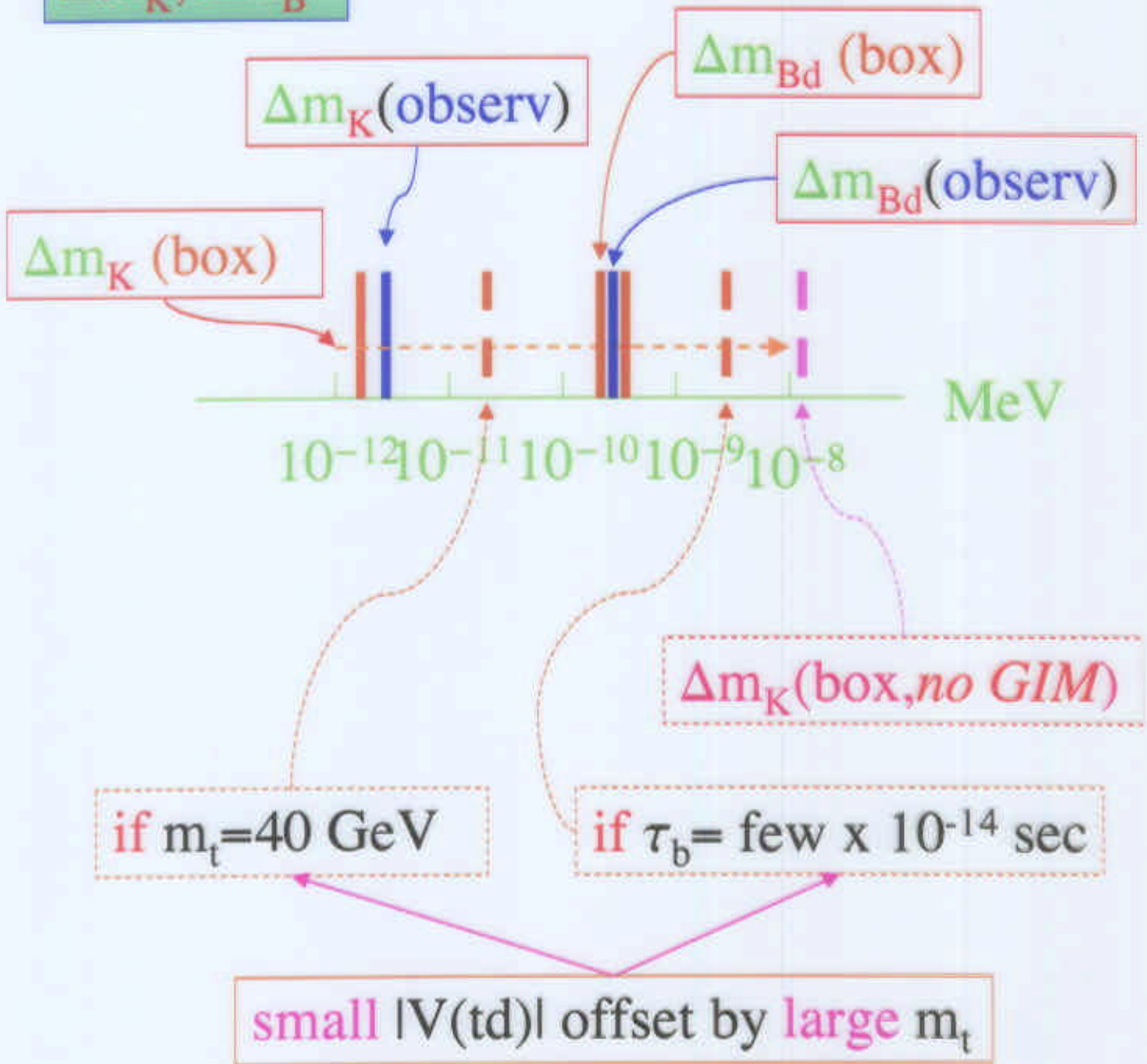
- $\tau(b) \sim 10^{-12} \text{ sec} \gg 10^{-14} \text{ sec}$
- $|b \Rightarrow c|^2 \gg |b \Rightarrow u|^2$

$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$



I.1 The unreasonable success of the CKM description

$\Delta m_K, \Delta m_B$



ϵ_K

could always be “accommodated” --

whether

$$|V(td)| \sim \lambda^2, |V(ts)| \sim \lambda \text{ and } m_t \sim 40 \text{ GeV}$$

or

$$|V(td)| \sim \lambda^3, |V(ts)| \sim \lambda^2 \text{ and } m_t \sim 180 \text{ GeV}$$

yet

$$|V(td)| \sim \lambda^2, |V(ts)| \sim \lambda \text{ and } m_t \sim 180 \text{ GeV}$$

or

$$|V(td)| \sim \lambda^3, |V(ts)| \sim \lambda^2 \text{ and } m_t \sim 40 \text{ GeV}$$

would have been a clear inconsistency!

I.2 The (important) Details

PDG 2000:

- **without** imposing 3 family unitarity

$ V(ud) $	$ V(us) $	$ V(ub) $...
0.9735 ± 0.0013	0.220 ± 0.004	0.003 ± 0.002	
$ V(cd) $	$ V(cs) $	$ V(cb) $	
0.226 ± 0.007	0.880 ± 0.096	0.040 ± 0.003	
$ V(td) $	$ V(ts) $	$ V(tb) $	
0.05 ± 0.04	0.28 ± 0.27	0.5 ± 0.49	
...			...

- **with** imposing 3 family unitarity

$ V(ud) $	$ V(us) $	$ V(ub) $	
0.9750 ± 0.0008	0.223 ± 0.004	0.003 ± 0.002	
$ V(cd) $	$ V(cs) $	$ V(cb) $	
0.222 ± 0.003	0.9742 ± 0.0008	0.040 ± 0.003	
$ V(td) $	$ V(ts) $	$ V(tb) $	
0.009 ± 0.005	0.039 ± 0.004	0.9992 ± 0.0002	

I.2.a. Theoretical Technologies for QCD

s	$m_s < \Lambda$	chiral pert. th.
c	$m_c > \Lambda$	$1/m_c$ expans. (?)
b	$m_b \gg \Lambda$	$1/m_b$ expans. !
t	$\Gamma_t \sim O(\Lambda)$	perturb. dynamics

↑
lattice
QCD

○ $1/m_Q$ expansions

○ lattice QCD (⇒ Kenway's lecture)

○ quark models (properly used)

→ ground prepared for fruitful feedback

→ both defined in Euclidean space

→ both "mature"

→ similar as well as different expansion parameters

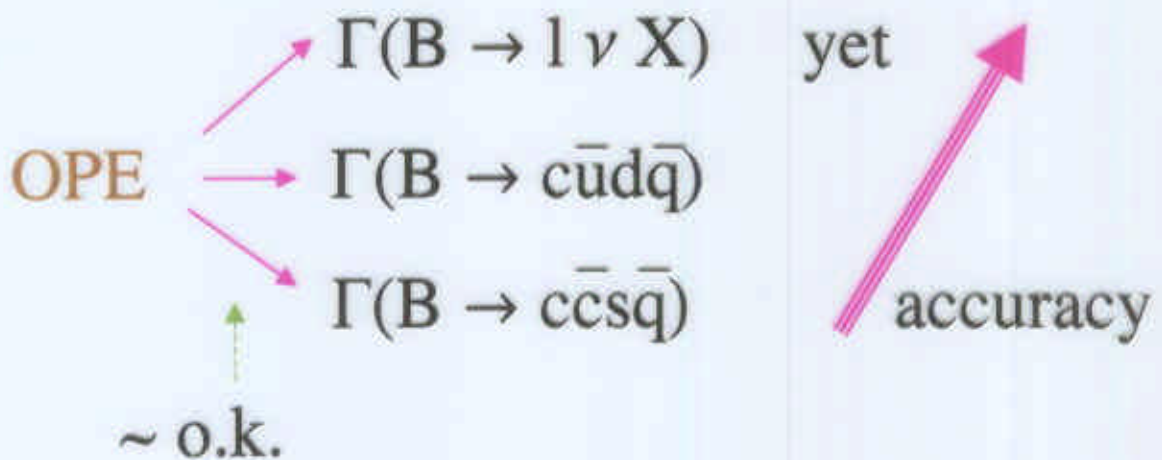
→ lattice QCD provides input to $1/m_Q$ expansion

I.2.b. Extracting CKM Parameters

main tool (so far): $1/m_Q$ expansions

○ **conceptual** convergence

e.g.:



○ **numerical** convergence in basic quantities

$$(\text{kinetic}) \ m_b(1 \text{ GeV}) = \begin{cases} 4.56 \pm 0.06 \text{ GeV} & \text{MeYe} \\ 4.57 \pm 0.04 \text{ GeV} & \text{Ho} \\ 4.59 \pm 0.06 \text{ GeV} & \text{Be Si} \end{cases}$$

error estimates quite possibly overly
optimistic, but not foolish

Caveat:

all three analyses based on Y(4S) region;
i.e. not truly independent!

essential quality/selfconsistency check

shape of spectra in SL B decays

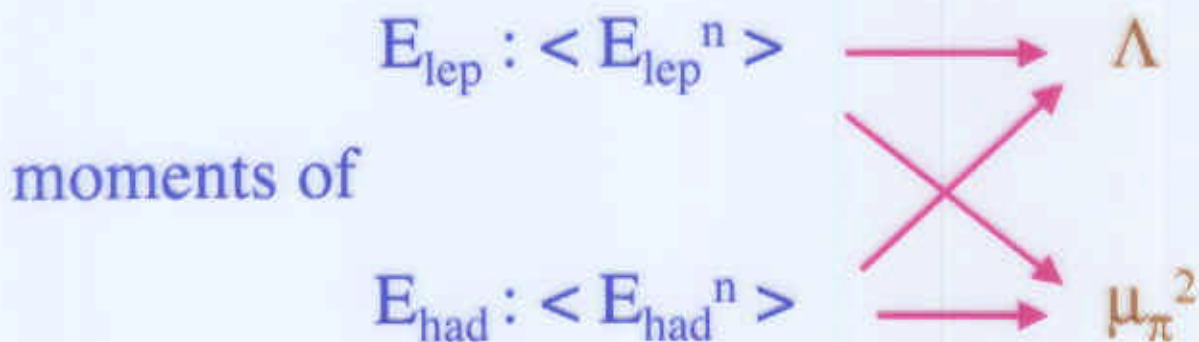
depend on m_b , $m_b - m_c$, $\mu_\pi^2 [\lambda_1]$

$$\rightarrow \bar{\Lambda}(\mu) \equiv M(H_b) - m_b(\mu) \text{ as } m_b \rightarrow \infty$$

$$\rightarrow \mu_\pi^2 \equiv \langle B | \bar{b}(iD)^2 b | B \rangle / 2M_B$$

$B \rightarrow l \nu X$

Fa Lu Sa



jury still out ...

V(cb)

2 methods with best theoretical justification

⇒ 'exclusive'

$B \Rightarrow l \nu D^*$ at zero recoil

$$|V(cb) F_{D^*}(0)|$$

$$= 1 + O(1/m_c^2) + O(\alpha_s)$$

the challenge!

$$F_{D^*}(0) = \begin{cases} 0.89 \pm 0.08 & \text{Uraltsev et al.} \\ 0.913 \pm 0.042 & \text{BaBar Book} \end{cases}$$

$$0.935 \pm 0.03$$

prelim. lattice:

$$(0.935 \pm 0.022^{+0.008}_{-0.011} \pm 0.008 \pm 0.020)$$

AK et al.

~~XXXXXXXXXX~~

will use: $F_{D^*}(0) = 0.90 \pm 0.05$

data:

- CLEO for ICHEP2000

$$|V(\text{cb}) F_{D^*}(0)| = (42.4 \pm 1.8|_{\text{stat}} \pm 1.9|_{\text{syst}}) \times 10^{-3}$$

$$|V(\text{cb})| = (47.1 \pm 2.0|_{\text{stat}} \pm 2.1|_{\text{syst}} \pm 2.1|_{\text{th}}) \times 10^{-3}$$

- LEP for ICHEP2000:

$$|V(\text{cb}) F_{D^*}(0)| = (34.9 \pm 0.7|_{\text{stat}} \pm 1.6|_{\text{syst}}) \times 10^{-3}$$

$$|V(\text{cb})| = (38.8 \pm 0.8|_{\text{stat}} \pm 1.8|_{\text{syst}} \pm 1.7|_{\text{th}}) \times 10^{-3}$$

→ $\Delta |V(\text{cb}) F_{D^*}(0)| = 7.5 \times 10^{-3}$

→ keeps me sitting on the fence

concerning the observation of B_s
oscill.

Future:

very hard to reduce **theoretical** uncertainty!

⇒ 'inclusive' -- total SL B width

$$\Gamma_{SL}(B) \propto m_b^5 (1 + O(1/m_b^2) + O(\alpha_s))$$

→ the challenge!

[actually $\Gamma_{SL}(B) \propto (m_b - m_c)^3 m_b^2$]

lots of new CLEO data on tape -- but not analyzed!

new LEP data:

$$|V(cb)|_{incl} = (40.76 \pm 0.41|_{exp} \pm 2.0|_{th}) \times 10^{-3}$$

$$|V(cb)| = 0.0411 \sqrt{\frac{1.55}{0.105} \Gamma(b \rightarrow l \nu X_c)} \times$$

$$\times \left(1 - 0.024 \left(\frac{\mu_\pi^2 - 0.5}{0.2} \right) \right) \times$$

Bi Sh Ur Va

$$\left(1 \pm 0.030|_{pert} \pm 0.020|_{m_b} \pm 0.024|_{1/m_b^3} \right)$$

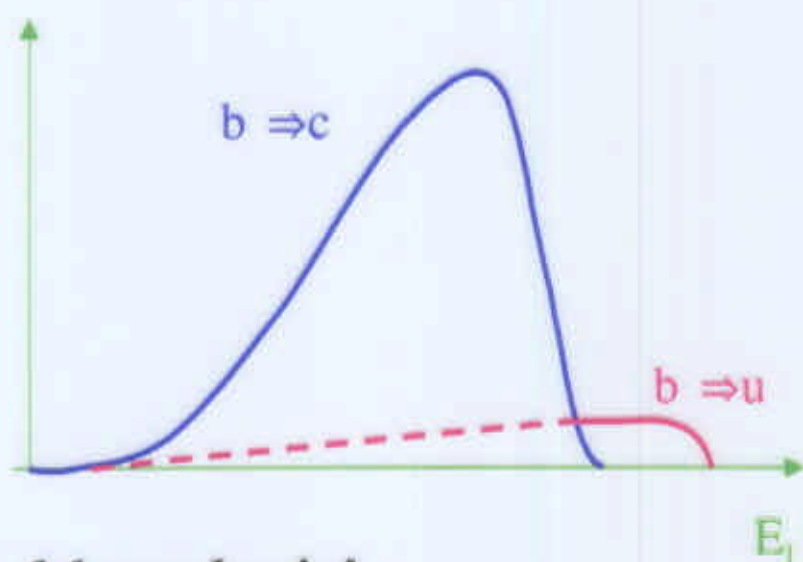
Future: not unrealistic to reduce theoretical uncertainty down to ~2%!

V(ub)

① first methods

□ energy endpoint spectrum

(i.e., kinematical discrimination)



'old analysis'

$$V(\text{ub})|_{\text{endp}} = (3.2 \pm 0.8) \times 10^{-3}$$

▷ strong model dependence *so far*

▷ yet can be reduced considerably!

▷ endpoint different for B_d and $B_{16}^{+!}$

- exclusive semileptonic modes

$$B \Rightarrow l \nu \pi, l \nu \rho$$

$$V(ub)|_{\text{excl}} =$$

$$(3.25 \pm 0.14|_{\text{stat}} \pm 0.27|_{\text{syst}} \pm 0.55|_{\text{th}}) \times 10^{-3}$$

- ⇒ strong model dependence *so far*
- ⇒ input from QCD sum rules
- ⇒ waiting for lattice QCD

② new method on the scene:

determine $\Gamma_{SL}(B \rightarrow l\nu X_u)$

$$V(ub)|_{\Gamma_{SL}} = (4.04 \pm 0.44|_{\text{stat}} \pm 0.46|_{b \rightarrow c, \text{syst}} \pm \pm 0.25|_{b \rightarrow u, \text{syst}} \pm 0.02|_{\tau_b} \pm 0.19|_{\text{HQE}}) \times 10^{-3}$$

⇒ good theoretical control BiShUrVa

⇒ experimentally very challenging

③ future method

measure the hadronic recoil mass spectrum

$$\frac{d}{dM_X} \Gamma(B \rightarrow l\nu X)$$

Ba Ph Ki

QCD compatible descript:

DiUr, BiUrDi, FaLiWi

mild dependance on cut-off $M_{X, \text{max}}$ for

$$M_{X, \text{max}} \sim 1.6 \text{ GeV}$$

- ▷ theoretical refinements under construction to improve experimental feasibility Ba Li Lu
- ▷ large statistics required
- ▷ 10% uncertainty not unrealistic

resume on $V(ub)$

- considerable improvements in the near future quantitatively and qualitatively

more reliable error estimates

- an uncertainty not exceeding 10 % appears achievable through dedicated efforts

- one can entertain hopes (dreams?) beyond that in the long run

V(td)

- B_d vs. B_s oscillations

$$x_d/x_s \approx |V(td)/V(ts)|^2 \times |Bf(B_d)/Bf(B_s)|^2$$

➤ could be sensitive to New Physics!

- $K^+ \Rightarrow \pi^+ \nu \nu$

its width is dominated by virtual top quark contribution

$$\Gamma(K^+ \Rightarrow \pi^+ \nu \nu) \Rightarrow |V(td)V(ts)|^2$$

- exclusive radiative decays

$$\Gamma(B \Rightarrow \rho \gamma) / \Gamma(B \Rightarrow K^* \gamma) \sim |V(td)/V(ts)|^2$$

➤ could be sensitive to New Physics

➤ could be quite sensitive to long distance dynamics

II. Nonleptonic Heavy Flavour Decays

II.1 Quark-hadron duality

systematic **theoretical** uncertainties?

quark-hadron duality!

$$\langle d\sigma(\text{quark\&gluon d.o.f.}) \rangle = \langle d\sigma(\text{hadr.d.o.f.}) \rangle$$

 duality

general expectations:

- duality cannot be exact
- limitations to duality will depend on the process
- duality violations larger in **NL** than **SL** decays

beyond?

lots of folklore

fruitful concepts -- but no theory

3 phases of QCD


- ① perturb. effects
- ② nonpert. effects -- in heavy flavour decays
- ③ limitations to duality **a new frontier!**

different realizations of duality

- global duality
 - duality
 - local duality
- averaging/"smearing"

physical origins of limitations to duality

- exact positions of hadronic thresholds
implementation through 'oscillating terms'

Euclidean $\exp\{-m_Q/\Lambda\}$ 
Minkowskian $\sin(m_Q/\Lambda)$

- sensitivity to 'distant cuts'
- validity of $1/m_c$ expans. in $B \rightarrow l \nu D^*$

theoretical tools

- OPE insensitive to duality violations
(yet indirect qualitative lessons)
- exactly solvable model field theories
e.g., 't Hooft model

QCD in 1+1 dimensions, $N_C \rightarrow \infty$

Le Gr numerical analysis suggested
significant or even large violations
of duality

Bi Sh Ur Va analytical analysis
→ truly tiny violations only!

Le Ur duality obeyed even in
spectra etc.

other tools? **redundant** determinations!

determine m_b , $m_b - m_c$, μ_π^2 in **different** ways

○ extract $V(cb)$, $V(ub)$, $V(td)/V(ts)$ from B_s

decays ‘hic Rhodus, hic salta!’

more telling than B_d vs. B_u

$V(cb)$

→ $\Gamma_{SL}(B_s)$

→ lepton spectra/moments in B_s decays

→ $B_s \Rightarrow l \nu D_s^*$ at zero recoil

$V(ub)$

→ hadronic recoil mass spectrum in

$$B_s \Rightarrow l \nu X$$

→ exclusive semileptonic modes

$$B_s \Rightarrow l \nu K, B_s \Rightarrow l \nu K^*$$

II.2 Weak lifetimes as validation studies

Charm

$$\tau(D^+) > \tau(D^0) \sim \tau(D_s^+) \geq \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^+)$$

	1/mc expect.	comments	data
$\tau(D^+)/\tau(D^0)$	~ 2 + 10-20% from WA	PI in $\tau(D^+)$ consistent treatment of momenta in 'wavefunction'	2.55 ± 0.034 updated BELLE $2.51 \pm 0.06 \pm 0.04$
$\tau(D_s)/\tau(D^0)$	1.0 – 1.07 0.9 – 1.3 B&U 1.08 ± 0.04 Cheng & Yang	without WA with WA “ “ use QCD SR for ME	“old” 1.125 ± 0.042 “new WA” 1.180 ± 0.017 BELLE $1.15 \pm 0.04 \pm 0.02$
$\tau(\Lambda_c^+)/\tau(D^0)$	~ 0.5	Quark model matrix elem.	0.489 ± 0.008 updated
$\tau(\Xi_c^+)/\tau(\Lambda_c)$	~ 1.3	“	1.75 ± 0.36
$\tau(\Xi_c^+)/\tau(\Xi_c^0)$	~ 2.8	“	3.57 ± 0.91
$\tau(\Xi_c^+)/\tau(\Omega_c)$	~ 4	“	3.9 ± 1.7
$2y =$ $\Delta\Gamma/\Gamma _{D^0}$	$\leq O(1\%)$	test bed for duality	$-12 < 2y < 0.6\%$

○ observed pattern reproduced/predict. semiquantitatively!

○ PI main effect for mesons

□ “old” data on $\tau(D_s)/\tau(D^0)$

⇒ WA *not* leading, possibly irrelevant

□ “new” data

⇒ WA -- while not leading --

still significant in D decays!

○ description for baryonic widths helped by generous errors

$$\delta\tau(\Xi_c)/\tau(\Lambda_c) \sim 10 - 15 \%$$

highly informative!

○ semileptonic BR's for baryons do **not** reflect lifetime ratios!

$$\Gamma_{SL}(D) \neq \Gamma_{SL}(\Lambda_c) \neq \Gamma_{SL}(\Xi_c) \neq \Gamma_{SL}(\Omega_c)$$

constructive PI in SL Ξ_c and Ω_c decays →

$$BR_{SL}(\Xi_c^0) \sim BR_{SL}(\Lambda_c) \quad \text{vs.} \quad \tau(\Xi_c^0) \sim 0.5 \cdot \tau(\Lambda_c)$$

$$BR_{SL}(\Xi_c^+) \sim 2.5 \cdot BR_{SL}(\Lambda_c) \quad \text{vs.} \quad \tau(\Xi_c^+) \sim 1.3 \cdot \tau(\Lambda_c)$$

$$BR_{SL}(\Omega_c) < 15 \%$$

beauty

	1/m _b predict.	comment	data
$\tau(B^+)/\tau(B_d)$	$1 + 0.05(f_B/200 \text{ MeV})^2$	PI in $\tau(B^+)$ factorization at low scale (1 GeV)	1.070 ± 0.020 updated ICHEP
$\tau(B_s)/\tau(B_d)$	$1 \pm O(0.01)$		0.945 ± 0.039 updated ICHEP
$\Delta\Gamma(B_s)/\Gamma(B_s)$	$0.18(f_B/200 \text{ MeV})^2$	Voloshin et al., Sov. J. Nucl. Phys. 46 (1987)112	< 0.31 (95 % C.L.)
$\tau(B_c)$	$\sim 0.5 \text{ psec}$	largest lifetime diff. !	$0.46 \pm 0.17 \text{ psec}$
$\tau(\Lambda_b)/\tau(B_d)$	$0.9 - 1.0$	quark model matrix elem.	0.79 ± 0.05

- Predictions for **meson** life times on the mark!
- recent lattice study (Di Piero & Sachrajda):

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.03 \pm 0.02 \pm 0.03$$

○ need more precise data on $\tau(B_s)$ (see later)

○
$$\frac{\tau(B_c)}{\tau(B_d)} \sim \frac{1}{3}$$

○ largest lifetime difference by far!

○ absence of $1/m_Q$ contribution crucial!

○ serious challenge from “short” baryon lifetime

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \equiv 1 - \Delta$$

○ $\Delta_m \approx 0.03 - 0.12$ quark model Uraltsev
 $\Delta_m \approx 0.13 - 0.21$ QCD SR Huang et al.
 $\Delta_{\text{exp}} \approx 0.21 \pm 0.05$

○ pilot lattice study

$$\Delta_{\text{lattice}} \approx (0.07 - 0.09) \pm ?$$

○ $\tau(\Xi_b^-)$ vs. $\tau(\Lambda_b)$ vs. $\tau(\Xi_b^0)$?

○ $\tau(\Xi_b^-) > \tau(\Lambda_b), \tau(\Xi_b^0)$

○ $[\tau(\Xi_b^-) - \tau(\Lambda_b)] / \tau(\Lambda_b) \sim 14\%$ Volosh., Gub. et al.

from observed $\tau(\Xi_c^+) - \tau(\Lambda_c)$

Heresy

- $\Gamma(H_Q) \propto M^5(H_Q)$ rather than m^5_Q Al et al.

⇒ $\tau(\Lambda_b)/\tau(B_d) \sim 0.75$

- **anathema to the OPE!**

➔ represents large contribution of $O(1/m_Q)$

➔ sum rules in OPE enforce Bi Sh Ur Va

quark phase space + nonperturb. corrections

=

hadronic phase space + boundstate corrections

- ⇒ would constitute rather massive violation of
- duality

- other predictions

⊃ $\tau(B_s)/\tau(B_d) \sim 0.94 !$

⊃ $\tau(B_c) ??$

⊃ $\frac{\tau(\Xi_b^0)}{\tau(\Lambda_b)} = \frac{\tau(\Xi_b^-)}{\tau(\Lambda_b)} = \left(\frac{M(\Xi_b)}{M(\Lambda_b)} \right)^5 \sim 0.86 !$

II.3 Exclusive NL decays $B \rightarrow M_1 M_2$

- guidance by **symmetry** considerations

SU(2) [SU(3)]

- phenomenological models

central *assumption* --

concept of **factorization**

$$\langle M_1 M_2 | J J' | B \rangle \approx \langle M_1 | J | B \rangle \langle M_2 | J' | 0 \rangle$$

- invoke $1/N_C$ counting rules to justify

factorization;

however

- ▣ no realistic hope to evaluate

nonleading terms (no **FSI!**)

- ▣ $N_C = 3$

- yet models still very useful if used with awareness and common sense

host of **well-measured BR's**: referee

for the games

Fl, Ho, HY C

- **theoretical** treatment -- **new frontier!**

puts the bar higher for models

- *conditio sine qua non*

large energy release ⇒ **hard** process

- 'colour transparency' ...

elements have been around -- but now are combined into comprehensive and detailed framework

2 groups

○ Beneke Buchalla Neubert Sachrajda

‘QCD factorization’

○ Keum Li Sanda

‘pQCD factorization’

ring of truth

factorization theorem:

- non-universal
- sometimes not the leading effect

apparent differences between 2 approaches

- **BBNS**: FSI mostly small in $B \rightarrow K\pi, \pi\pi$;
WA suppressed
- **KLS**: WA important, FSI not small

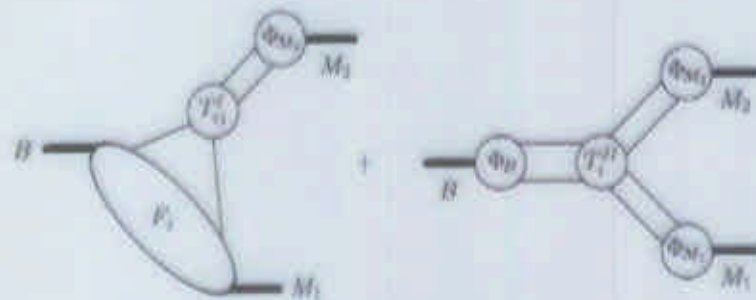
QCD factorization formula

Hard gluon effects ($k > m_b$) can be calculated and lead to the **effective hamiltonian**:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i^{\text{CKM}} C_i(\mu) Q_i(\mu)$$

Principal idea: factorize systematically the remaining hard effects ($k \sim m_b$) from long-distance effects ($k \sim \Lambda_{\text{QCD}}$) – **heavy quark expansion**. Result is:

$$\begin{aligned} \langle \pi K | Q_i | B \rangle &= f_+^{B \rightarrow \pi}(0) f_K T_{K,i}^I * \Phi_K \\ &+ f_+^{B \rightarrow K}(0) f_\pi T_{\pi,i}^I * \Phi_\pi + \underbrace{f_B f_K f_\pi T_i^{II}}_{\text{spectator interaction}} * \Phi_B * \Phi_K * \Phi_\pi \end{aligned}$$



- long-distance: form factor, decay constant, **light-cone distribution amplitudes**
- short-distance: kernels $T^{I,II} = \alpha_s^0 + \alpha_s^1 + \dots$, contain all **"non-factorizable"** corrections and **strong phases**.

III: CP averaged branching fraction ratios

Despite significant corrections to naive factorization, the **qualitative pattern** that emerges for the set of $\pi\pi$ and πK decay modes is **similar to that of naive factorization**:

the **penguin-tree interference** is constructive (destructive) in $B \rightarrow \pi^+\pi^-$ ($B \rightarrow \pi^-K^+$) decays if $\gamma < 90^\circ$.

Taking the currently favoured range $\gamma = (60 \pm 20)^\circ$, we find [CLEO, hep-ex/0001010 in brackets]:

$$\begin{aligned} \frac{\text{Br}(\pi^+\pi^-)}{\text{Br}(\pi^\mp K^\pm)} &= 0.5-1.9 \quad [0.25 \pm 0.10] \quad 0.36 \pm 0.26 \text{ Br} \\ & \quad 0.74 \pm 0.29 \text{ Br} \\ \frac{\text{Br}(\pi^\mp K^\pm)}{2\text{Br}(\pi^0 K^0)} &= 0.9-1.4 \quad [0.59 \pm 0.27] \quad 0.91 \pm 0.32 \text{ Br} \\ \frac{2\text{Br}(\pi^0 K^\pm)}{\text{Br}(\pi^\pm K^0)} &= 0.9-1.3 \quad [1.27 \pm 0.47] \quad 2.26 \pm 1.92 \text{ Br} \\ & \quad [R_s^{-1} - \text{Neubert-Rosner}] \\ \frac{\tau_{B^+}}{\tau_{B^0}} \frac{\text{Br}(\pi^\mp K^\pm)}{\text{Br}(\pi^\pm K^0)} &= 0.6-1.0 \quad [1.00 \pm 0.30] \quad 1.44 \pm 0.32 \text{ Br} \\ & \quad [R - \text{Fleischer-Mannel}] \end{aligned}$$

The near equality of the second and the third ratios is a result of **isospin symmetry**.

We find (almost independently of γ):

$$\text{Br}(B \rightarrow \pi^0 K^0) = (4.5 \pm 2.5) \times 10^{-6} (V_{cb}/0.039)^2 (f_+^{B \rightarrow \pi}(0)/0.3)^2$$

W.S. Hou et al. (99) factorization approach

$m_s \sim 4.0 \text{ GeV}$

FIGURES

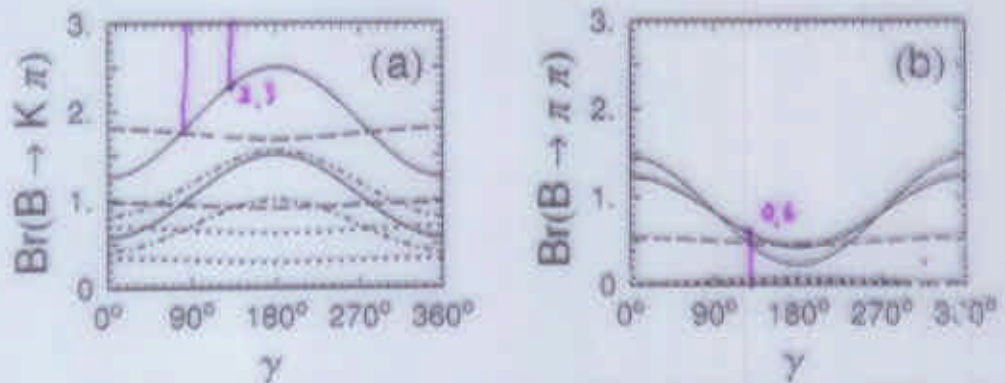


FIG. 1: (a) Solid, dash, dotdash and dots for $B \rightarrow K^+ \pi^-, K^0 \pi^+, K^+ \pi^0$ and $K^0 \pi^0$, for $m_s = 105$ (upper curves) and 200 MeV . (b) Solid, dash and dots for $B \rightarrow \pi^+ \pi^-, \pi^+ \pi^0$ and $\pi^0 \pi^0$ for $m_s = 2m_c = 3$ and 6.4 MeV , where the lower (upper) curve at $\gamma = 180^\circ$ for $\pi^+ \pi^-$ ($\pi^0 \pi^0$) is for lower m_s . In all figures Br s are in units of 10^{-6} , and $|V_{cb}/V_{cb}| = 0.08$.

CLEO data (central values)

PQCD ($\phi_3 \sim 90^\circ$)

$$\begin{aligned}
 \mathcal{B}(B^{\pm} \rightarrow K^0 \pi^{\pm}) &\sim 18.2 \times 10^{-6} && (20 \times 10^{-6}) \text{ Kamf, Li, Senlu} \\
 \mathcal{B}(B^0 \rightarrow K^{\pm} \pi^{\mp}) &\sim 17.2 \times 10^{-6} && (19 \times 10^{-6}) \\
 \mathcal{B}(B^0 \rightarrow \pi^{\pm} \pi^{\mp}) &\sim 4.3 \times 10^{-6} && (4.6 \times 10^{-6}) \text{ Lu, Ukai, Yang} \\
 &&& (6.3 \times 10^{-6}, \text{ Belle } (P, N) \text{ no}^{\pm}, \text{ BaBar}) \\
 R = \frac{\mathcal{B}(B^0 \rightarrow K^{\pm} \pi^{\mp})}{\mathcal{B}(B^{\pm} \rightarrow K^0 \pi^{\mp})} &\sim 1 && \Rightarrow \phi_3 \sim 90^\circ \\
 R_{\pi} = \frac{\mathcal{B}(B^0 \rightarrow K^{\pm} \pi^{\mp})}{\mathcal{B}(B^0 \rightarrow \pi^{\pm} \pi^{\mp})} &\sim 4 && \Rightarrow \phi_3 \sim 130^\circ \\
 &&& (3)
 \end{aligned}$$

The $\Delta I = 1/2$ Saga

Pa,So,HY
Ch,Fa

?



phrased as concise rule --

compact explanation?



after large value for ϵ'/ϵ was established (and for some heretics even before):

Sanda

face up to this challenge!



several dynamical enhancements found -- never of sufficient size, though

lack of success rationalized: in ϵ'/ϵ ignore origin of $\Delta I=1/2$ rule (wait for lattice QCD)



exact value of ϵ'/ϵ -- much harder question (and history does not represent one of the glory pages of theor. HEP)!

- $\mathcal{L}(\Delta S=1)$ under good theoretical control

Bu et al.

Fabb: 'educated guess'

$$\epsilon'/\epsilon \sim 10^{-3}$$

- considerable uncertainties in the size of **hadronic matrix elements**
- magnified by the presence of several relevant operators.

III. Rare $\Delta S=1$ & $\Delta B=1$ Decays

$$K^+ \rightarrow \pi^+ \nu \nu$$

BNL E787, Komat.

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \nu) = (1.5 + 3.4 - 1.2) \times 10^{-10}$$

sensitivity will be 0.7×10^{-10}

E949 sensitivity expected $(8-14) \times 10^{-12}$

$$\text{SM: } (0.82 \pm 0.32) \times 10^{-10}$$

$$B \rightarrow \gamma X, l^+ l^- X$$

the first ~ correctly predicted Penguin!

$$\text{BR}(H_b \rightarrow \gamma X_{\text{no charm}}) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4} \quad \text{CLEO}$$

$$\text{BR}(H_b \rightarrow \gamma X_{\text{no charm}}) = (3.34 \pm 0.5 \pm 0.35 \pm 0.28) \times 10^{-4} \quad \text{BELLE}$$

$$\text{BR}(B \rightarrow \gamma X_s) \Big|_{\text{SM}} = (3.29 \pm 0.33) \times 10^{-4}$$

- central value & uncertainty hardly changed
 many new calculations } impressive theor. machinery
 new contrib. \sim cancel
- careful analysis of γ spectrum
- $\text{BR}(B \rightarrow l^+ l^- X_s) \Big|_{\hat{s} \in [0.05, 0.25]} = (1.46 \pm 0.19) \times 10^{-6}$
- impact of New Physics different
 for $B \rightarrow \gamma X$ and $B \rightarrow l^+ l^- X$

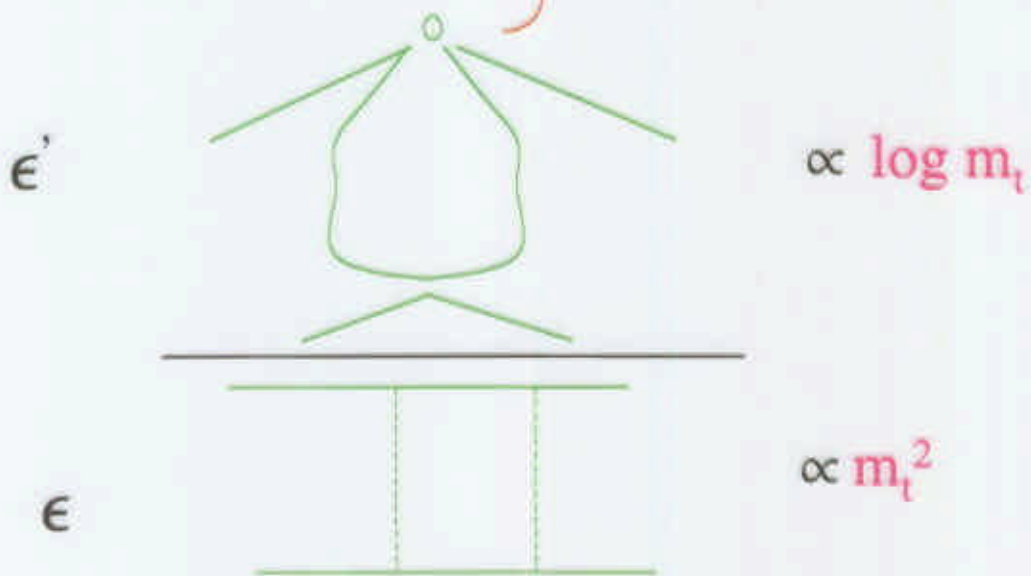
IV. CP Violation in $\Delta S \neq 0$ Decays

ϵ' / ϵ

$$\eta_{+-} = \epsilon + \epsilon', \quad \eta_{00} = \epsilon - 2\epsilon'$$

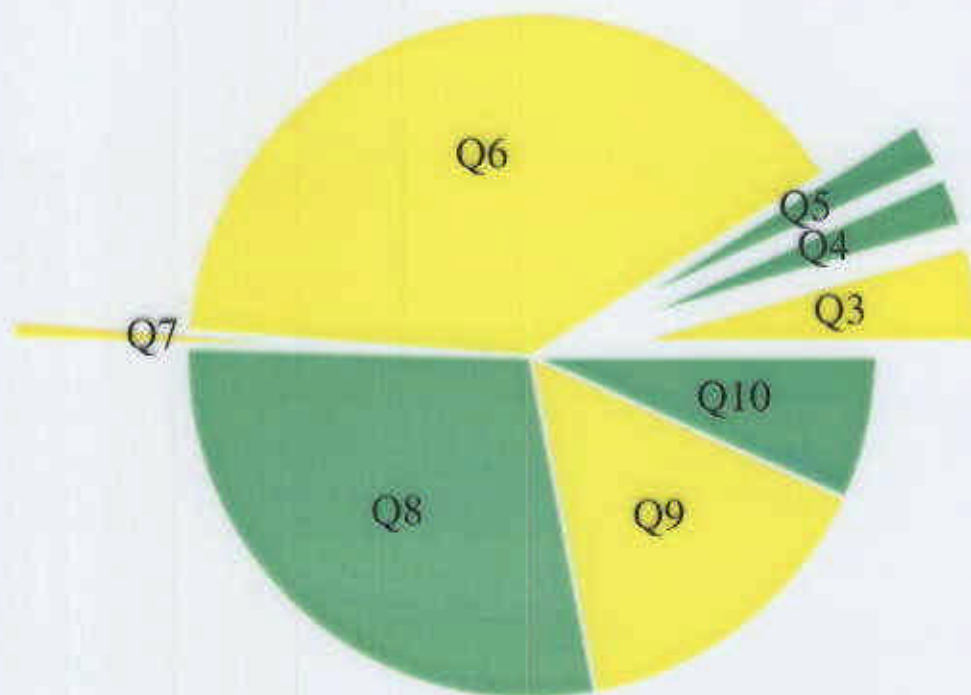
$$\frac{\epsilon'}{\epsilon} = \frac{1}{\sqrt{2}} \left\{ \frac{\langle (\pi\pi)_{I=2} | H_W | K_L \rangle}{\langle (\pi\pi)_{I=0} | H_W | K_L \rangle} - \frac{\langle (\pi\pi)_{I=2} | H_W | K_S \rangle}{\langle (\pi\pi)_{I=0} | H_W | K_S \rangle} \right\}$$

- $\Delta I = 1/2$ rule
 - single KM phase
 - m_t large
- } $0 \neq \epsilon'/\epsilon \ll 1/20$



THE ε'/ε PIE

Gluonic Penguins

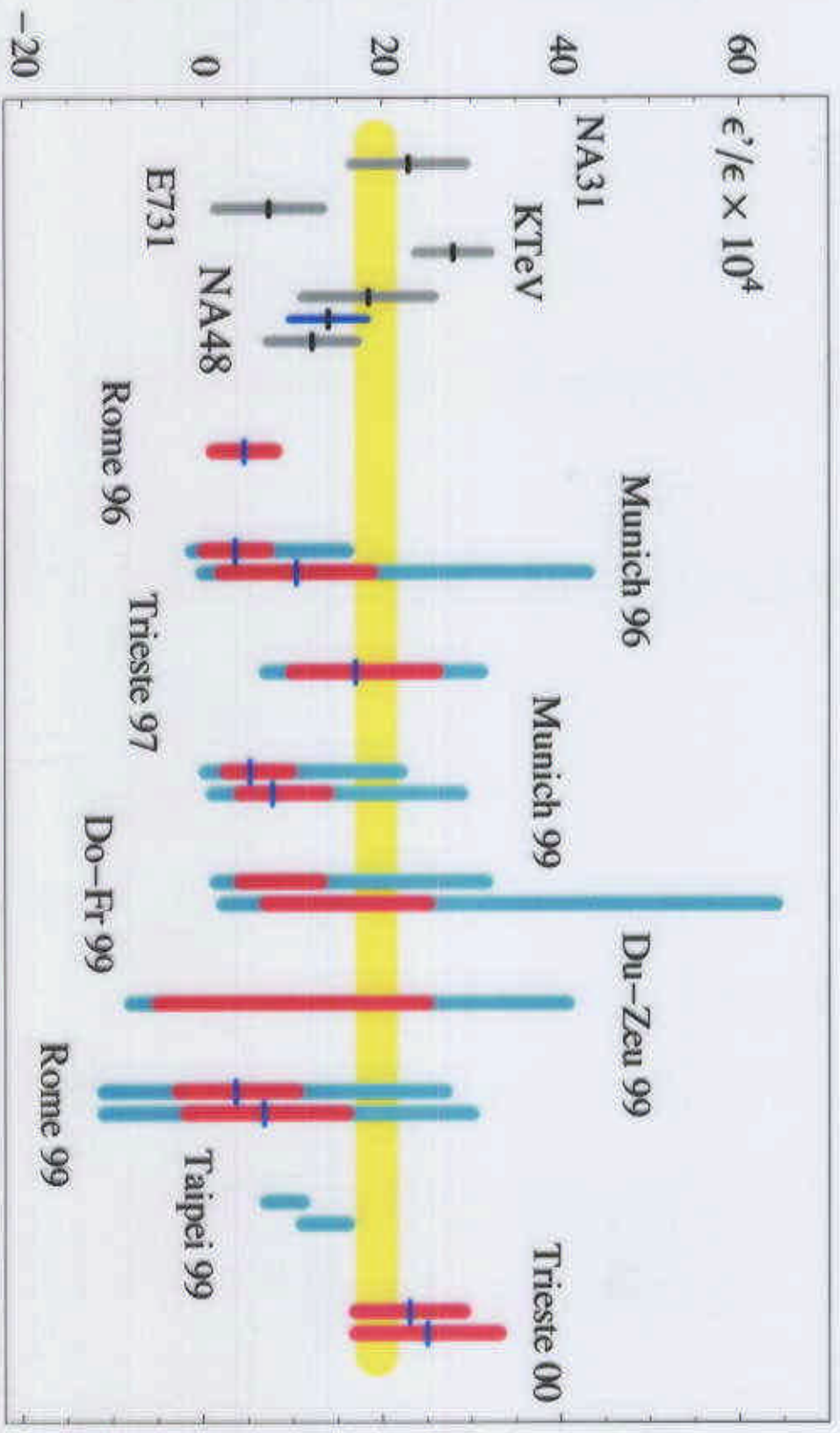


Electroweak Penguins and Box

Vacuum Saturation Approximation to the hadronic matrix elements of the effective four-quark operators.

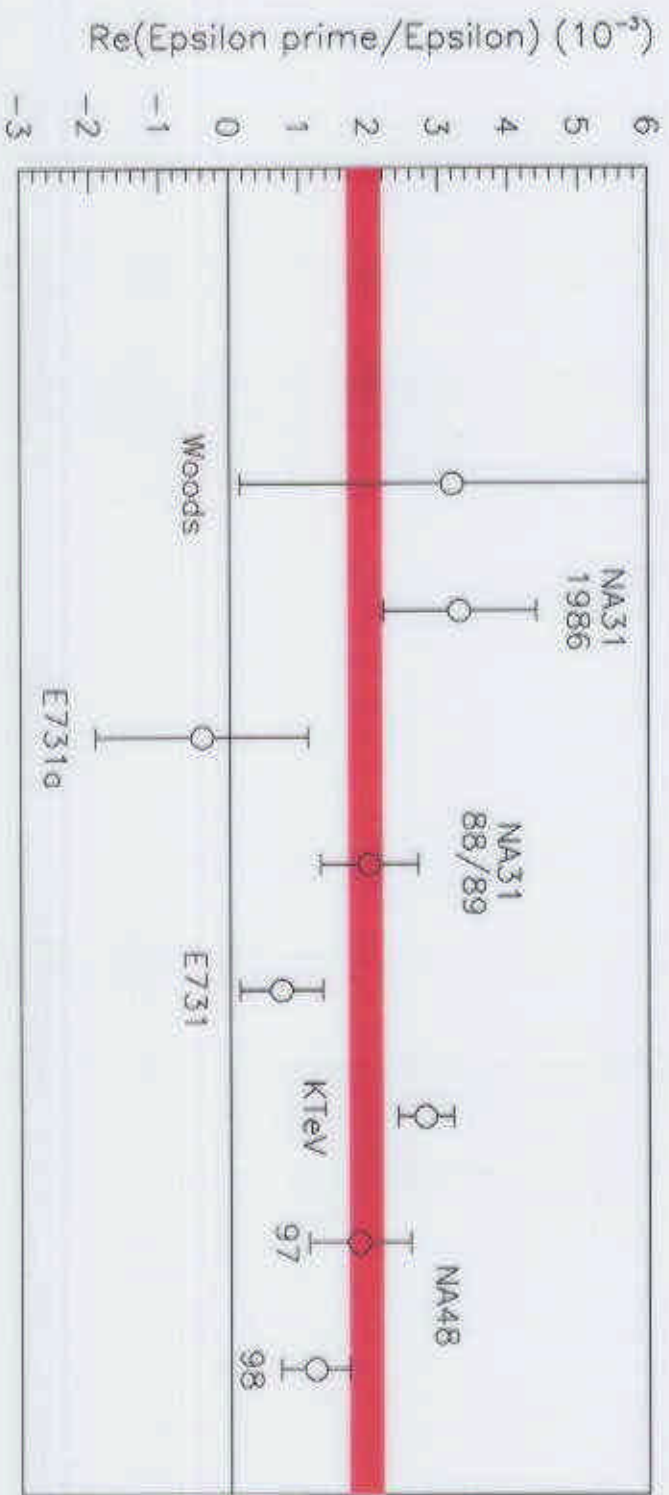
In **Yellow** (**Green**) the **Positive** (**Negative**) contributions.

Experiment vs. Theory



Experiments versus Theory

New world average



New world average:

$$\text{Re}(\epsilon'/\epsilon) = (19.3 \pm 2.4) \times 10^{-4}$$

$$(\chi^2/\text{ndf} = 11.1/5)$$

Direct Evidence for T Violation

of course ~~CP~~ \Rightarrow ~~T~~

↑
⋮
CPT ✓

$K^0 \Rightarrow K^0$ vs. $\bar{K}^0 \Rightarrow \bar{K}^0$

The 'Kabir Test'

associated production ✓

$$A_T = \frac{\Gamma(K^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \rightarrow K^0)}{\Gamma(K^0 \rightarrow \bar{K}^0) + \Gamma(\bar{K}^0 \rightarrow K^0)}$$

semilept. $K_{\text{neut}} \rightarrow l^\pm \nu \pi$, CPT ✓

CPLEAR:

$A_T = (6.6 \pm 1.3 \pm 1.0) \times 10^{-3} \neq 0$

vs. $(6.54 \pm 0.24) \times 10^{-3}$ ✓

$$K_L \rightarrow \pi^+ \pi^- e^+ e^-$$

and its T odd correlation

$$\phi = \angle(\mathbf{n}_1, \mathbf{n}_\pi)$$

$$\mathbf{n}_1 = \mathbf{p}(e^+) \times \mathbf{p}(e^-) / |\mathbf{p}(e^+) \times \mathbf{p}(e^-)|$$

$$\mathbf{n}_\pi = \mathbf{p}(\pi^+) \times \mathbf{p}(\pi^-) / |\mathbf{p}(\pi^+) \times \mathbf{p}(\pi^-)|$$

$$d\Gamma/d\phi = \Gamma_1 \cos^2\phi + \Gamma_2 \sin^2\phi + \Gamma_3 \cos\phi \sin\phi$$

↓ T,CP

$$- \cos\phi \sin\phi$$

$$A = \frac{2\Gamma_3}{\pi(\Gamma_1 + \Gamma_2)} = \begin{cases} (13.6 \pm 2.5 \pm 1.2)\%, \text{ KTeV} \\ (14.3 \pm 1.3)\%, \text{ Se Wa} \end{cases}$$

η₊₋ effect!

can be made consistent with T \checkmark --
 Bi Sa at the price of ~~CPT~~ $\sim 10^{-3}$ in

$$K^\pm \rightarrow \pi^\pm \pi^0 !$$

V. CP Violation in $\Delta B \neq 0$ Decays

wide-spread attitude:

- observing a CP asymmetry in

$$B_d \rightarrow \psi K_S$$

no big deal since expected -- unless its value falls clearly outside predicted range;

- observing a CP asymmetry in

$$B_d \rightarrow \pi^+ \pi^-$$

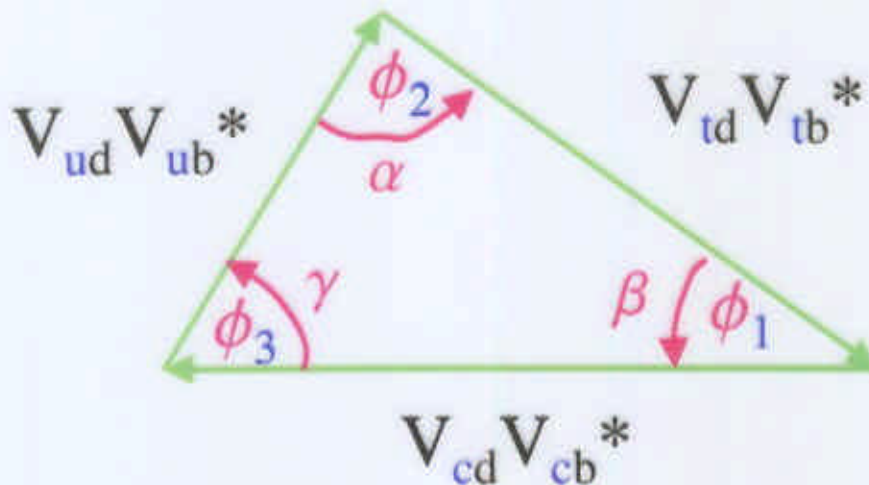
no big deal either, since it cannot be interpreted cleanly in terms of CKM parameters and its value is hardly constrained.

such sentiments miss the **paradigmatic** character of such observations:

- an asymmetry in $B_d \rightarrow \psi K_S$
 - ▷ would be the first CP violation directly observed outside K_L decays,
 - ▷ would have to be big to be established in the near future and
 - ▷ would establish the KM ansatz as a -- if not the -- major agent of CP violation!
- likewise an asymmetry in $B_d \rightarrow \pi^+ \pi^-$
 - ▷ again would have to be big to be established in the near future and
 - ▷ would probably reveal direct CP violation to be huge in beauty decays!

V.1 CKM Trigonometry, Part I

'the' KM unitarity triangle



ϕ_i opposite side with $V_{id} V_{ib}^*$

$$\phi_1 = \pi - \arg \left(\frac{-V_{tb}^* V_{td}}{-V_{cb}^* V_{cd}} \right)$$

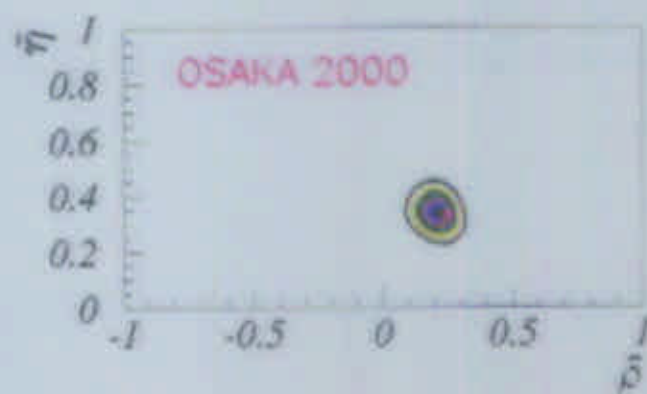
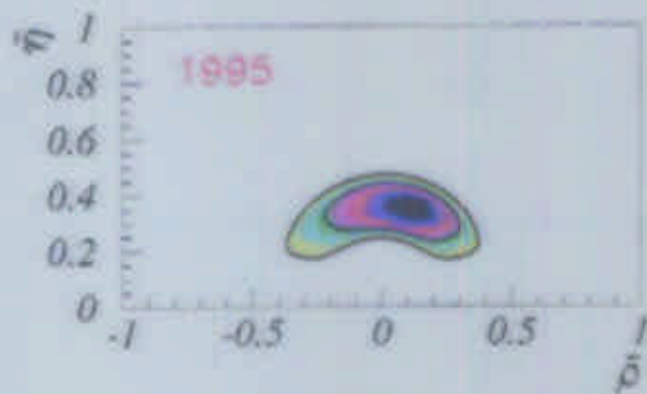
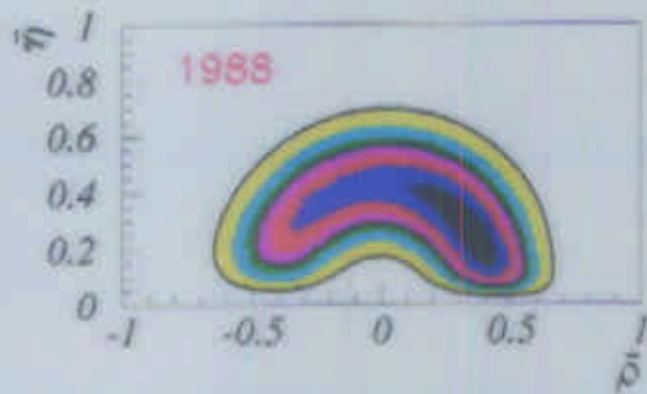
$$\phi_2 = \arg \left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}} \right)$$

$$\phi_3 = \arg \left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}} \right)$$

2 classes of observables:

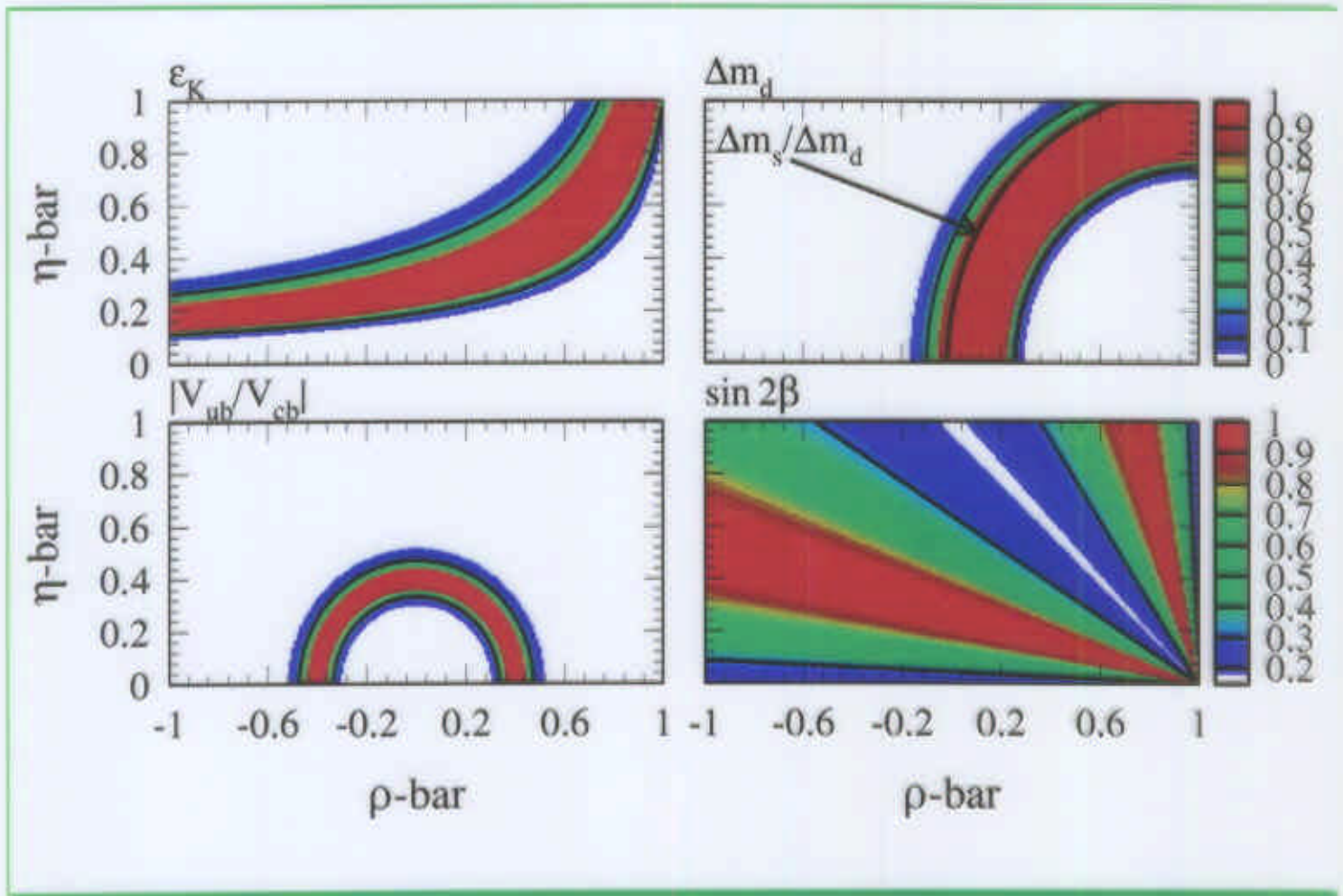
- CP insensitive rates \Rightarrow sides
 - $\Rightarrow b \rightarrow l \nu u / b \rightarrow l \nu c \Rightarrow |V_{ub}/V_{cb}|$
 - $\Rightarrow \Delta m(B_d)/\Delta m(B_s) \Rightarrow |V_{td}/V_{ts}|$
 - $\Rightarrow K^+ \rightarrow \pi^+ \nu \nu \Rightarrow |V_{td}/V_{ts}|$
 - CP asymmetries \Rightarrow angles
 - $\Rightarrow \epsilon_K/\Delta m(B_d) \Rightarrow \sim \phi_1$
-
- $\Rightarrow B \rightarrow \psi K_S \Rightarrow \phi_1$

plots



CKM-FITTER:

CONSTRAINTS IN THE $\rho - \eta$ PLANE



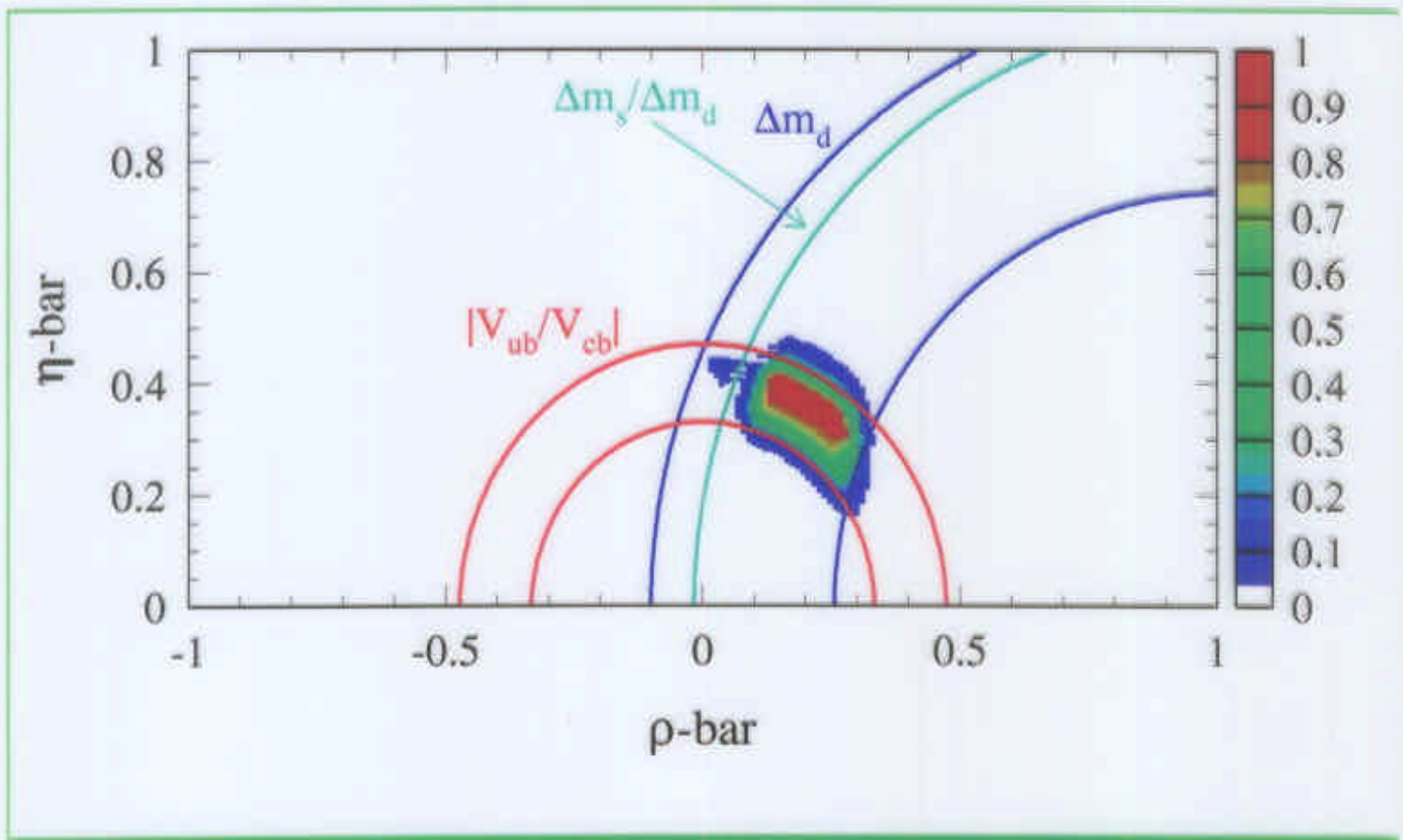
Andreas Höcker, Heiko Lacker, Sandrine Laplace and Francois Le Diberder

CKM-FITTER:

CONSTRAINTS IN THE $\rho - \eta$ PLANE

Indirect Evidence for CP-violation:

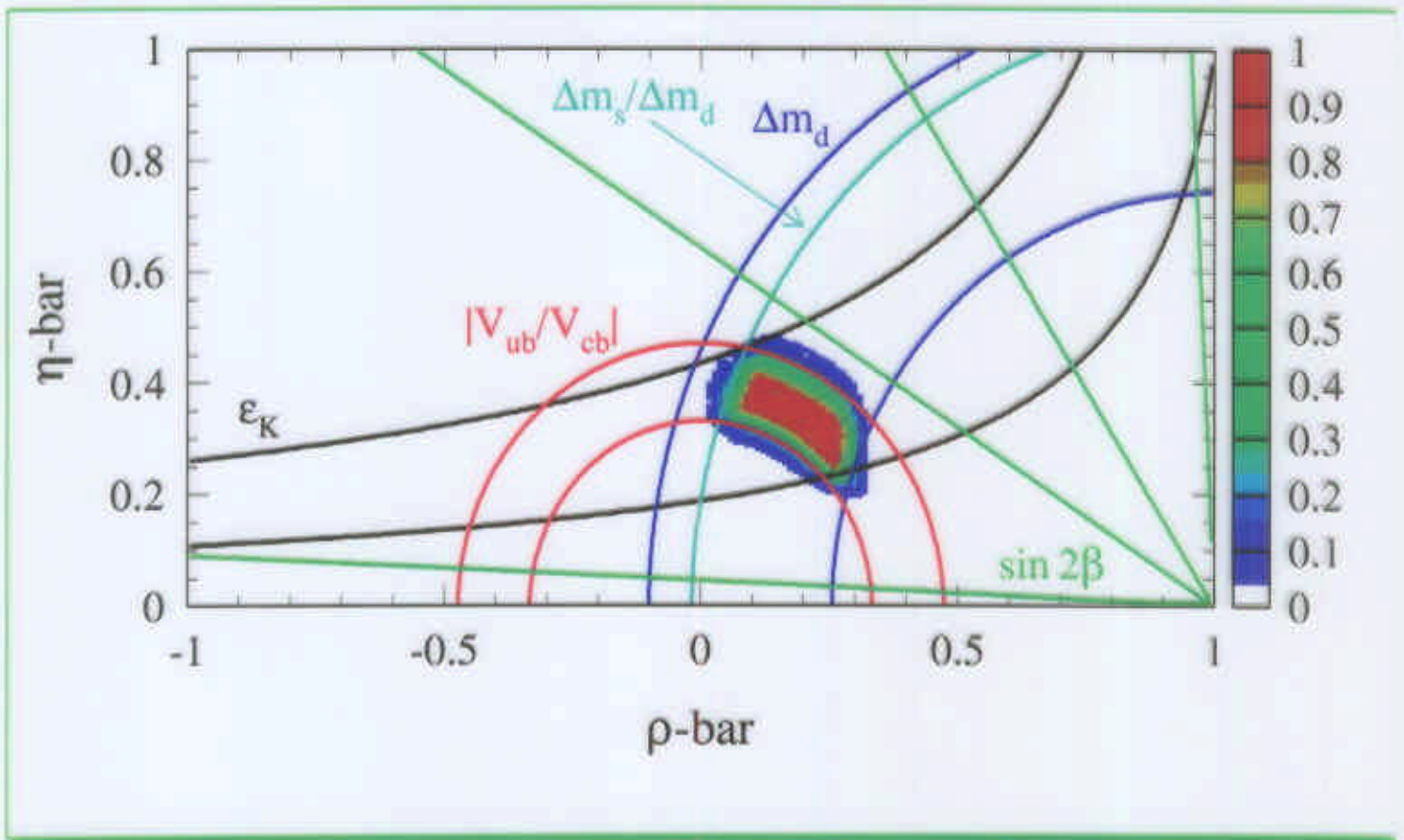
- Constraints without ϵ_K and $\sin 2\beta$



Andreas Höcker, Heiko Lacker, Sandrine Laplace and Francois Le Diberder

CKM-FITTER:

CONSTRAINTS IN THE $\rho - \eta$ PLANE



Andreas Höcker, Heiko Lacker, Sandrine Laplace and Francois Le Diberder

with caveats to be discussed below

present analyses (e.g. Stoc, DiLe) suggest:

$$\sin 2\phi_1 = 0.716 \pm 0.070 \leftrightarrow 0.7 \pm 0.2$$

$$\sin 2\phi_2 = -0.26 \pm 0.28 \leftrightarrow -0.25 \pm 0.7$$

$$\phi_3 = 59.3^\circ \pm 7.3^\circ$$

$B \rightarrow \psi K_S$

○ CDF: $\sin 2\phi_1 = 0.79 \pm 0.44$

○ BELLE: $\sin 2\phi_1 = 0.45$ $\left\{ \begin{array}{l} + 0.43 + 0.07 \\ - 0.44 - 0.09 \end{array} \right.$

○ BaBar: $\sin 2\phi_1 = 0.12 \pm 0.37 \pm 0.09$

what **if** `Michelson-Morley outcome`

$$\sin 2 \phi_1 \leq 0.1 \text{ !?}$$

➔ CKM ruled **out as major** player in

$$K_L \rightarrow \pi \pi$$

no plausible deniability!

➔ **new puzzle:** why is CKM phase so
small?

VI. Probing for New Physics

- historic precedent $\Delta S \neq 1,2$
 - ⇒ instrumental for emergence of SM through 'qualitative' discrepancies
- can happen again
 - ▣ $\Delta C \neq 0$
 - ▣ EDM's
 - ▣ $K_{\mu 3} \dots$
- different situation in $\Delta B \neq 1,2$

more complex
↙ ↘
more opportunities more challenges

quantitative discrepancies

control over uncertainties!

VI.1 KM Trigonometry, Part II

3x3 unitary matrix \Rightarrow 6 triangles with **same** area!

① *bd* triangle



$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = \delta_{bd} = 0$$

② *tu* triangle



$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = \delta_{tu} = 0$$

③ *bs* triangle



$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = \delta_{bs} = 0$$

④ *tc* triangle



$$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = \delta_{tc} = 0$$

⑤ *sd* triangle



$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = \delta_{sd} = 0$$

⑥ *cu* triangle



$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = \delta_{cu} = 0$$

there are **three classes** of angles:

- ① angles of **order unity** like ϕ_1, ϕ_2 and ϕ_3 ;

they differ from each other in $O(\lambda^2)$.

- ② angles that are themselves $\sim O(\lambda^2)$;

the most accessible representative is an angle

in the bs triangle, often referred to as χ :

$$\chi = \phi_1^{bs} = \pi + \arg \left(\frac{V_{cs}^* V_{cb}}{V_{ts}^* V_{tb}} \right) \approx \lambda^2 \eta$$

it controls the CP asymmetry in

$$B_s \rightarrow \psi \eta, \psi \phi$$

B S '80

- ③ angles that are themselves $\sim O(\lambda^4)$;

the least un accessible representative is an

angle in the cu triangle, often referred to as χ' :

$$\chi' = \phi_3^{cu} = \arg \left(\frac{-V_{ud}^* V_{cd}}{V_{us}^* V_{cs}} \right) \approx -\lambda^4 A^2 \eta$$

B '89

it controls CP asymmetries in **D decays**

⇒ 3 step strategy:

- ➔ measure the large angles ϕ_1, ϕ_2, ϕ_3 and their 'cousins' and check their correlations with the sides

- ➔ check whether the small [tiny] angle χ [χ'] are indeed small
- ➔ measure as many of the angles as possible and analyze their correlations.

search for New Physics

probe of features of New Physics

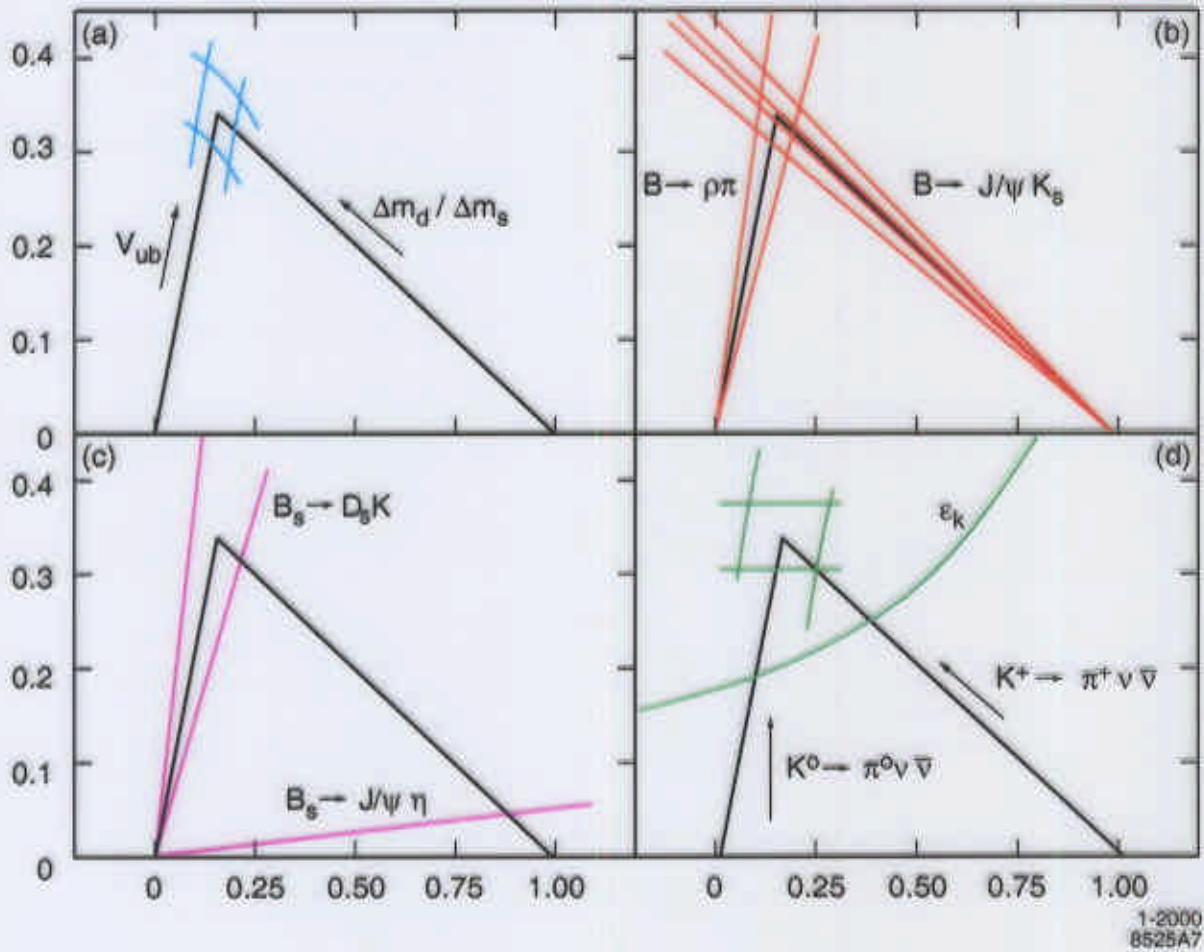


Figure 8: Illustration of four determinations of the unitarity triangle, by (a) non-CP observables, (b) B asymmetries, (c) B_s asymmetries, (d) K rare decays. See the text for more details.

easier said than done

- systematic experimental uncertainties!
- experiments could be wrong!

‘Combiner’ Hoecker, Laplace, Le Diberder

- theoretical uncertainties!

meaning of theoretical uncertainties?

my ‘definition’ for stating theor.
uncertainties:

“I would be very surprised if the true value
would fall outside the stated range.”

obviously hard to quantify

remember ϵ'/ϵ !

Be Fa plot

- theoret. uncertainties mostly like **systematic** uncertainties with hidden correlations
- they can be evaluated only through **overconstraints** or **'redundant'** extractions
- **prior** to that predictions should be considered **'preliminary'**

a triangle analysis better be much more sturdy than a bunraku stage!

'good decisions derive from experience that often is based on bad decisions'

VI.2 D^0 - D^0 Oscillations and CP Violation

charm usually seen as the dull cousin of beauty

- CKM parameters V_{cs} & V_{cd} 'known'
due to unitarity
 - slow $D^0 - D^0$ oscillations
 - tiny CP asymmetries
- ➔ ~ zero background search for New Physics

however ...

- ? how slow is slow?
- ? how tiny is tiny?

VI.2.a D^0 - D^0 Oscillations

2 quantities control D^0 - D^0 oscillations:

$$x_D = \frac{\Delta m_D}{\Gamma_D} \quad y_D = \frac{\Delta \Gamma_D}{2\Gamma_D}$$

$$\Gamma_D = \frac{\Gamma(D^0 \rightarrow l^- X)}{\Gamma(D^0 \rightarrow l^+ X)} \approx \frac{x_D^2 + y_D^2}{2} \quad \text{for } x_D, y_D \ll 1$$

- x_D and y_D Cabibbo suppressed
 - $x_D = 0 = y_D$ in the $SU(3)_{Fl}$ limit
- ➔ $x_D, y_D < 0.05$

a conservative estimate

$$x_D, y_D \sim \mathcal{O}(0.01)$$

popular claims

- $x_D(\text{SM})|_{\text{OPE}} \ll x_D(\text{SM})|_{\text{LD}}$
 $y_D(\text{SM})|_{\text{OPE}} \ll y_D(\text{SM})|_{\text{LD}}$
- $x_D(\text{SM})|_{\text{LD}}, y_D(\text{SM})|_{\text{LD}} \sim 10^{-4} - 10^{-3}$

general expectations

- $\Delta\Gamma$: on-shell contributions
 - ➔ \sim insensitive to New Physics
- Δm : virtual intermediate states
 - ➔ sensitive to New Physics
 - $x_D \sim O(\text{few } \%)$ conceivable

more careful analysis

- $x_D(\text{SM})|_{\text{OPE}}, y_D(\text{SM})|_{\text{OPE}} \sim O(10^{-3})$
- central theoretical issue: does quark-hadron duality hold at the charm scale already?
- more averaging in x_D than in y_D
 - ➔ duality better in x_D than in y_D

data

E791

$$y_{CP} = 0.8 \pm 2.9 \pm 1.0 \%$$

CLEO

$$-5.8 \% < y'_{CP} < 1 \% \quad (95 \% \text{ C.L.})$$

FOCUS

$$y_{CP} = 3.42 \pm 1.39 \pm 0.74 \%$$

if $y_D \sim 0.01$ { for $x_D \leq \text{few} \times 10^{-3}$: $1/m_c$ expansion.
okay!
for $x_D \sim 0.01$: theor. conundrum

sobering lesson:

case for New Physics based on x_D uncertain!

VI.2.b CP Violation in Charm Decays

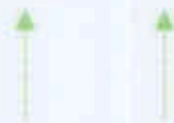
- ~~CP~~ involving $D^0 - \bar{D}^0$ oscillations

➔ $D^0 \rightarrow K^+K^-$ vs. $\bar{D}^0 \rightarrow K^+K^-$

➔ $D^0 \rightarrow K^+\pi^-$ vs. $\bar{D}^0 \rightarrow K^-\pi^+$

CP asymmetry given by

$$\sin \Delta m_D t \operatorname{Im} \rho(D \rightarrow f)$$



small

in

SM KM

➔ strong case
for New Physics!

- direct ~~CP~~ in Cabibbo supp. modes

KM → $O(10^{-3})$ effects

unclear case ?!

VI.3 Probe for New Physics in $\Delta S = 1$

$$P_T(\mu) \text{ in } K^+ \Rightarrow \mu^+ \pi^0 \nu$$

T odd correlation

$$P_{\perp} \equiv \langle \mathbf{s}_{\mu} \cdot (\mathbf{p}_{\mu} \times \mathbf{p}_{\pi}) / |\mathbf{p}_{\mu} \times \mathbf{p}_{\pi}| \rangle$$

can be 'faked' by final state interactions

$$P_{\perp} \propto \text{Im } \xi, \quad \xi = f_{-}/f_{+}$$



New Physics ~ Higgs-X!

o 'ancient' result:

$$\text{Im } \xi = -0.01 \pm 0.019 \Leftrightarrow P_{\perp} = (-1.85 \pm 3.6) \cdot 10^{-3}$$

o $P_{\perp} \sim 10^{-6}$ from final state interactions

o new result:

Aoki

$$\text{Im } \xi = -0.013 \pm 0.016 \pm 0.003$$

VI.4 Electric Dipole Moments

energy shift $\Delta\mathcal{E}$ of system inside electric field \mathbf{E} :

$$\Delta\mathcal{E} = d_i E_i + d_{ij} E_i E_j + \dots$$

linear in E

$\mathbf{d} \propto \mathbf{s} \Rightarrow \mathbf{d} \neq 0 \Leftrightarrow \text{T violation!}$

o experim. bounds

$\Rightarrow d_{\text{neutron}} < 9.7 \cdot 10^{-26} \text{ e cm}$

$\Rightarrow d_{\text{elektron}} = (-0.3 \pm 0.8) \cdot 10^{-26} \text{ e cm}$

o theoret. expectations

$\Rightarrow \text{CKM: } d_{\text{neutron}} < 10^{-30} \text{ e cm}$

$\Rightarrow \text{SM: } d_{\text{neutron}} \sim 10^{-9} \cdot \theta_{\text{QCD}} \text{ e cm}$

\Rightarrow many New Physics scenarios:

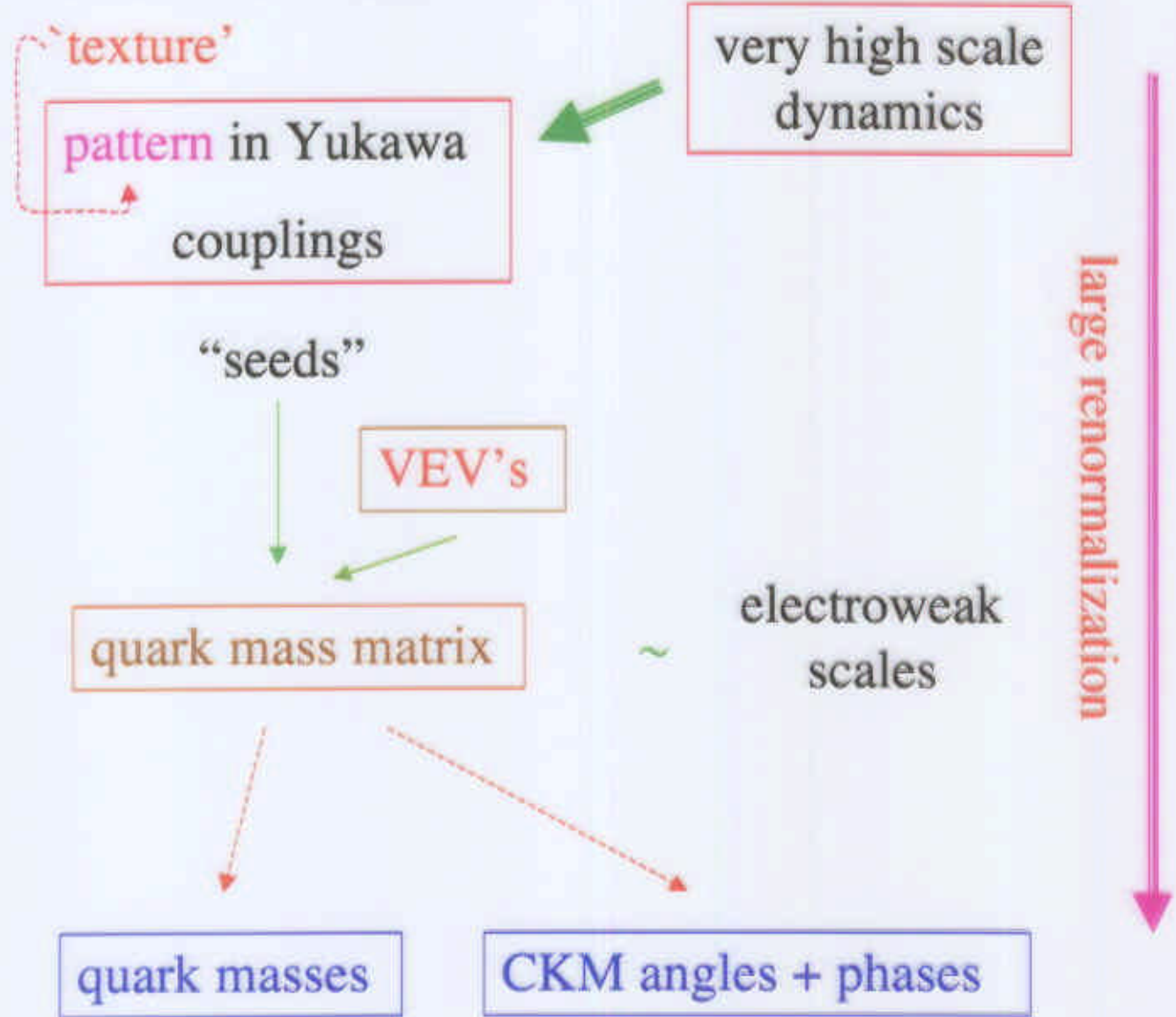
$d_{\text{neutron}}, d_{\text{elektron}} > 10^{-27} \text{ e cm}$

\rightarrow a definite must for experimental study!

VI.5 Glazing into the Crystal Ball

- my own expectation on beauty physics:
 - ➔ various large ($> 10\%$) CP asymmetries will be found in B decays
 - ➔ including **direct** CP violation
 - ➔ that agree with the CKM expectation to first order
 - ➔ yet exhibit some smallish, though definite deviations \Rightarrow New Physics!
- however it is conceivable that the whole future beauty phenomenology can be accommodated in the CKM ansatz --
were our efforts for naught then?

an emphatic **NO!**



personal conjecture/bias:

simple pattern \Rightarrow special CKM parameters!

yet

high scale \longrightarrow low scale

washes out

VII. Conclusions & Outlook

exciting & even **decisive** phase in flavour dynamics

- phenom. success of CKM description quite unreasonable \Rightarrow hidden message!
- **new (sub-)paradigms** established or about to be established:

① **direct** ~~CP~~ in $\Delta S=1$: $\epsilon'/\epsilon \neq 0$

② first evidence for ~~CP~~ **outside** K_L decays

③ CKM predictions for ~~CP~~ in B decays about to be **tested**

high sensitivity probes of dynamics

many possible portals for New Physics!

○ basic quantities known with good accuracy --
and **the promise for even better:**

⇒ $\delta m_b \sim 1.5\%$ most precise quark mass

⇒ $\delta m_t \sim 3\%$ [10%] **direct** [rad.corr.]

⇒ $\delta V(cb) \sim 5+\%$ → 2% feasible

⇒ $\delta V(ub) \sim 40\%$ → 10% feasible

→ 5% not impossible

⇒ $\delta V(td) \sim 60\%$ → 10% not impossible

high precision probes of dynamics!

○ greatly improved **practical** theoret. technologies

⇒ increasing sophistication in treating **SL** &
radiat. decays

⇒ **new frontier:** exclusive **NL B** → $M_1 M_2$

□ compreh. classif. scheme based on QCD

□ promise to evaluate even non-leading

effects in the **real world**

- need **experimental** program that allows **precise** measurements in a comprehensive way

- *other portals for New Physics:*

- ➔ d_N, d_e : any improvement in **experim.**

- ~~sensitivity~~ could reveal effect!

- ➔ P_{\perp} in $K \rightarrow \mu \nu \pi$

- ➔ charm decays:

- CP & D^0 - \bar{D}^0 oscillations

... and: evidence for neutrino oscillations

- ➔ neutrino masses **non**-degenerate
- ➔ lepton **flavour** ES \neq lepton **mass** ES
- ➔ as analogue of CKM matrix

the **M**(aki-)**N**(akagawa-)**S**(akata) matrix

“leptons exactly like quarks -- only different”

colour singlets

ν masses **extremely tiny**

↖ see-saw mechanism

➔ atm. ν 's $\Leftrightarrow \tau \rightarrow \mu \gamma$

solar ν 's $\Leftrightarrow \mu \rightarrow e \gamma$

Okada

(H)OPE: the Effective Lagrangian

$$\mathcal{L}_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu)$$

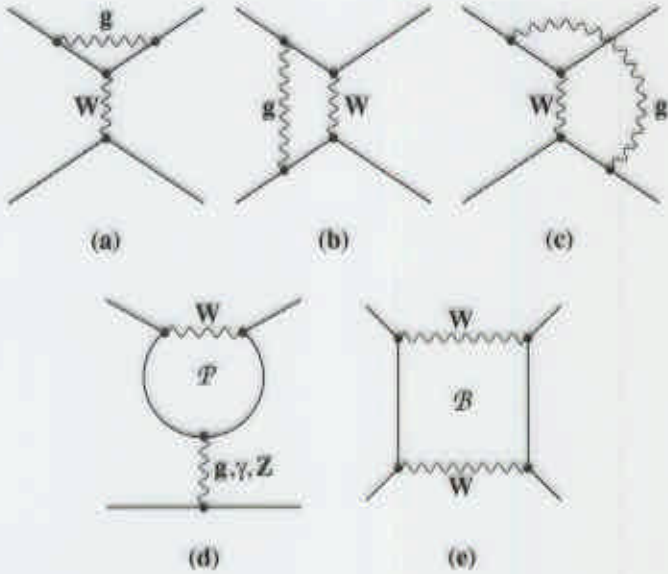
$$\tau = -V_{td} V_{ts}^* / V_{ud} V_{us}^*$$

For $\mu < m_c$ ($q = u, d, s$):

$\left. \begin{aligned} Q_1 &= (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \\ Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \end{aligned} \right\}$	Current-Current
$\left. \begin{aligned} Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} \\ Q_{4,6} &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V\mp A} \end{aligned} \right\}$	Gluon "penguins"
$\left. \begin{aligned} Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_q \hat{e}_q (\bar{q}q)_{V\pm A} \\ Q_{8,10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q \hat{e}_q (\bar{q}_\beta q_\alpha)_{V\pm A} \end{aligned} \right\}$	Electroweak "penguins"

"Penguins" feel all three quark families in the loop:
they are sensitive to the CP phase.

Standard model contributions to the matching of the quark operators in the effective flavor-changing Lagrangian



$\mu = m_W$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Tree		✓								
Tree + g	✓	✓								
Tree + γ		✓								
\mathcal{P}_g			✓	✓	✓	✓				
\mathcal{P}_γ							✓			
\mathcal{P}_Z							✓			
\mathcal{B}									✓	✓

Calculation of four-quark matrix elements

The ideal approach

- A: Consistent definition of renormalized operators: correct scheme and scale matching with short-distance.
- B: Self-contained calculation of all hadronic matrix elements (including B_K).
- C: It reproduces simultaneously the $\Delta I = 1/2$ selection rule and ε'/ε .

- **VSA:**

$$\begin{aligned} \langle \pi^+ \pi^- | Q_6 | K^0 \rangle &= 2 \langle \pi^- | \bar{u} \gamma_5 d | 0 \rangle \langle \pi^+ | \bar{s} u | K^0 \rangle \\ &\quad - 2 \langle \pi^+ \pi^- | \bar{d} d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle \\ &\quad + 2 \left[\langle 0 | \bar{s} s | 0 \rangle - \langle 0 | \bar{d} d | 0 \rangle \right] \langle \pi^+ \pi^- | \bar{s} \gamma_5 d | K^0 \rangle \end{aligned}$$

- **Generalized Factorization:** Scale and scheme independent Wilson coefficients, matched with factorized matrix elements at the scale μ_F (H-Y Cheng, 1999).
- **Phenomenological 1/N:** Fix some of the matrix elements by fitting the $\Delta I = 1/2$ rule and vary others around the 1/N values (München).
- **Chiral Quark Model:** All matrix elements at $O(p^4)$ in terms of $\langle \bar{q} q \rangle$, $\langle \frac{\alpha_s}{\beta} G G \rangle$, M , phenomenologically fixed via the $\Delta I = 1/2$ rule (Trieste).
- **Phenomenological NJL:** Chiral loops up to $O(p^6)$ and fit to the $\Delta I = 1/2$ rule. It includes scalar, vector and axial-vector resonances (Dubna).
- **1/N:** Chiral loops regularized via cutoff, partial $O(p^4)$ (Dortmund).
- **1/N and NJL:** It includes scalar, vector and axial-vector resonances, good scale stability (Bijnens and Prades, 1999).
- **Lattice:** $K \rightarrow \pi$ matrix elements of four-quark operators. Use chiral symmetry to obtain $K \rightarrow \pi\pi$ (Roma, RBC).

