Conformal Anomaly and c-function from AdS/CFT.

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Sachiko Ogushi (Ochanomizu University)

S. Nojiri (National defence academy)
S. D. Odintsov (Tomsk Pedagogical Univ.)

- This talk is based on the works hep-th/9912191, hep-th/0001122, hep-th/0005197.

- Plan of the talk.

1. Introduction (AdS/CFT).
2. Conformal Anomaly from AdS/CFT.
3. c-function from Supergravity.
4. Conclusions and discussions.

1. Introduction

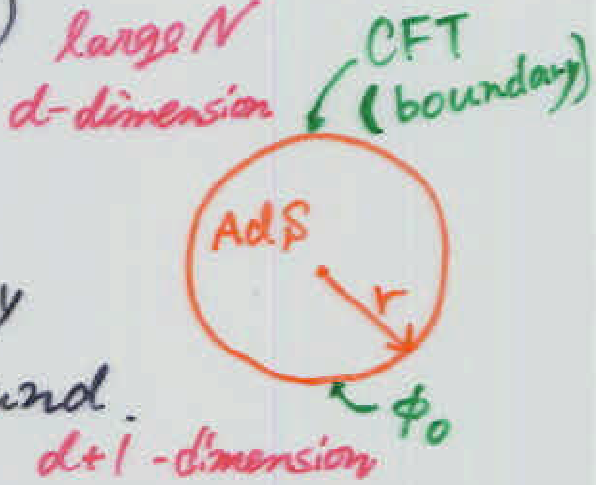
♦ AdS/CFT Correspondence

J. Maldacena 1997~

- $\mathcal{N}=4$, $d=4$ Super Yang-Mills Theory
  (Conformal Field Theory)

  ⬍ dual

- Classical Supergravity
  in $AdS_5 \times S^5$ background.



* CFT Correlation function from Supergravity

$R_s = (4\pi g_s N)^{\frac{1}{4}}$

S. Gubser, I. Klebanov and A.M. Polyakov,
E. Witten (1998~)

$$Z_{CFT} = \langle \exp(\int_d dx\, \underbrace{\phi_0}_{\text{Source}} \underbrace{\mathcal{O}}_{\text{Operator}}) \rangle$$

$$\underset{AdS/CFT}{\simeq} \exp(-S_{sg}(\phi^{cl}(\phi_0)))$$

- The Operator in CFT ⟷ The field in AdS

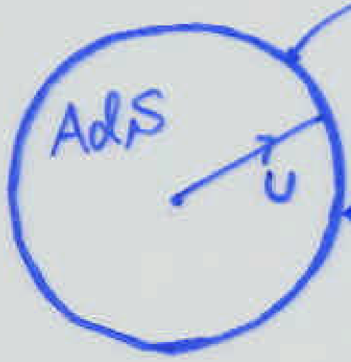
* AdS/CFT Correspondence.

· AdS	· CFT
Field	↔ Operator
$AdS_5 \times S^5$ (d+1-dim.)	↔ CFT_4 ($N=4$, $d=4$ SYM) (d-dim.)

★ Deformation of AdS ↔ Deformation of CFT.

$$S_{CFT} \to S_{CFT} + \int d^d x\, \underbrace{\phi(0)}_{d-\Delta}\, \underbrace{\Phi(\phi)}_{\Delta}$$

boundary



field $\phi(0)$

Coupling constant

conformal dimension

R-G-flow

radial coordinate U dependence of $\phi(0)$ ↔ The running of the Coupling constant

Near the boundary ($U \to \infty$)

$$\phi(\vec{x}, U) \to U^{\Delta - d} \phi_0(\vec{x})$$

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + l^2 m^2}$$

The equation of motion

$$(\Box - \boxed{m^2})\,\phi = 0$$

$$\Box = \frac{1}{\sqrt{G}} \partial_\mu \left(G^{\mu\nu} \sqrt{G}\, \partial_\nu \phi \right)$$

AdS Metric

Potential terms

$\Delta < d$: relevant

$\Delta > d$: irrelevant

$\Delta = d$: marginal

AdS

The stational point of Scalar potential $\Phi(\phi)$

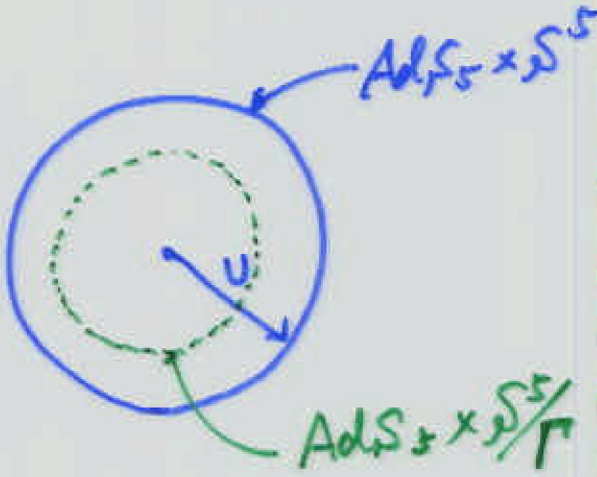
$$\left(\frac{d\Phi}{d\phi} = 0 \right)$$

m can be determined by expanding ϕ around the stational point.

⬇

Then, $\phi_0(\vec{x})$ depends on U.

$\boxed{?}$



The Anomaly calculation from gravity action which include Scalar potential (AdS/CFT)

↓

C-function.

CFT

$\frac{d\phi}{dU} = 0$ → fixed point.

coupling

↓

CFT.

Central charge is determined in this point.

* R-G flow

The changing of C

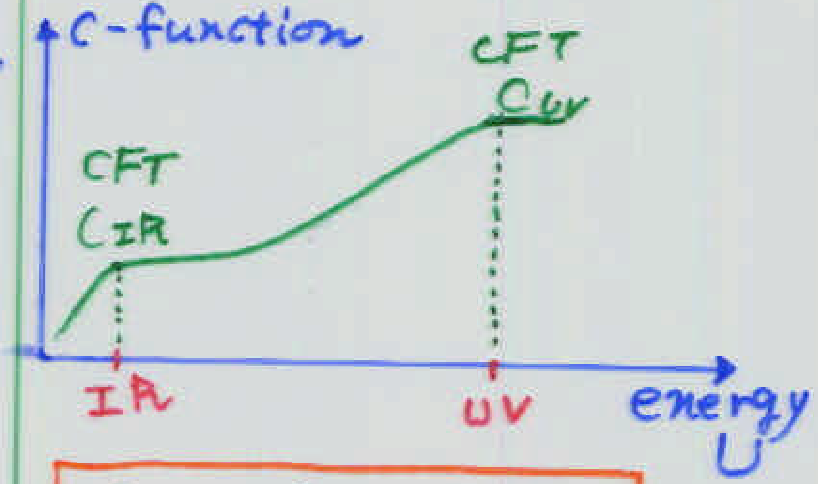
$C_{UV} \rightarrow C_{IR}$

C-function

* C-theorem

(Zamolodchikov 2D)

1. Positive
2. The monotonically increasing function of the energy scale.
3. The function has fixed points. (central charge)



$$\boxed{\langle T_\mu{}^\mu \rangle = \frac{c}{24\pi} R}$$

2. Conformal Anomaly from AdS/CFT.

★ M. Henningson and K. Skenderis, JHEP 9807, 023 (1998), hep-th/9806087.

★ S. Nojiri and S.D. Odintsov, Phys. Lett. B444 (1998) 92, hep-th/9810008. ← dilaton

◆ UV/IR duality.

UV divergence in CFT ↔ IR divergence in AdS.

◆ Conformal anomaly in CFT from Supergravity in AdS.

$$\langle T_\mu{}^\mu \rangle_{CFT}$$

• d+1-dim. asymptotically AdS metric

($g_{ij} \to \eta_{ij}$ (flat) AdS)

$$\hat{G}_{\mu\nu} dx^\mu dx^\nu = \frac{\ell^2}{4} \rho^{-2} d\rho d\rho + \sum_{i=0}^{d-1} \rho^{-1} g_{ij} dx^i dx^j$$

AdS radial coordinate $\rho^{-\frac{1}{2}} = U$

The boundary of AdS ($\rho \to 0$) ⇒ d-dim. CFT

This expression is invariant under the scale transformation

$$\delta\rho = \underline{\underline{\delta\sigma}}\,\rho \,, \quad \delta g_{ij} = \underline{\underline{\delta\sigma}}\, g_{ij}$$

constant parameter

* d+1 (=5)-dimensional dilatonic gravity action

$$S_{SG} = \frac{1}{16\pi G}\Big[\int_{M_{d+1}} d^{d+1}x\sqrt{-\hat{G}}\,\{\hat{R} + X(\phi)(\hat{\nabla}\phi)^2 + Y(\phi)\hat{\Delta}\phi + 4\lambda^2\} + \int_{M_d} d^d x\sqrt{-\hat{g}}\,(2\hat{\nabla}_\mu n^\mu + \alpha)\Big]$$

- M_{d+1} : d+1-dimensional manifold ~ AdS_{d+1}
- M_d : d-dimensional manifold (the boundary of M_{d+1})
- n_μ : The unit normal vector to M_d
- $4\lambda^2$: Cosmological constant $= \frac{d(d-1)}{\ell^2}$

We expand ϕ and g_{ij} in the power series of ρ:

$$\phi = \phi_{(0)} + \rho\phi_{(1)} + \rho^2\phi_{(2)} + \cdots,\quad g_{ij} = g_{(0)ij} + \rho g_{(1)ij} + \rho^2 g_{(2)ij} + \cdots$$

The $\underbrace{\text{square}}_{\text{root}}$ of $-\hat{G}$ ($= -\det \hat{G}_{\mu\nu}$) has the following form

$$\sqrt{-\hat{G}} = \rho^{-d/2-1}\sqrt{-g}$$

The action diverges in general since the action contains the infinite volume integration of M_{d+1}.

⬇

The action is regularized by introducing the infrared cutoff ε and replacing

$$\int d^{d+1}x \to \int d^d x \int_\varepsilon d\rho,\quad \int_{M_d} d^d x(\cdots) = \int d^d x(\cdots)\Big|_{\rho=\varepsilon}$$

Then we get

$$S = S_0(\phi_{(0)}, g_{(0)ij})\,\epsilon^{-\frac{d}{2}} + S_1(\phi_{(0)}, \phi_{(1)}, g_{(0)ij}, g_{(1)ij})\,\epsilon^{-\frac{d}{2}-1}$$

$$+ \cdots + \underline{S_{ln} \ln \epsilon} + S_{fin}$$

The term propotional to $\ln \epsilon$ appears when d is even. The system is originally invariant under the conformal transformation

$$\delta g_{(0)\mu\nu} = 2\delta\sigma\, g_{(0)\mu\nu}, \quad \delta\epsilon = 2\delta\sigma\,\epsilon$$

If we subtract the terms proportional the negatina power of ρ, the system is still invariant under the transformation.

But If we subtract the term proportional to $\ln\epsilon$, the invariance is broken since the variation of $\ln\epsilon$ term is finite:

$$\delta(\ln\epsilon) = 2\delta\sigma$$

The original system is invariant under the transformation, the variation of $\ln\epsilon$ term should be cancelled by

$$\delta S_{fin} = -2\delta\sigma$$

→ We can find the Weyl anomaly T from the expression of S_{ln}:

$$\boxed{S_{ln} = -\frac{1}{2}\int d^4x \sqrt{-g_{(0)}}\, T}$$

Then we find, when $d=4$

$$S_{\ell n} = \frac{\ell^3}{16\pi G}\int d^4x\sqrt{-g_{(0)}}\Big\{\underline{\frac{1}{8}R_{(0)ij}R^{ij}_{(0)} - \frac{1}{24}R^2_{(0)}}$$
$$\underline{+\frac{1}{2}R^{ij}_{(0)}\partial_i\phi_{(0)}\partial_j\phi_{(0)} - \frac{1}{6}R_{(0)}g^{ij}_{(0)}\partial_i\phi_{(0)}\partial_j\phi_{(0)}}$$
$$+\frac{1}{4}\Big\{\frac{1}{\sqrt{-g_{(0)}}}\partial_i\left(\sqrt{-g_{(0)}}\,g^{ij}_{(0)}\partial_j\phi_{(0)}\right)\Big\}^2 + \frac{1}{3}\left(g^{ij}_{(0)}\partial_i\phi_{(0)}\partial_j\phi_{(0)}\right)^2\Big\}$$

◆ The field theory calculation

- H. Liu, A.A. Tseytlin, Nucl. Phys. B533 (1998) 88 hep-th/9804083.

$\mathcal{N}=4$. $d=4$ Supersymmetric $SU(N)$ Yang-Mills theory coupled with $\mathcal{N}=4$ conformal supergravity.

$$T = -\frac{N^2}{4(4\pi)^2}\Big\{2\underline{\left(R_{ij}R^{ij} - \frac{1}{3}R^2\right)} + F^{ij}F_{ij}$$
$$+4\Big\{\underline{2\left(R^{ij} - \frac{1}{3}Rg^{ij}\right)\partial_i\varphi^*\partial_j\varphi} + |\Delta\varphi|^2\Big\} + \cdots\Big\}$$

$$\frac{\ell^3}{16\pi G} = \frac{2N^2}{(4\pi)^2}, \quad \varphi = \phi + e^{\phi}\chi, \quad \varphi^* = -\phi + e^{\phi}\chi$$

dilaton, axion.

* Axion-dilatonic Conformal Anomaly

S. Nojiri, S.D. Odintsov, S. Ogushi, A. Sugamoto. and M. Yamamoto, Phys. Lett. B465 (1999) 128 hep-th/9908066.

3. The c-function from supergravity.

★ The action of d+1-dimensional dilaton gravity with the potential $\Phi(\phi)$

$$S=\frac{1}{16\pi G}\int_{M_{d+1}}d^{d+1}x\sqrt{\underbrace{-\hat{G}}_{\hat{G}:\ \text{AdS Metric}}}\left\{\hat{R}+X(\phi)(\hat{\nabla}\phi)^2+Y(\phi)\hat{\Delta}\phi+\Phi(\phi)+\underbrace{4\lambda^2}_{\text{cosmological constant}}\right\}$$

◆ First we consider the case of $d=2$ (3-dimensional SG.). The anomaly term S_{ln} proportional to $\ln\varepsilon$ in the action is

$$S_{\ln}=-\frac{1}{16\pi G}\frac{l}{2}\int d^2x\sqrt{-g_{(0)}}\left\{R_{(0)}+X(\phi_{(0)})(\nabla\phi_{(0)})^2+Y(\phi_{(0)})\Delta\phi_{(0)}+\phi_{(1)}\Phi'(\phi_{(0)})+\frac{1}{2}g^{ij}_{(0)}g_{(1)ij}\Phi(\phi_{(0)})\right\}. \qquad ' = \frac{d}{d\phi}$$

The equation of motion given by variation of the action with respect to ϕ and $G^{\mu\nu}$ lead to $g_{(1)ij}$, $\phi_{(1)}$ in terms of $g_{(0)ij}$, $\phi_{(0)}$.

$$\begin{aligned}g_{(1)ij}=&\Big[-R_{(0)ij}-V(\phi_{(0)})\partial_i\phi_{(0)}\partial_j\phi_{(0)}-g_{(0)ij}\Phi'(\phi_{(0)})\phi_{(1)}\\&+\frac{g_{(0)ij}}{l^2}\left\{2\Phi'(\phi_{(0)})\phi_{(1)}+R_{(0)}+V(\phi_{(0)})g^{kl}_{(0)}\partial_k\phi_{(0)}\partial_l\phi_{(0)}\right\}\\&\times\left(\Phi(\phi_{(0)})+\frac{2}{l^2}\right)^{-1}\Big]\times\Phi(\phi_{(0)})^{-1}\end{aligned}$$

$$V(\phi_{(0)})\equiv X(\phi_{(0)})-Y'(\phi_{(0)})$$

$$\phi_{(1)} = \left[V'(\phi_{(0)}) g^{ij}_{(0)} \partial_i \phi_{(0)} \partial_j \phi_{(0)} + 2\frac{V(\phi_{(0)})}{\sqrt{-g_{(0)}}} \partial_i \left(\sqrt{-g_{(0)}} g^{ij}_{(0)} \partial_j \phi_{(0)} \right) \right.$$
$$\left. + \frac{1}{2} \Phi'(\phi_{(0)}) \left(\Phi(\phi_{(0)}) + \frac{2}{l^2} \right)^{-1} \left\{ R_{(0)} + V(\phi_{(0)}) g^{ij}_{(0)} \partial_i \phi_{(0)} \partial_j \phi_{(0)} \right\} \right]$$
$$\times \left(\Phi''(\phi_{(0)}) - \Phi'(\phi_{(0)})^2 \left(\Phi(\phi_{(0)}) + \frac{2}{l^2} \right)^{-1} \right)^{-1}$$

Here $V(\phi) \equiv X(\phi) - Y'(\phi)$

◆ The terms proportional to $\ln \epsilon \Rightarrow$ Anomaly in CFT

$$T = \frac{1}{8\pi G} \frac{l}{2} \left\{ R_{(0)} + X(\phi_{(0)}) (\nabla \phi_{(0)})^2 + Y(\phi_{(0)}) \Delta \phi_{(0)} \right.$$
$$+ \frac{1}{2} \left\{ \frac{3\Phi'(\phi_{(0)})}{l^2} \left(\Phi''(\phi_{(0)}) \left(\Phi(\phi_{(0)}) + \frac{3}{l^2} \right) - \Phi'(\phi_{(0)})^2 \right)^{-1} - \Phi(\phi_{(0)}) \right\}$$
$$\times \left(R_{(0)} + V(\phi_{(0)}) g^{ij}_{(0)} \partial_i \phi_{(0)} \partial_j \phi_{(0)} \right) \left(\Phi(\phi_{(0)}) + \frac{3}{l^2} \right)^{-1}$$
$$+ \frac{3\Phi'(\phi_{(0)})}{l^2} \left(\Phi''(\phi_{(0)}) \left(\Phi(\phi_{(0)}) + \frac{3}{l^2} \right) - \Phi'(\phi_{(0)})^2 \right)^{-1}$$
$$\left. \times \left(V'(\phi_{(0)}) g^{ij}_{(0)} \partial_i \phi_{(0)} \partial_j \phi_{(0)} + 2\frac{V(\phi_{(0)})}{\sqrt{-g_{(0)}}} \partial_i \left(\sqrt{-g_{(0)}} g^{ij}_{(0)} \partial_j \phi_{(0)} \right) \right) \right\}$$

We introduce the candidate c-function by the coefficient of $R_{(0)}$.

$$c = \frac{3}{2G} \left[l + \frac{l}{2} \left\{ \frac{3\Phi'(\phi_{(0)})}{l^2} \left(\Phi''(\phi_{(0)}) \left(\Phi(\phi_{(0)}) \right.\right.\right.\right.$$
$$\left.\left.\left.\left. + \frac{3}{l^2} \right) - \Phi'(\phi_{(0)})^2 \right)^{-1} - \Phi(\phi_{(0)}) \right\} \times \left(\Phi(\phi_{(0)}) + \frac{3}{l^2} \right)^{-1} \right]$$

* C-function (2-dimension)

• 4 - dimension

◆ The terms proportional to $\ln \varepsilon$ → Anomaly in CFT.

$$
\begin{aligned}
S_{\ln} = \frac{1}{16\pi G}\int d^4x\sqrt{-g_{(0)}}\Big[&\frac{-1}{2l}g_{(0)}^{ij}g_{(0)}^{kl}\left(g_{(1)ij}g_{(1)kl}-g_{(1)ik}g_{(1)jl}\right)\\
&+\frac{l}{2}\left(R_{(0)}^{ij}-\frac{1}{2}g_{(0)}^{ij}R_{(0)}\right)g_{(1)ij}\\
&-\frac{2}{l}V(\phi_{(0)})\phi_{(1)}^2+\frac{l}{2}V'(\phi_{(0)})\phi_{(1)}g_{(0)}^{ij}\partial_i\phi_{(0)}\partial_j\phi_{(0)}\\
&+lV(\phi_{(0)})\phi_{(1)}\frac{1}{\sqrt{-g_{(0)}}}\partial_i\left(\sqrt{-g_{(0)}}g_{(0)}^{ij}\partial_j\phi_{(0)}\right)\\
&+\frac{l}{2}V(\phi_{(0)})\left(g_{(0)}^{ik}g_{(0)}^{jl}g_{(1)kl}-\frac{1}{2}g_{(0)}^{kl}g_{(1)kl}g_{(0)}^{ij}\right)\partial_i\phi_{(0)}\partial_j\phi_{(0)}\\
&-\frac{l}{2}\left(\frac{1}{2}g_{(0)}^{ij}g_{(2)ij}-\frac{1}{4}g_{(0)}^{ij}g_{(0)}^{kl}g_{(1)ik}g_{(1)jl}+\frac{1}{8}(g_{(0)}^{ij}g_{(1)ij})^2\right)\Phi(\phi_{(0)})\\
&-\frac{l}{2}\left(\Phi'(\phi_{(0)})\phi_{(2)}+\frac{1}{2}\Phi''(\phi_{(0)})\phi_{(1)}^2+\frac{1}{2}g_{(0)}^{kl}g_{(1)kl}\Phi'(\phi_{(0)})\phi_{(1)}\right)\Big]
\end{aligned}
$$

We can get the anomaly (in CFT) in terms of $g_{(0)ij}$ and $\phi_{(0)}$ which are the boundary values by using the equations of motion.

$$
T = -\frac{1}{8\pi G}\left[h_1 R_{(0)}^2 + h_2 R_{(0)ij}R_{(0)}^{ij} + \cdots\right]
$$

In the following, we choose $l=1$, denote $\Phi(\phi_{(0)})$ by Φ and abbreviate the index (0) for the simplicity.

Potential.

Here

the coefficient of $R^2_{(0)}$

$$h_1 = \Big[3\ \Big\{(24-10\ \Phi)\ \Phi'^6$$
$$+(62208+22464\ \Phi+2196\ \Phi^2+72\ \Phi^3+\Phi^4)\ \Phi''\ (\Phi''+8\ V)^2$$
$$+2\ \Phi'^4\ \Big\{(108+162\ \Phi+7\ \Phi^2)\ \Phi''+72\ (-8+14\ \Phi+\Phi^2)\ V\Big\}$$
$$-2\ \Phi'^2\ \Big\{(6912+2736\ \Phi+192\ \Phi^2+\Phi^3)\ \Phi''^2$$
$$+4\ (11232+6156\ \Phi+552\ \Phi^2+13\ \Phi^3)\ \Phi''\ V$$
$$+32\ (-2592+468\ \Phi+96\ \Phi^2+5\ \Phi^3)\ V^2\Big\}$$
$$-3\ (-24+\Phi)\ (6+\Phi)^2\ \Phi'^3\ (\Phi'''+8\ V')\Big\}\Big]\ /$$
$$\Big[16\ (6+\Phi)^2\ \Big\{-2\ \Phi'^2+(24+\Phi)\ \Phi''\Big\}\ \Big\{-2\ \Phi'^2$$
$$+(18+\Phi)\ (\Phi''+8\ V)\Big\}^2\Big]$$

the coefficient of $R^{ij}_{(0)}R_{ij(0)}$

$$h_2 = -\frac{3\ \{(12-5\ \Phi)\ \Phi'^2+(288+72\ \Phi+\Phi^2)\ \Phi''\}}{8\ (6+\Phi)^2\ \{-2\ \Phi'^2+(24+\Phi)\ \Phi''\}}$$

In the limit of $\Phi \to 0$, we obtain

$$h_1 \ \to\ \frac{3\cdot 62208\Phi''(8V)^2}{16\cdot 6^2\cdot 24\cdot 18\Phi''(8V)^2} = \frac{1}{24}$$
$$h_2 \ \to\ -\frac{3\cdot 288\Phi''}{8\cdot 6^2\cdot 24\Phi''} = -\frac{1}{8}$$

→ $\mathcal{N}=4$ SYM Conformal Anomaly

- Since we have two functions h_1 and h_2, there are two ways to define the candidate c-functions.

$$c_1 = \frac{24\pi h_1}{G},\quad c_2 = -\frac{8\pi h_2}{G}\quad > 0$$

If we redefine the potential

$\mathcal{V}(\phi) \equiv \underbrace{4\lambda^2}_{\text{Cosmological constant}} + \Phi(\phi)$, then

$4\lambda^2 = \dfrac{d(d-1)}{\ell^2}$

$$l = \left(\frac{12}{\mathcal{V}(0)}\right)^{\frac{1}{2}}$$

We can restore ℓ by changing.

$$h_1 \to l^3 h_1, \quad h_2 \to l^3 h_2,$$
$$\phi' \to l\phi', \quad \phi'' \to l^2\phi'', \quad \phi''' \to l^3\phi''' .$$

Then in the limit of $\Phi \to 0$, one gets

$$c_1 \,, \quad c_2 \to \frac{\pi}{G}\left(\frac{12}{\mathcal{V}(0)}\right)^{\frac{3}{2}}$$

which agrees with the proposal of the previous work in the limit.

* L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni hep-th/9810126 ; JHEP 9812 (1998) 022.

- In the asymptotically AdS region, we need to require $\underset{(0)}{\Phi \to \text{constant}}$ and $\Phi' \to 0$ when $\underbrace{\rho \to 0}_{\text{boundary}}$.

* As a trial, if we put $\Phi' = 0$, we obtain

· $d = 2$

$$c = \frac{3}{2G}\left[\frac{l}{2} + \frac{3}{2l}\frac{1}{\Phi + 3}\right]$$

· $d = 4$

$$c_1 = \frac{2\pi}{3G}\frac{62208 + 22464\Phi + 2196\Phi^2 + 72\Phi^3 + \Phi^4}{(6+\Phi)^2(24+\Phi)(18+\Phi)}$$
$$c_2 = \frac{3\pi}{G}\frac{288 + 72\Phi + \Phi^2}{(6+\Phi)^2(24+\Phi)}$$

◆ We can now check the monotonicity in the c-functions. For this purpose, we consider some examples.

★ D. Freedman, S. Gubser, K. Pilch and N.P. Warner, hep-th/9906194.

$$4\lambda^2 + \Phi_{\rm FGPW}(\phi) = 4\left(\exp\left[\left(\frac{4\phi}{\sqrt{6}}\right)\right] + 2\exp\left[-\left(\frac{2\phi}{\sqrt{6}}\right)\right]\right)$$
$$4\lambda^2 + \Phi_{\rm GPPZ}(\phi) = \frac{3}{2}\left(3 + \left(\cosh\left[\left(\frac{2\phi}{\sqrt{3}}\right)\right]\right)^2 + 4\cosh\left[\left(\frac{2\phi}{\sqrt{3}}\right)\right]\right)$$

* L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, hep-th/9909047.

★ Properties of the Φ

- Φ is monotonicaly increasing function of the absolute value $|\phi|$. ← dilaton

- ϕ is the monotonically decreasing function of the energy scale $U = \rho^{-\frac{1}{2}}$ and $\phi = 0$ at the UV limit corresponding to the boundary.

- Φ have a minimum $\Phi = 0$ at $\phi = 0$.

the equation of motion (classical properties)

$$\frac{d(\ln c_1)}{d\Phi} = -\frac{18\,(622080 + 383616\Phi + 64296\Phi^2 + 4548\Phi^3 + 130\Phi^4 + \Phi^5)}{(6+\Phi)(18+\Phi)(24+\Phi)(62208 + 22464\Phi + 2196\Phi^2 + 72\Phi^3 + \Phi^4)} \le 0$$

$$\frac{d(\ln c_2)}{d\Phi} = -\frac{5184 + 2304\Phi + 138\Phi^2 + \Phi^3}{(6+\Phi)(24+\Phi)(288 + 72\Phi + \Phi^2)} < 0\,.$$

$$\frac{dc}{dU} = \underbrace{\frac{dc}{d\Phi}}_{<0}\,\underbrace{\boxed{\frac{d\Phi}{d\phi}}}_{\geq 0}\,\underbrace{\boxed{\frac{d\phi}{dU}}}_{<0} > 0$$

→ Therefore the c-function c_1 and c_2 (also c in 2-dimension) are monotonically increasing functions of the energy scale U.

4. Conclusions and discussions.

Conformal anomaly of boundary QFT is calculated from 5-dim. and 3-dim Super gravity with arbitrary dilatonic potential with the use of AdS/CFT correspondence

→ We suggested the candidate c-function for 2-dim. and 4-dim. boundary QFT. It is shown that such proposal gives monotonic and positive c-function for few examples of dilatonic potential.

· Discussions.

★ We can consider large number of scalars and construct the corresponding anomaly from the bulk side.
(we considered in hep-th/0005197.)

★ The generalization of the discussions for higher dimensions (ex) $d = 7, 9$) is possible.