Modular Invariant Partition Function for TypeII Strings on a Conifold

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Superstrings on a Singular Calabi-Yau

- Critical dimension $= 10 \Rightarrow$ Compactification
- Size, shape and other structures of the space compactified $\Rightarrow$ Massless scalars

How are the Moduli fixed?

Strings on a conifold

- The structure of the singularity is characterized locally. $\Rightarrow$ If the neighborhood of the singularity is zoomed in on, the system becomes independent of the detail of the original CY.

Less-moduli models

Models using abstract CFTs

Gepner models

Flat $D = 4$ spacetime $\Rightarrow$ Internal $c = 9$ CFT as a tensor product of minimal $N = 2$ SCFTs (Gepner)

$$c = \frac{3k}{k+2} \quad (k = 1, 2, \ldots)$$

- Corresponds to a special point of the moduli space of CY compactification.
- $N = 2$ minimal $\cong \underbrace{\frac{SU(2)}{U(1)}}_{\text{parafermions}} \times \underbrace{U(1)}_{\text{fermions}}$

"Noncompact" Gepner models

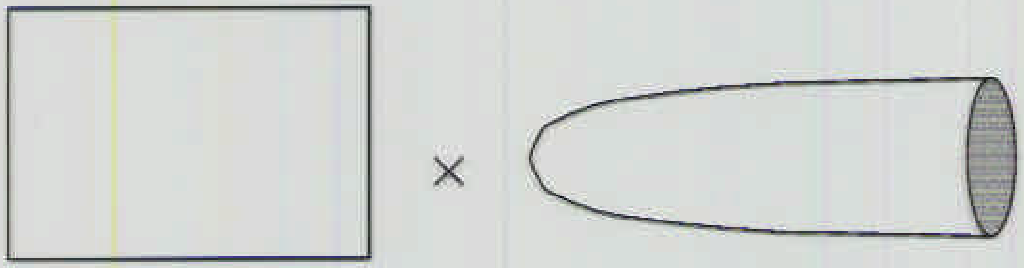
Replace $SU(2)/U(1)$ with $SL(2,\mathbb{R})/U(1)$.

$$c = \frac{3k}{k-2} \quad (k > 0)$$

- A single $SL(2,\mathbb{R})/U(1)$ CFT with $k = 3$ supplies the necessary central charge.
- Corresponds to a conifold.(Ghoshal,Vafa)

$SL(2,\mathbb{R})/U(1)$		Gauged $SL(2,\mathbb{R})$
Kazama-Suzuki	$\cong$	WZW model
model		2D black hole

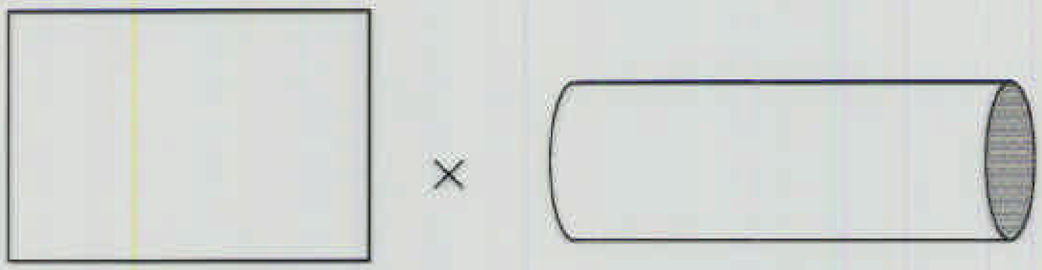
(Schnitzer; Witten)



4D Minkowski　　　2D black hole

$\Longrightarrow$ 6D critical string

c.f. Liouville theory



$\longrightarrow$ Linear dilaton

4D Minkowski　　　$S^1 \times \mathbb{R}, c = 1$Liouville

$\Longrightarrow$ 6D noncritical string

Modular Invariant Partition Function

$N = 2, c > 3$ characters

NS non-degenerate characters:

$$\mathrm{Tr} q^{L_0} = q^{h+\frac{1}{8}} \frac{\vartheta_3(0|\tau)}{\eta^3(\tau)}.$$

Monomial factor $q^{h+\frac{1}{8}}$ makes the modular behavior awful .

Idea 1. Consider (a countable set of) infinitely many primary fields so that the sum of their $q^{h+1/8}$ factors form a certain theta function !

Numbers of eta and theta functions do not balance.

$\Rightarrow \sqrt{\tau}$ factors do not cancel in the modular S transformation.

Idea 2. Assume that the "internal" $N = 2$ CFT has a degree of freedom of the center-of-mass motion along the "cigar"!

Agrees with the picture of CFTs on singular Calabi-Yau spaces.(Witten;Ooguri,Vafa)

Theta identities

In type II string theories, the key equation was Jacobi's abstruse identity:

$$\vartheta_3^4(0|\tau) - \vartheta_4^4(0|\tau) - \vartheta_2^4(0|\tau) = 0.$$

Any such nice theta identities?

$$\Theta_{1,1}(\tau,0)\left(\vartheta_3^2(0|\tau) + \vartheta_4^2(0|\tau)\right) - \Theta_{0,1}(\tau,0)\,\vartheta_2^2(0|\tau) = 0$$
$$\equiv \Lambda_1(\tau).$$

(Bilal,Gervais)

$$\Theta_{0,1}(\tau,0)\left(\vartheta_3^2(0|\tau) - \vartheta_4^2(0|\tau)\right) - \Theta_{1,1}(\tau,0)\,\vartheta_2^2(0|\tau) = 0$$
$$\equiv \Lambda_2(\tau).$$

Modular S transform of $\Lambda_1(\tau)$

- Modular properties

$$\begin{aligned}
\Lambda_1(\tau+1) &= i\Lambda_1(\tau), \\
\Lambda_2(\tau+1) &= -\Lambda_2(\tau), \\
\Lambda_1(\tau) &= \frac{\exp(3\pi i/4)}{\sqrt{2}\tau^{3/2}}\left(-\Lambda_1(-1/\tau) + \Lambda_2(-1/\tau)\right), \\
\Lambda_2(\tau) &= \frac{\exp(3\pi i/4)}{\sqrt{2}\tau^{3/2}}\left(+\Lambda_1(-1/\tau) + \Lambda_2(-1/\tau)\right).
\end{aligned}$$

$\Rightarrow \left|\Lambda_1(\tau)/\eta^3(\tau)\right|^2 + \left|\Lambda_2(\tau)/\eta^3(\tau)\right|^2$ is modular invariant!

GSO projection

- NS sector

Keep, in both left and right sectors, only the odd fermion excited states if $\epsilon = \overline{\epsilon} = 0$, while even fermion excited states if $\epsilon = \overline{\epsilon} = 1/2$.

m and ϵ, both of which label the $U(1)$ charge, are separately GSO projected.

- R sector

States with $\epsilon = \overline{\epsilon}$ are similarly paired, but the left-right chirality may or may not be the same.

Same chirality $\Rightarrow$ IIB-like model

Opposite chirality $\Rightarrow$ IIA-like model

Total partition function

$$Z = \int \frac{d\tau d\overline{\tau}}{\mathrm{Im}\tau} (\mathrm{Im}\tau)^{-2} |\eta(\tau)|^{-4} (\mathrm{Im}\tau)^{-\frac{1}{2}} |\eta(\tau)|^{-2} \left[\left| \frac{\Lambda_1(\tau)}{\eta^3(\tau)} \right|^2 + \left| \frac{\Lambda_2(\tau)}{\eta^3(\tau)} \right|^2 \right].$$

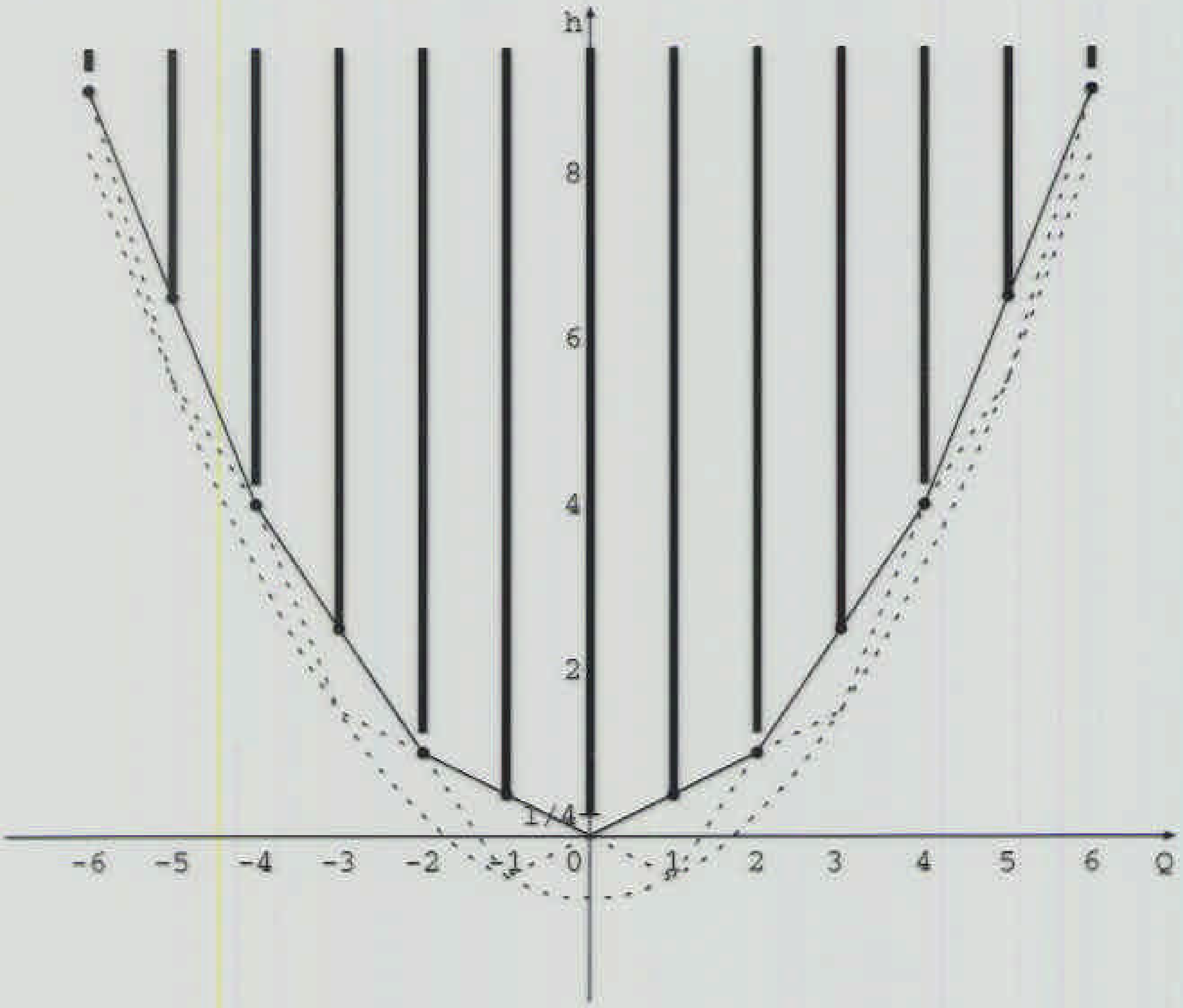


Figure 1: (Boucher,Friedan,Kent) The NS sector. The thick lines and dots show the $N = 2$ unitary representations used as the internal CFT. The former correspond to the propagating modes along the cigar, while the latter are the bound states .

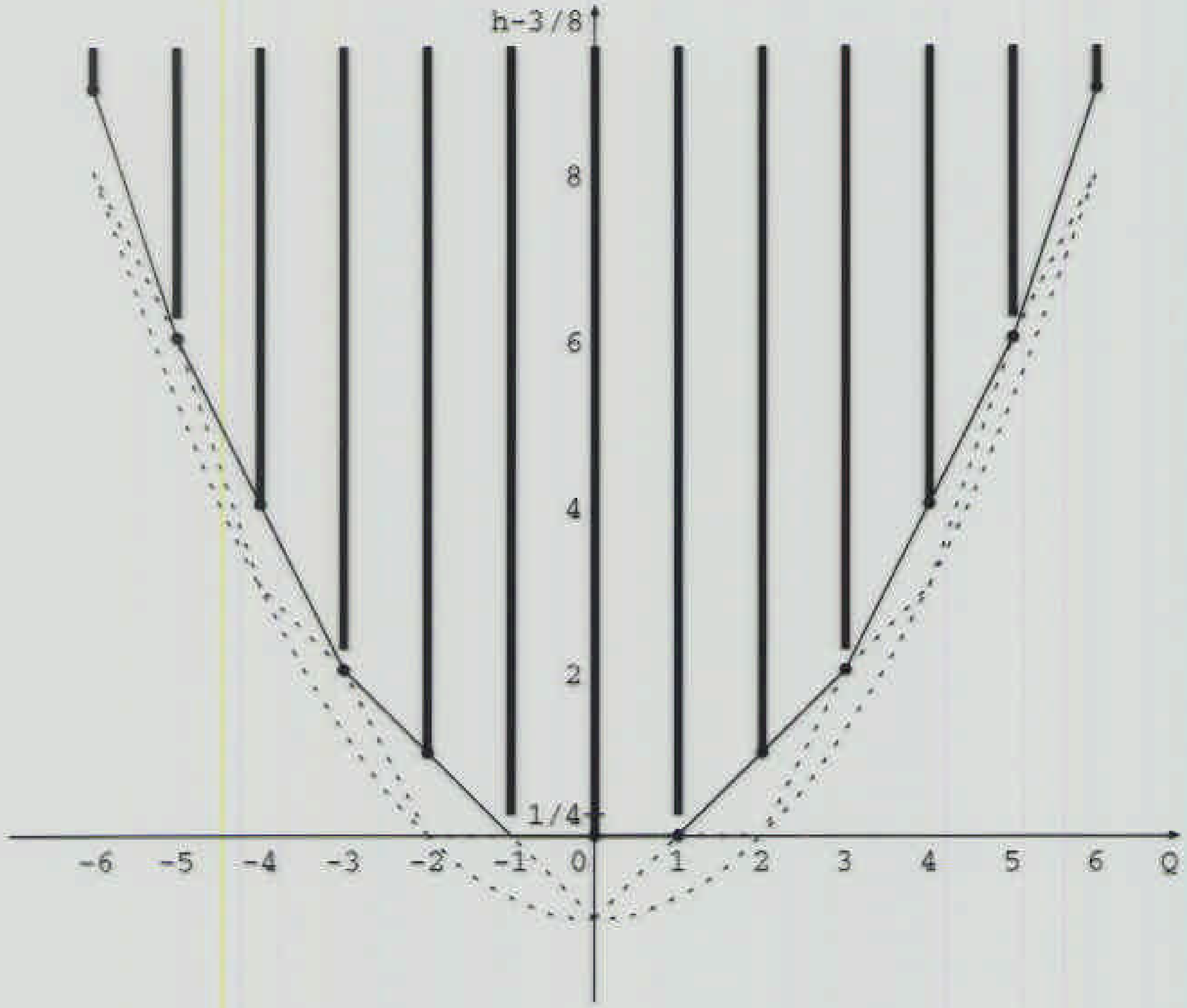


Figure 2: The R sector.

- Notable features of Z

1. *It is modular invariant.* Modular invariance has been achieved by integrating the "Liouville momenta" p. Consequently, spacetime, which was supposed to be four-dimensional, turns five-dimensional effectively; any "particle" in the four-dimensional world has a continuous spectrum.

2. *It is unitary and tachyon free.*

3. *It is spacetime supersymmetric.* Z is zero in reality. Since the vanishing $\Lambda_{1,2}(\tau)$ are a consequence of the ordinary spectral flow, the spacetime supercharge must be given by the usual construction. This is in contrast to the supercharges containing a contribution from the "longitudinal" boson. (Kutasov,Seiberg)

4. *It has a graviton.* The tensor product of the NS transverse fermion excitations yields a graviton, a dilaton and an anti-symmetric two-form field. They survive the GSO projection. Those fields are massive in the sense that they have $L_0 = \overline{L_0} = 1/4$ even when $p = \overline{p} = 0$.
   Identified as perturbative mass due to the negative cosmological constant.
   $\Rightarrow$ Gauged supergravity

5. *It contains bound states in the spectrum.* The partition function has a contribution from the representations made out of the discrete series of $SL(2, \mathbb{R})$. They do not have a momentum along the cigar and are regarded as the bound states.
   (Eguchi, Taormina; Dijkgraaf, Verlinde, Verlinde)

Mass Spectra

	IIB-like	IIA-like
$\|\Lambda_1\|^2$ $(L_0=\overline{L}_0=0)$	4D $N=2$ $U(1)$ vector +hypermultiplet	4D $N=2$ $U(1)$ vector +hypermultiplet
$\|\Lambda_2\|^2$ $(L_0=\overline{L}_0=\frac{1}{4})$	Dimensional reduction of 6D $(2{,}0)$ graviton +tensor multiplet	Dimensional reduction of 6D $(1{,}1)$ graviton multiplet

$$(p = \overline{p} = 0)$$

- For either projection, the upper row coincides with the spectrum of 4D tensionless strings! (Hanany,Klebanov)
- The right colomn is the field content of 6D $N=4$ gauged supergravity! $(= S^1$ reduction of 7D $N=2$ gauged supergravity) (Townsend, van Nieuwenhuizen; Giani, Pernici, van Nieuwenhuizen; Romans)

Conclusion

- We have constructed a modular invariant partition function of superstrings on 4D Minkowski space $\times$ 2D black hole using the $N=2$, $c=9$ characters. Our model may be thought of as a modular invariant extension of Bilal-Gervais's model and describes type II strings on a conifold. It is unitary, tachyon free and has a continuous spectrum in the 4D sense.
- In the $\alpha' \to 0$ limit the cigar becomes very thin and can be replaced by a thin cylinder $\mathbb{R} \times S^1$ since one would need very high energy to see the effect of the sharp tip. In this case one is left with a 4D $N=2$ non-gravitational theory with a vector multiplet and a hypermultiplet.

An example of holography

(Aharony,Berkooz,Kutasov,Seiberg; Giveon,Kutasov,Pelc)

- 4D massless spectrum (with $p = 0$) coincides with that of the tensionless strings which arise on the 4D intersection of two M5-branes. Including the second lightest states, the field content is the same as that of 7D gauged supergravity, of which the near-horizon geometry of two interseting 5-branes is a supersymmetric solution. (Cowdall, Townsend)

$\Rightarrow$A dual description of type II theories on a conifold in terms of two intersecting NS5-branes (Bershadsky, Sadov, Vafa)

- Prospects
  - Three intersecting 5-branes — dual to a singular CY four-fold?
  - Heterotic string CFT on singular CYs?