NONCOMMUTATIVE AND ORDINARY SUPER YANG-MILLS

ON THE (D($p-2$), Dp) BOUND STATES

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CONTENTS

(References: hep-th/9910092, 0001213 with N. Ohta)

I. INTRODUCTION

$$
\begin{array}{ccc}
\text{Dp Branes} & \stackrel{B}{\Longrightarrow} & \text{(D(p}-\text{2), Dp) Bound States} \\
 & \Downarrow & \\
\text{OSYM} & \Longrightarrow & \text{NCSYM}
\end{array}
$$

To discuss

(i) the relation between D(p-2)-branes and Dp-branes;

(ii) the relation between OSYM and NCSYM.

II. BLACK Dp-BRANES WITH NS B FIELDS

The (D(p-2),Dp) Bound States:

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$$ds^2 = H^{-1/2}[-f dt^2 + dx_1^2 + \cdots + dx_{p-2}^2 + h(dx_{p-1}^2 + dx_p^2)] + H^{1/2}(f^{-1}dr^2 + r^2 d\Omega_{8-p}^2),$$

$$e^{2\phi} = g^2 H^{\frac{3-p}{2}} h, \quad B_{p-1,p} = \tan\theta H^{-1} h,$$

$$A^p_{012\cdots p} = g^{-1}(H^{-1} - 1) h \cos\theta \coth\alpha,$$

$$A^{p-2}_{012\cdots(p-2)} = g^{-1}(H^{-1} - 1)\sin\theta \coth\alpha,$$

$$H = 1 + \frac{r_0^{7-p}\sinh^2\alpha}{r^{7-p}}, \quad f = 1 - \left(\frac{r_0}{r}\right)^{7-p}, \quad h^{-1} = \cos^2\theta + H^{-1}\sin^2\theta.$$

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Two special cases:

(i) $\theta = \pi/2$, D(p-2)-brane with smeared two coordinates

(ii) θ =0, Dp-brane solution

Some quantities associated with the solution:

$$Q_p = \frac{1}{2\kappa^2}\int_{\Omega_{8-p}} *F_{p+2} = \frac{(7-p)\Omega_{8-p}\cos\theta}{2\kappa^2 g} r_0^{7-p}\sinh\alpha\cosh\alpha,$$
$$Q_{p-2} = \frac{1}{2\kappa^2}\int_{V_2\times\Omega_{8-p}} *F_p = \frac{(7-p)\Omega_{8-p}V_2\sin\theta}{2\kappa^2 g} r_0^{7-p}\sinh\alpha\cosh\alpha.$$

The D(p-2)-brane charge density on the worldvolume of Dp-brane

$$\tilde{Q}_{p-2} = \frac{Q_{p-2}}{V_2}$$

The relation of the B field and charges:

$$\tan\theta = \frac{\tilde{Q}_{p-2}}{Q_p} = \frac{Q_{p-2}}{V_2 Q_p} = \frac{1}{V_2}\frac{T_{p-2}}{T_p}\frac{N_{p-2}}{N_p}.$$

Thermodynamic quantities:

$$M = \frac{(8-p)\Omega_{8-p}V_p r_0^{7-p}}{2\kappa^2 g^2}\left(1 + \frac{7-p}{8-p}\sinh^2\alpha\right),$$
$$T = \frac{7-p}{4\pi r_0\cosh\alpha}, \quad S = \frac{4\pi\Omega_{8-p}V_p}{2\kappa^2 g^2} r_0^{8-p}\cosh\alpha. \tag{1}$$

which satisfy the first law of black hole thermodynamics:

$$dM = TdS + \mu_p dq_p + \mu_{p-2}dq_{p-2}$$
$$= TdS + \mu_p V_p T_p dN_p + \mu_{p-2}V_{p-2}T_{p-2}dN_{p-2},$$

$$\mu_p = \cos\theta\tanh\alpha/g, \quad \mu_{p-2} = \sin\theta\tanh\alpha/g.$$

Decoupling limit:

$$\alpha' \to 0: \quad \tan\theta = \frac{\tilde{b}}{\alpha'}, \quad x_{0,1,\cdots,p-2} = \tilde{x}_{0,1,\cdots,p-2}, \quad x_{p-1,p} = \frac{\alpha'}{\tilde{b}}\tilde{x}_{p-1,p},$$
$$r = \alpha' u, \quad r_0 = \alpha' u_0, \quad g = \alpha'^{(5-p)/2}\tilde{g},$$

where $\tilde{b}$, u, u_0, $\tilde{g}$, and $\tilde{x_\mu}$ kept fixed. The solution becomes

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$$ds^2 = \alpha'\left[\left(\frac{u}{R}\right)^{(7-p)/2}\left(-\tilde{f}dt^2 + d\tilde{x}_1^2 + \cdots + d\tilde{x}_{p-2}^2 + \tilde{h}(d\tilde{x}_{p-1}^2 + d\tilde{x}_p^2)\right) + \left(\frac{R}{u}\right)^{(7-p)/2}\left(\tilde{f}^{-1}du^2 + u^2 d\Omega_{8-p}^2\right)\right],$$
$$e^{2\phi} = \tilde{g}^2\tilde{b}^2\tilde{h}\left(\frac{R}{u}\right)^{(7-p)(3-p)/2}, \quad B_{p-1,p} = \frac{\alpha'}{\tilde{b}}\frac{(au)^{7-p}}{1+(au)^{7-p}},$$

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where

$$\tilde{f} = 1 - \left(\frac{u_0}{u}\right)^{7-p}, \quad \tilde{h} = \frac{1}{1+(au)^{7-p}}, \quad a^{7-p} = \tilde{b}^2/R^{7-p},$$
$$R^{7-p} = \frac{1}{2}(2\pi)^{6-p}\pi^{-(7-p)/2}\Gamma[(7-p)/2]\tilde{g}\tilde{b}N_p.$$

$$\left\{ \tan\theta = \frac{\tilde{b}}{\alpha'} = \frac{(2\pi)^2\tilde{b}^2}{\alpha'\tilde{V}_2}\frac{N_{p-2}}{N_p} \quad \Longrightarrow \quad \frac{N_{p-2}}{N_p} = \frac{\tilde{V}_2}{(2\pi)^2\tilde{b}} \right\}.$$

(i) $au << 1$;

(ii) $au >> 1$.

Thermodynamics (independent of a!):

$$E = \frac{(9-p)\Omega_{8-p}\tilde{V}_p}{2(2\pi)^7(\tilde{g}\tilde{b})^2}u_0^{7-p},$$
$$T = \frac{7-p}{4\pi}R^{-\frac{7-p}{2}}u_0^{\frac{5-p}{2}},$$
$$S = \frac{2\Omega_{8-p}\tilde{V}_p}{(2\pi)^6(\tilde{g}\tilde{b})^2}R^{\frac{7-p}{2}}u_0^{(9-p)/2}.$$

Here $\tilde{V}_p = V_{p-2}\tilde{V}_2$ is the spatial volume of the Dp-brane after taking the decoupling limit, and $\tilde{V}_2 = V_2\tilde{b}^2/\alpha'^2$ is the area of the torus. The free energy, defined as $F = E - TS$, of the thermal excitations:

$$\begin{aligned} F &= -\frac{(5-p)\Omega_{8-p}\tilde{V}_p}{2(2\pi)^7(\tilde{g}\tilde{b})^2}u_0^{7-p} \\ &= -\frac{\Omega_{8-p}V_{p-2}\tilde{V}_2}{(2\pi)^7\tilde{g}^2\tilde{b}^2}\frac{5-p}{2}\left(\frac{4\pi}{7-p}\right)^{\frac{2(7-p)}{5-p}}R^{\frac{(7-p)^2}{5-p}}T^{\frac{2(7-p)}{5-p}}, \end{aligned}$$

Question:

$$au \ll 1 \Longrightarrow \text{the case for OSYM}$$

$$au \gg 1 \Longrightarrow \quad ?$$

III. INFINITELY MANY DELOCALIZED D(p-2)-BRANES AND U(∞) SYM

When $au \gg 1$:

The decoupling solution $\Longrightarrow$ *nearhorizon geometry of* $D(p-2)$

with smeared coordinates x_{p-1} and x_p and zero B field.

Boundary condition of open string:

$$\partial_\sigma X^i + B^i_j \partial_\tau X^j = 0, \quad \delta X^a = 0.$$

D(p-2)-brane solution:

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$$ds^2 = H^{-1/2}[-f dt^2 + dx_1^2 + \cdots + dx_{p-2}^2 + H(dx_{p-1}^2 + dx_p^2)] + H^{1/2}(f^{-1} dr^2 + r^2 d\Omega_{8-p}),$$

$$e^{2\phi} = g^2 H^{(5-p)/2}, \quad A^{p-2}_{01\cdots(p-2)} = g^{-1}(H^{-1} - 1)\coth\alpha, \quad B_{p-1,p} = 0,$$

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Under the usual decoupling limit:

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$$ds^2 = \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(-\tilde{f} dt^2 + d\tilde{x}_1^2 \cdots + d\tilde{x}_{p-2}^2 + \frac{1}{(au)^{7-p}} (d\tilde{x}_{p-1}^2 + d\tilde{x}_p^2) \right) \right.$$
$$\left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(\tilde{f}^{-1} du^2 + u^2 d\Omega_{8-p}^2 \right) \right],$$
$$e^{2\phi} = \tilde{g}^2 \tilde{b}^{5-p} (au)^{(7-p)(p-5)/2}, \quad B_{p-1,p} = 0,$$

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where

$$R^{7-p} = \frac{1}{2} (2\pi)^{6-p} \pi^{-(7-p)/2} \Gamma[(7-p)/2] \tilde{g} \tilde{b} N_{p-2} \times \frac{(2\pi)^2 \tilde{b}}{\tilde{V}_2}.$$

$$\left(\frac{N_{p-2}}{N_p} = \frac{\tilde{V}_2}{(2\pi)^2 \tilde{b}} \right).$$

Under T-duality on $\tilde{x}_{p-1}$ and $\tilde{x}_p$:

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$$ds^2 = \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(-\tilde{f} dt^2 + d\tilde{x}_1^2 + \cdots + d\tilde{x}_{p-2}^2 + dx_{p-1}^2 + dx_p^2 \right) \right.$$
$$\left. + \left(\frac{R}{u}\right)^{(7-p)/2} \left(\tilde{f}^{-1} du^2 + u^2 d\Omega_{8-p}^2 \right) \right],$$
$$e^{2\phi} = \frac{(2\pi)^4 \tilde{g}^2 \tilde{b}^4}{\tilde{V}_2^2} \left(\frac{u}{R}\right)^{(7-p)(p-3)/2}, \quad \tilde{B}_{p-1,p} = 0.$$

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The dual torus with area:

$$\hat{V}_2 = (2\pi)^4 \tilde{b}^2 / \tilde{V}_2.$$

What is the result?

$\Longrightarrow U(N_{p-2})$ OSYM in p+1 Dimensions $[N_{p-2} = \frac{\tilde{V}_2}{(2\pi)^2 \tilde{b}} N_p]$

When $\tilde{V}_2 \to \infty$:

U(∞) OSYM in p-1 Dimensions: $g^2_{YM} = (2\pi)^{p-4} \tilde{g}$.

While NCSYM in p+1 Dimensions: $g^2_{YM} = (2\pi)^{p-2} \tilde{g}\tilde{b}$.

From the point of view of Dp-brane:

Define:

$$\rho = \frac{\tilde{V}_2}{(2\pi)^2 \alpha'} \left(\tilde{B}_{p-1,p} + i\sqrt{G_{(p-1)(p-1)} G_{pp}} \right).$$

The relevant T-duality transformation is given by the $SL(2,Z)$ transformation

$$\rho \to \hat{\rho} = \frac{a\rho + b}{c\rho + d},$$

where $ad - bc = 1$. Acting this transformation to the decoupling solution yields

$$ds^2 = \alpha' \left[\left(\frac{u}{R}\right)^{(7-p)/2} \left(-\tilde{f} dt^2 + d\tilde{x}_1^2 + \cdots + d\tilde{x}_{p-2}^2 + dx_{p-1}^2 + dx_p^2\right) + \left(\frac{R}{u}\right)^{(7-p)/2} \left(\tilde{f}^{-1} du^2 + u^2 d\Omega_{8-p}^2\right)\right],$$

$$e^{2\phi} = \frac{(2\pi)^4 \tilde{g}^2 \tilde{b}^4}{\tilde{V}_2^2} \left(\frac{u}{R}\right)^{(7-p)(p-3)/2}, \quad \tilde{B}_{p-1,p} = \frac{\alpha'}{\tilde{b}},$$

for $c = -1$, $d = \tilde{V}_2/(2\pi)^2 \tilde{b} \equiv N_{p-2}/N_p$.

Here the dual torus has area

$$\hat{V}_2 = (2\pi)^4 \tilde{b}^2 / \tilde{V}_2$$

The Yang-Mills coupling:

$$\hat{g}_{\rm YM}^2 = \frac{(2\pi)^p \tilde{g} \tilde{b}^2}{\tilde{V}_2} \Longrightarrow (2\pi)^{p-4} \tilde{g} \ \ in\ p-1\ dimensions$$

The number of Dp-branes:

$$\frac{\tilde{V}_2}{(2\pi)^2 \tilde{b}} N_p = N_{p-2}.$$

However, the 't Hooft coupling constant:

$$\hat{\lambda} = 2 \hat{g}_{\rm YM}^2 N_p \frac{\tilde{V}_2}{(2\pi)^2 \tilde{b}} = \lambda.$$

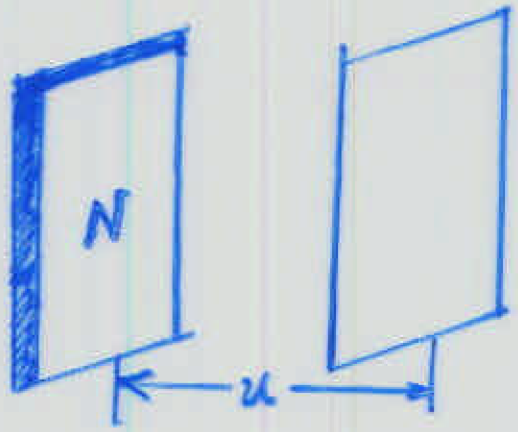
$\Longrightarrow$ Morita Equivalence!

IV. THERMODYNAMICS OF TWO D-BRANE PROBES

Motivation:

Single Center Configuration ⇔ SYM in Higgs Branch

Multi-center Configuration ⇔ SYM in Coulomb Branch



$U(N+1) \to U(N) \times U(1)$

u as a mass scale in SYM

(i) A (D(p-2),Dp) Bound State Probe

The starting point is the DBI action:

$$S_p = -T_p \int d^{p+1}x e^{-\phi}\sqrt{-\det(G_{ab} - \mathcal{F}_{ab})} - T_p \int A^p - T_p \int A^{p-2} \wedge \mathcal{F},$$

where $T_p = 1/(2\pi)^p \alpha'^{(p+1)/2}$ is the tension of Dp-brane.

$$S_p = -\frac{T_p V_p}{g\cos\theta}\int d\tau H^{-1}[\sqrt{f} - 1 + H_0 - H]$$

where

$$H = 1 + \frac{r_0^{7-p}\sinh^2\alpha}{r^{7-p}}, \quad H_0 = 1 + \frac{r_0^{7-p}\cosh\alpha\sinh\alpha}{r^{7-p}}.$$

In the extremal background $f = 1$:

The interaction potential vanishes!

(a) In the asymptotically flat limit:

$$U_p|_{r=r_0} = \frac{T_p V_p}{g\cos\theta}(1 - \tanh\alpha)$$

$\Longrightarrow$ This process satisfies the first law of thermodynamics and the potential converts into heat energy of the source: $U_3|_{r=r_0} = TdS$.

$$dM = TdS + \mu_p T_p V_p dN_p + \mu_{p-2} T_{p-2} V_{p-2} dN_{p-2}$$

with $dM = m_p$, $dN_p = 1$, and $dN_{p-2} = \tan\theta V_p T_p dN_p/(V_{p-2}T_{p-2})$.

(b) In the decoupling limit:

$$F_p = \frac{\tilde{V}_p}{(2\pi)^p \bar{g}\bar{b}}\left(\frac{u}{R}\right)^{7-p}\left[\sqrt{\tilde{f}} - 1 + \frac{u_0^{7-p}}{2u^{7-p}}\right]$$

$\Longrightarrow$ The same as that of the case without B field !

At the horizon:

$$F_p|_{u=u_0} = \frac{dF}{dN_p}\delta N_p, \quad \delta N_p = 1$$

$$\Longrightarrow F_p|_{u=u_0} \approx F(N_p+1) - F(N_p)$$

Dp-branes at the horizon!

(ii) A D(p-2)-brane Probe

The action is:

$$S_{p-2} = -T_{p-2}\int d^{p-1}xe^{-\phi}\sqrt{-\det G_{ab}} - T_{p-2}\int A^{p-2},$$

which reduces to

$$S_{p-2} = -\frac{T_{p-2}V_{p-2}}{g}\int d\tau H^{-1}\left[H^{1/2}h^{-1/2}\sqrt{f} - (1-H_0)\sin\theta - H\right].$$

(a) In the asymptotically flat limit:

$$U_{p-2}|_{r=r_0} = \frac{T_{p-2}V_{p-2}}{g}(1-\sin\theta\tanh\alpha)$$

satisfying the first law

$$dM = TdS + \mu_p V_p T_p dN_p + \mu_{p-2}V_{p-2}T_{p-2}dN_{p-2}$$

with $dN_p = 0$, $dN_{p-2} = 1$ and $dM = m_{p-2}$.

(b) In the decoupling limit:

$$F_{p-2} = -\frac{V_{p-2}}{(2\pi)^{p-2}\bar{g}}\left(\frac{u}{R}\right)^{7-p}\left[\sqrt{\frac{1+(au)^{7-p}}{(au)^{7-p}}}\sqrt{\bar{f}} - 1 + \frac{u_0^{7-p}}{2u^{7-p}}\right]$$

When $au \gg 1$:

$$F_{p-2} = \frac{\delta N_{p-2}}{\delta N_p} F_p$$

with

$$\frac{\delta N_{p-2}}{\delta N_p} = \frac{\tilde{V}_2}{(2\pi)^2 \bar{b}}$$

Furthermore, at the horizon

$$F_{p-2}|_{u=u_0} = \frac{dF}{dN_{p-2}} \delta N_{p-2}, \quad \delta N_{p-2} = 1.$$

$$\Longrightarrow F_{p-2} \approx F(N_{p-2}+1) - F(N_{p-2})$$

D(p-2)-branes at the horizon as well!

V. SUMMARY

(1). Thermodynamics of black Dp-branes with B fields is the same as that without B fields; also the same as that of the black Dp-2)-branes with two smeared coordinates and zero B field; in the decoupling limit, one has $N_{p-2}/N_p = \tilde{V}_2/(2\pi)^2\bar{b}$.

(2). The free energy of a (D(p-2),Dp) bound state probe is the same as that of a Dp-brane probe in the background without B field; A D(p-2)-brane probe is found to be identified with a Dp-brane with B field provided $\delta N_{p-2}/\delta N_p = \tilde{V}_2/(2\pi)^2\bar{b}$.

$\Longrightarrow$ NCSYM $\approx$ OSYM at large N limit

(3). When $\tilde{V}_2 \to \infty$, there might be an equivalence between $U(\infty)$ OSYM in $p-1$ dimensions and a NCSYM in $p+1$ dimensions; their coupling constants $g^2_{\rm OYM} = (2\pi)^{p-4}\tilde{g}$ and $g^2_{\rm NCSYM} = (2\pi)^{p-2}\tilde{g}\bar{b}$.