L. Bertini

(Osaka 2000)

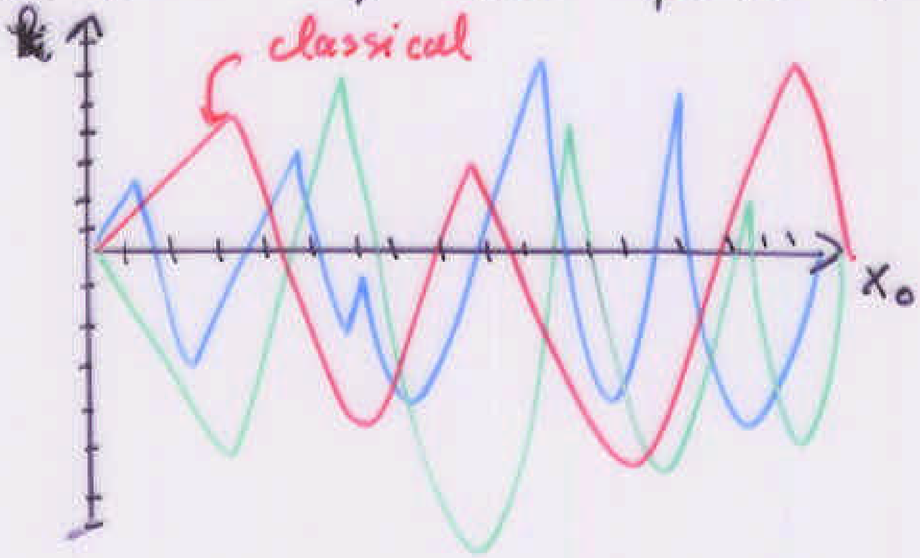
papers with D. Zwanziger

and A. Grassi + D. Zwanziger

(recent papers on hep-th + references therein)

Is our space $\overset{4+1}{\overset{\shortparallel}{\text{five}}}$ dimensional?

$t = x^5$ is invisible as the time that one uses for Monte-Carlo computations of the path integral.



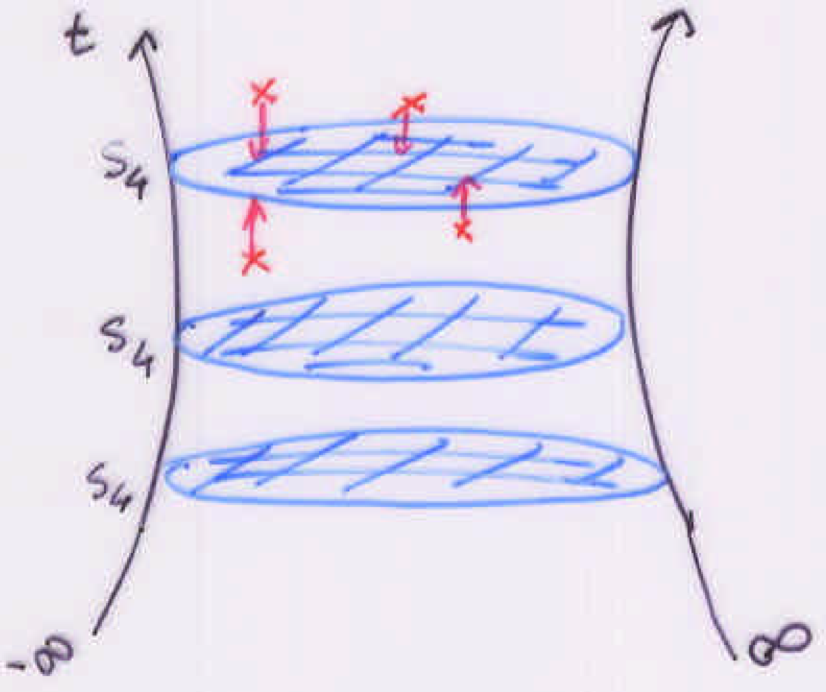
Goal: Do a "TQFT" that unifies t and the 4 space-time coordinates x^μ.

$$\int [d\varphi_x] \exp - S_4[\varphi(x)] \longleftrightarrow \int [d\varphi']_{x,t} \exp - \underset{TQFT_5}{I}[\varphi'(x,t)]$$

or

$$\begin{cases} \dot{\varphi}(x,t) = \dfrac{\delta S_4}{\delta \varphi(x,t)} + b(x,t) \\ \langle b(x,t), b(x',t') \rangle = \delta(t-t')\, \delta^4(x-x') \end{cases}$$

Introduce a fifth time t.



$$G_{QCD_4}(x_1,\dots x_n) = \lim_{t_1=t_2=\dots t_n} G_{TQFT_5}((x_1,t_1),\dots(x_n,t_n))$$

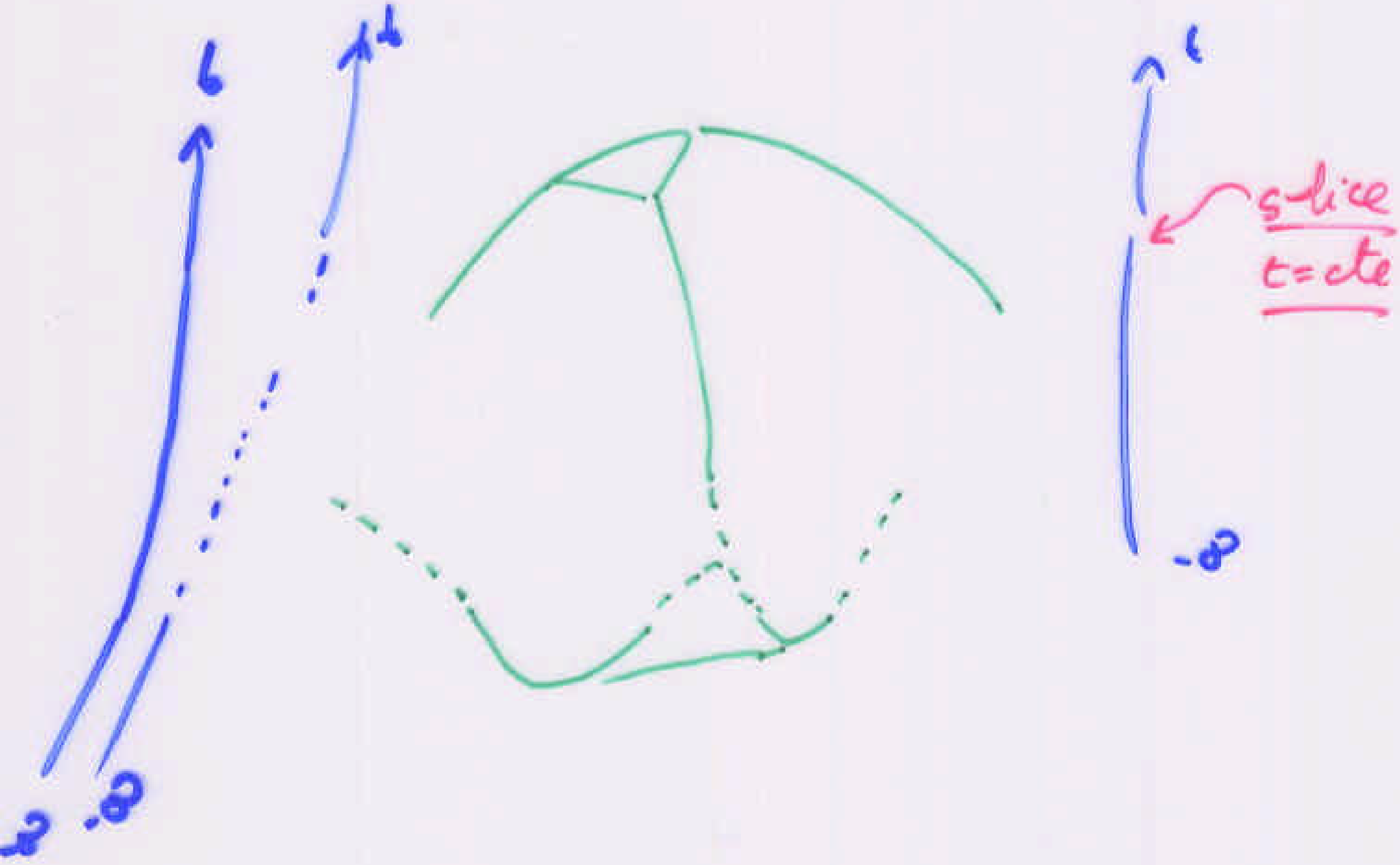
SUSY TERMS

$$I_{TQFT_5} = \int dt\, d^4x \left(F_{\mu 5}^2 + (D_\nu F^\nu_\mu)^2 + \dots\dots + b(a A_5 + \partial_\mu A_\mu)\right)$$

$$\begin{cases} \dim t = 2 = \dim^{-1} A_5 \\ \dim x^\mu = 1 = \dim^{-1} A_\mu \end{cases}$$ Renormalizable theory.

(equivalence for connected Green functions, <u>not</u> 1PI vertices)

$$\rightarrow \begin{cases} A(x,t),\ \Psi(x,t),\ \bar{\Psi}(x,t),\ b(x,t) \\ I_{TQFT_5}(A,\ \Psi,\ \bar{\Psi},\ b) \end{cases}$$



$$=$$

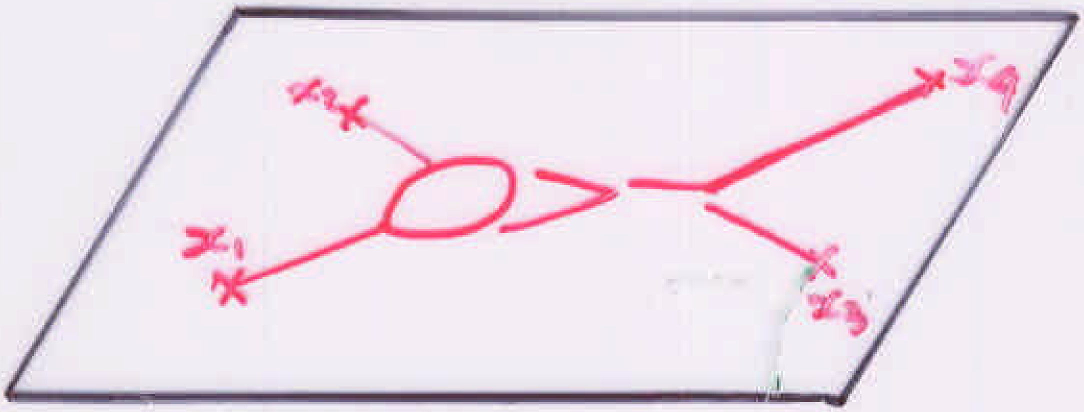
$$\sum_{i=1}^{4} \left\langle b(x_i,t)\, A(x_{i+1},t)\, A(x_{i+2},t)\, A(x_{i+3},t) \right\rangle^{I_{TQFT_5}}_{\text{amputated}}$$

$$\rightarrow \begin{cases} k_\mu^2 = m^2 \\ \omega = 0 \end{cases}$$

S- matrix

$$\begin{cases} A(x) \\ I = \int F_{\mu\nu}^2 \, dx \end{cases}$$

$$\sum_{i=1}^{4}$$



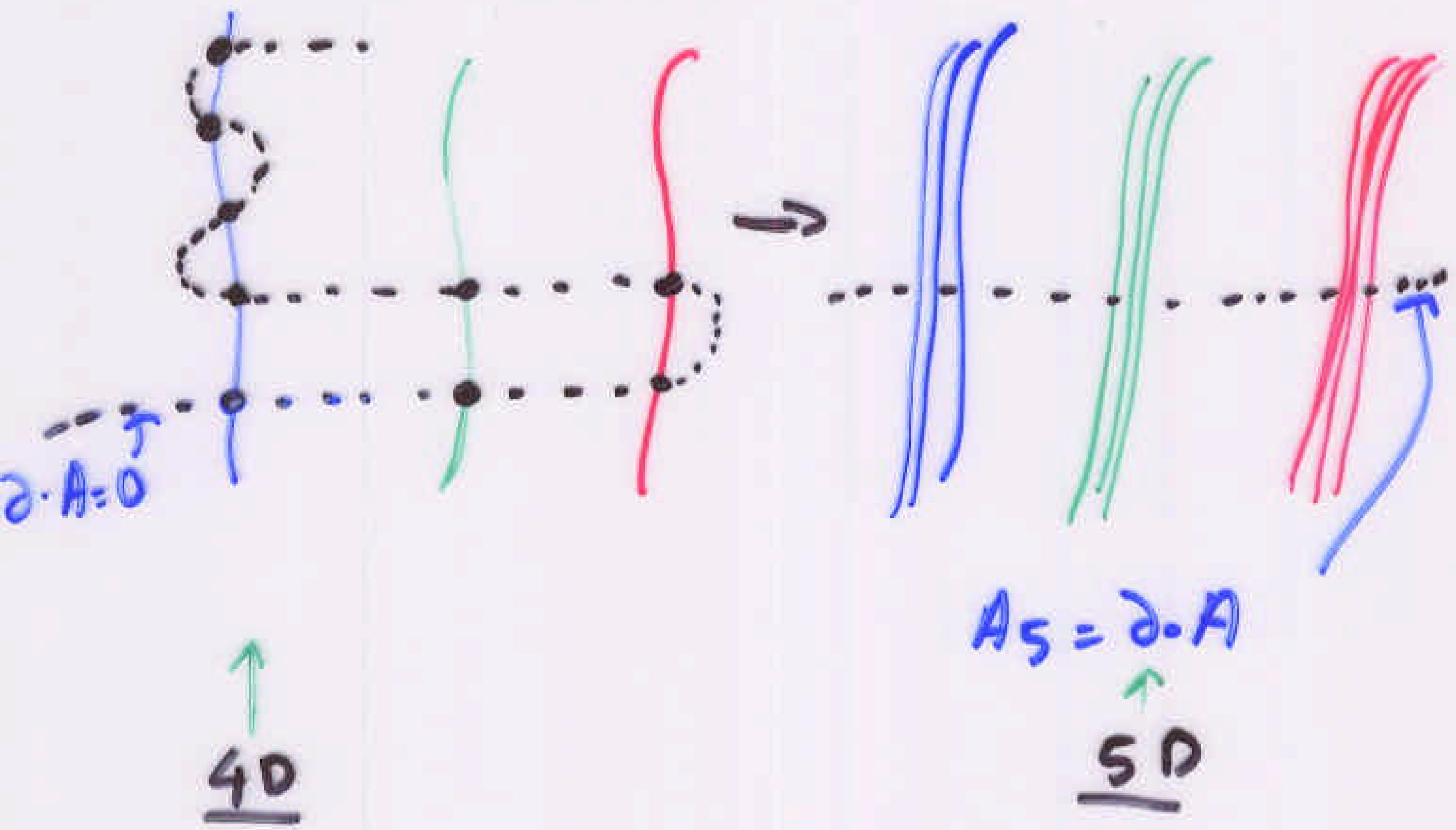
S- matrix in D= 4:

$$\langle A x_1, A x_2, A x_3, A x_4 \rangle^{F_{\mu\nu}^2}_{\text{amputated}}$$

mass - shell : $p_\mu^2 = m^2$

Justification:

(1) solves a mathematical question:
the Gribov question.



$$I_4=\int d^4x\left(F_{\mu\nu}^2+\text{gauge fixing}\right)\;\rightarrow\; I_5=\int dt\,d^4x\left(F_{\mu 5}^2+\left(D_\nu F^\nu_\mu\right)^2+\text{gauge-fixing}\right)$$

we (D.Z.+L.B.) call $x\rightarrow(x,t)$ adding
a "ghost-time".

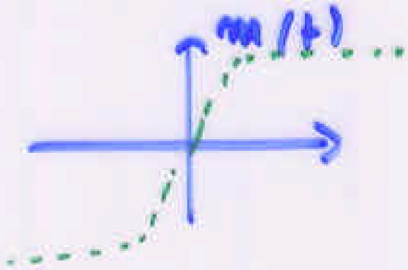
As natural as introducing the F.P. ghosts: $A\rightarrow(A,c,\bar{c},b)$

(2) Solves a physical question: put chiral fermions on a lattice.

D. Kaplan (1992)

$$q^\alpha(x) \to \left(q^\alpha_i(x),\ m_i\right) \qquad -\infty \le i \le \infty$$

$$\to \left(q^\alpha(x,t),\ m(t)\right)$$



t is a formal parameter.

However, from Neuberger:

$$\left(D_5 + \not{D} + m(t) \ldots\right) q(x,t) = 0$$

(3) Gauge Fields know about "topological ghosts" (since 1988).

$$A_\mu \to (A_\mu, \Psi_\mu, \bar{\Psi}_\mu, b_\mu, \dots)$$

The topological ghosts are invisible because one expresses the theory (tentatively) in $D = 4$.

In the 5D formulation they are necessary to express the invisibility of t.

The theory is just better defined.

t acts as regulator.
Gravity is interesting in this context.

General idea

Classically, one has:

$$I_4[\varphi_x] = \int dt \frac{d}{dt} I_4[\varphi_{x,t}] = \int dt\, \dot{\varphi}\, \frac{\delta I_4}{\delta t}$$

Then:

$$\int [d\varphi]_x \exp - I_4[\varphi]$$

$$= \int [d\varphi]_{x,t} [d\Psi][d\bar{\Psi}][db] \exp \cdot \int dt \Big(\dot{\varphi} \frac{\delta I_4}{\delta t}$$

$$+ \Big\{ Q, \bar{\Psi}\Big(\dot{\varphi} + \frac{\delta I_4}{\delta \varphi} + b\Big) \Big\} \Big)$$

In this way

$$\boxed{I_4 \to I_{TQFT_5} = \int \dot{\varphi}^2 + \frac{\delta S}{\delta \varphi} + \bar{\Psi}\Big(\dot{\Psi} + \frac{\delta^2 S}{\delta\varphi\delta\varphi}\Psi\Big)}$$

This is a $TQFT_5$ action:

$g_{55}, g_{\mu 5}$

$$\boxed{I_5 = \mathrm{Tr}\int dt \int d^4x \Big(\frac{d}{dt}(F_{\mu\nu})^2 + \cdot \Big\{ Q, \bar{\Psi}_\mu \big(D_\nu F^\nu_\mu + F_{\mu 5} + \frac{H_\mu}{2} \big) + \bar{\Phi}\,(\Psi_5 + D_\mu \Psi_\mu) + \bar{\Psi}_5 (a A_5 + \partial \cdot A) \Big\} \Big)}$$

$$\begin{cases} Q A_\alpha = \Psi_\alpha + D_\alpha c \\ Q \Psi_\alpha = D_\alpha \Phi - [c, \Psi] \\ Q \Phi = -[c, \Phi] \end{cases} \qquad \begin{cases} Q \bar{\Psi}_\mu = H_\mu \\ Q \bar{\Phi} = \bar{\eta} \end{cases}$$

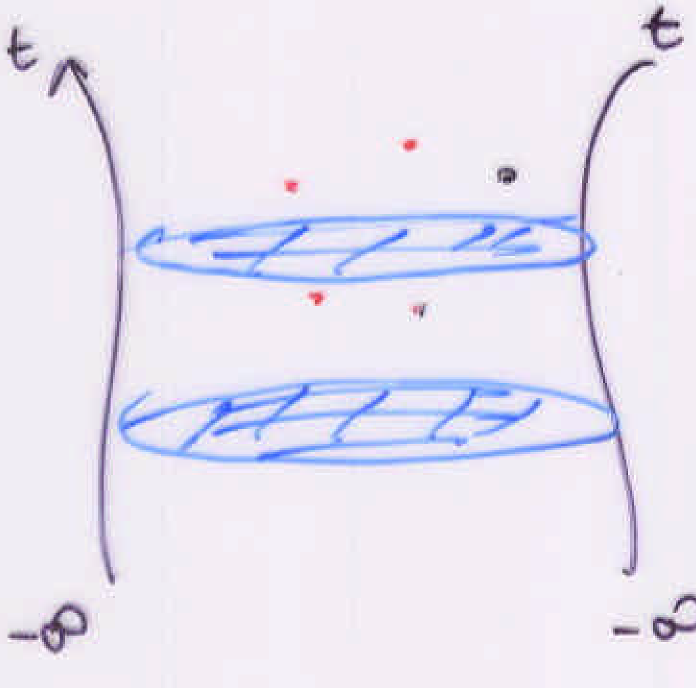
$$(A, \Psi, \bar{\Psi}, \phi, \bar{\phi}, \bar{\eta})$$

~ <u>susy multiplet</u>

It is obvious that:

(1) Gribov is not any more a complex question since:

$$\Leftrightarrow \begin{cases} a A_5 = \partial \cdot A & \text{(like Coulomb gauge)} \\ \bar{\Psi}_5 \left(a D_5 c + \partial D c \right) & \text{F.P. terms.} \end{cases}$$



perturbation theory is messy in 5D, but all ghost propagators are parabolic. This is conceptually much more simpler.

(fields vanish at $t=-\infty$)

Singer's argument that there cannot exist a globally well-defined gauge function doesn't apply (space is not compact)

(2) The theory is renormalizable:

$$\boxed{\begin{aligned} I_{TQFT5} = \int \Big\{ & s\Big(\bar\Psi_\mu \left(D_\nu F^\mu_\nu + F_{\mu 5} + H_\mu \right) \\ & \quad + \bar\Phi \left(\Psi_5 + D_\mu \Psi_\mu \right) \Big) \\ & + W s\left(\bar\mu \left(A_5 + \partial\cdot A \right)\right) \Big\} \det d^4x \end{aligned}}$$

$\dim A_5 = \dim^{-1} t = 2$ $\quad$ $\dim A_\mu = \dim^{-1} x^\mu = 1$

We have
$$\begin{cases} s A_\mu = D_\mu c + \Psi_\mu \\ s \Psi_\mu = - D_\mu \Phi - [c, \Psi_\mu] \\ \text{etc}\ldots \end{cases}$$

$$\boxed{\begin{aligned} &(s + W + d)(A + c + \lambda) + (A + c + \lambda)^2 \\ &\quad = F + \Psi + \Phi \end{aligned}}$$

$$\begin{cases} W A_\mu = D_\mu \lambda \\ W \lambda = -\frac{1}{2}[\lambda, \lambda] \\ W c = -\mu - [c, \lambda] \end{cases} \qquad s\lambda = \mu$$

$$\boxed{W^2 = s^2 = sW + Ws = 0} \quad I_R = s(\ldots) + Ws(\ldots)$$

$\to$ multiplicative renormalizability!

Spinors and anomalies

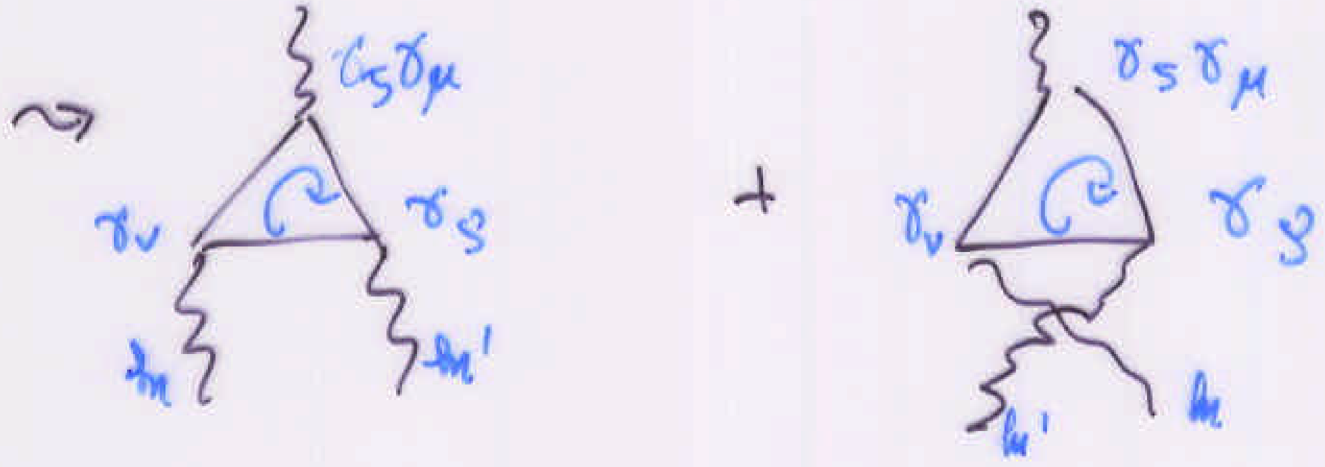
spinors propagate from $(x,t) \to (x',t')$

$$I_{TQFT} = \int_{D+1} \left\{ Q, \bar{\Psi}_q \left(D_5 q + \not{p} \left(\not{p} q + b_q \right) \right) \right\}$$

$$\rightsquigarrow \quad I_{TQFT} = \int dt\, dx \left\{ b_q^{\dagger} \left(D_5 q + \not{p} \not{p} q + \not{p} b_q \right) + \text{"susy"} \right\}$$

$$\rightsquigarrow \quad \begin{array}{c|cc} & b_q & q \\ \hline b_q & \not{p}, & D_5 + \not{p}^2 \\ q & -D_5 + \not{p}^2, & 0 \end{array} \quad \rightsquigarrow \quad \langle q, q \rangle = \frac{\not{p}}{\omega^2 + (p_\mu^2)^2}$$

$$J_\mu = \bar{q}\, \gamma^\mu \overset{L}{q} \quad \cdots \quad 1 + \gamma^5$$



$$\sim \int dp_1\, dp_2\, dp_3\, d\omega_1\, d\omega_2\, d\omega_3 \ \exp i(\omega_1 t_1 + \omega_2 t_2 + \omega_3 t_3)$$

$$\times \mathrm{Tr}\left(\gamma^5 \gamma^\mu \frac{\not{p}_3}{\omega_3^2 + (p_3^2)^2} \gamma^\nu \frac{\not{p}_2}{\omega_2^2 + (p_2^2)^2} \gamma^\rho \frac{\not{p}_3}{\omega_1^2 + (p_1^2)^2} + \leftrightarrow \right)$$

set $t_3 = 0$.

A) When $t_1 = t_2 = t_3 = 0$, use the Cauchy theorem $\frac{\not p}{\omega^2 + (p^2)^2} \sim \frac{\not p}{p^2}$ etc...
and one recovers the 4-D anomaly coefficient, with the subtility of the linear divergency.

B) Put $t_3 = 0$ and $t_1 \neq t_2$ non zero.
Each integral is convergent. Then

$$\boxed{\langle \partial_\mu J^{(5)}_\mu, J_\rho, J_\nu \rangle \sim \epsilon_{\rho\nu\alpha\beta} f^{\alpha} f^{\beta} \frac{|t_1| + |t_2|}{|t_1| + |t_2| + |t_2 - t_1|}}$$

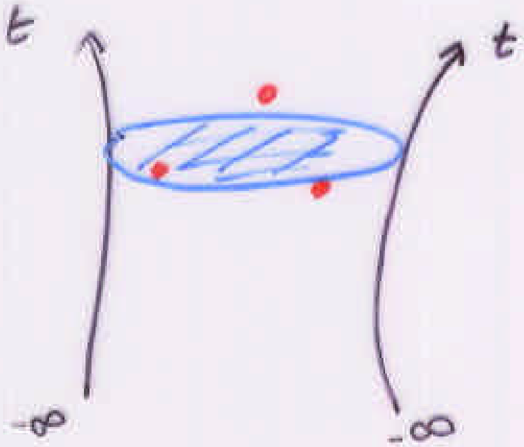
The limit $t_1, t_2 \to 0$ is not smooth.

- $(t_1 = t_2) \to 0 \quad \to 1$

- $t_1 = 0$, then $t_2 \to 0 \quad \to \frac{1}{2}$

Thus the coupling to a chiral Fermion breaks the convergence of the stochastic process.

Maybe it can be seen as a breaking of translation invariance in t.



$\rightarrow$

The case of gravity.

The bulk extension $(x^\mu) \to (x^\mu, t)$ allows one to express the idea that gravity is somehow a gauge theory for the Poincaré symmetry $ISO(D)$ (or $ISO(D-1,1)$), that is, for $SO(D+1)$, (or $SO(D,1)$, or $SO(D-1,2)$) up to a contraction.

This occurs in $D+1$ dimensions, and the bulk theory is not unitary, as a $TQFT_{D+1}$; only the equal time limit gives a unitary S-matrix.

For gauge theories:

$$\boxed{\dot{A}_\mu(x,t) = \frac{\delta}{\delta A_\mu(x,t)}\left(F_{\rho\sigma}\right)^2 + D_\mu A_5(x,t) + H_\mu(x,t)} \quad (1)$$

classical drift force ; diff force along gauge orbits ; gaussian noise

or $\rightsquigarrow$ $\boxed{F_{\mu 5} = D_\nu F^\nu_\mu + H_\mu}$ (1)

One can sum over all A_5, since gauge invariant observables are blind to the choice of A_5 -

(Zwanziger + Baulieu, Baulieu, Halpern)-

(1) Indicates the beginning of a (TQFT)$_5$-
(1) like self-duality equation)

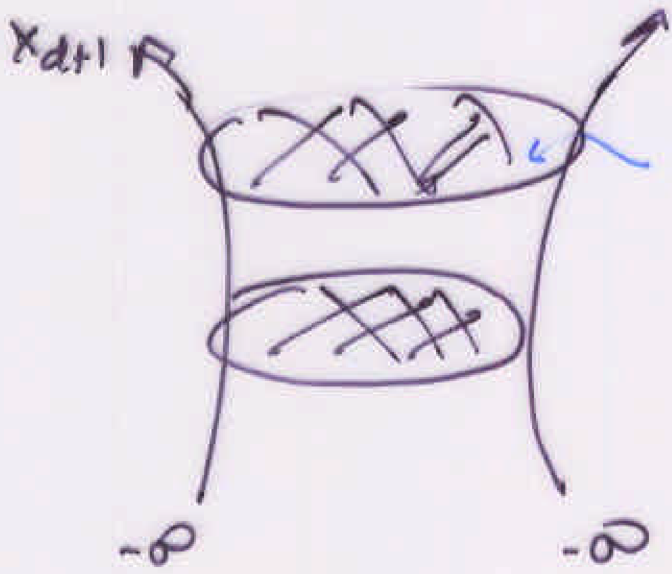
Complete by

$$\boxed{\Psi_5 = D_\mu \Psi_\mu} \quad (2)$$

$A_5 = \partial \cdot A$ (3) $\boxed{\cancel{A_5 = \partial_\mu A_\mu}}$ (3)

Gravity

$(\text{Topological gravity})_{D+1} \sim (\text{gravity})_D$



$S = \int_D (R\sqrt{g} + \gamma\sqrt{g})$

A priori: we need equations

- linear in $R_{\mu\nu}$
- which count for $\frac{d(d+1)}{2} - d$ conditions

(# $g_{\mu\nu}$ modulo reparametrisation

$$\rightarrow \boxed{\frac{\partial}{\partial t} g_{\mu\nu} = R_{\mu\nu} + \gamma g_{\mu\nu} + D_{\{\mu} A_{\nu\}} + b_{\mu\nu}}$$

g_{55}

$$\begin{cases} b_{\mu\nu} = 0 \equiv \text{Friedan equation for RG group} \\ \text{otherwise, with ghost time } A_\nu = g_{\nu 5}. \end{cases}$$

First order formalism

$$S = \int_{M_D} \epsilon_{ab\ldots cd}\; e^a \wedge \ldots \wedge e^b \wedge R^{cd}(\omega)$$

$$R = d\omega + \omega\omega \qquad T = de + \omega e$$

Then:

$$\begin{cases} \partial_{D+1} e^a_\mu = {}^*\left(\epsilon^a{}_{b\ldots cef}\; e^b \wedge \ldots \wedge e^c \wedge R^{ef}\right)_\mu + D_\mu e^a_{D+1} + \omega^{ab}_{D+1} e^b_\mu + H^a_\mu \\ \partial_{D+1} \omega^{ab}_\mu = {}^*\left(\epsilon^{ab}{}_{c\ldots ef}\; e^c \wedge \ldots \wedge e^e\, T^f\right)_\mu + D_\mu \omega^{ab}_{D+1} + H^{ab}_\mu \end{cases}$$

$$\rightsquigarrow \boxed{\begin{aligned} T^a_{[D+1,\mu]} &= {}^*\left(\epsilon^a{}_{b\ldots ef}\; e^b \wedge \ldots \wedge e^c \wedge R^{ef}\right)_\mu + H^a_\mu \\ R^{ab}_{[D+1,\mu]} &= {}^*\left(\epsilon^{ab}{}_{c\ldots ef}\; e^c \wedge \ldots \wedge e^e\, T^f\right)_\mu + H^{ab}_\mu \end{aligned}}$$

Set of self duality equations in $(D+1)$-space – $\rightarrow (TQFT)_{D+1}$.

This can be implemented as a (TQFT) on

$$\sim \boxed{\int d^{D+1}x \Big(T^a_{[D+1,\mu]} T^{a[D+1,\mu]} + T^a_{[\mu\nu]} T^{a[\mu\nu]} + R^{ab}_{[D+1,\mu]} R^{ab[D+1,\mu]} + R^{ab}_{[\mu\nu]} R^{ab[\mu\nu]} + \text{susy} \Big)}$$

This action is : $\int_{D+1} \{Q, \dots\} = \int_{D+1} \Big(F^M_{\alpha\beta} F^{M\alpha\beta} + \text{susy} \Big)$

The gauge symmetry of this action is ISO(D) or ISO(D-1,1) ; we have world indices α, β in D+1 dimensions.

$\longrightarrow$

stochastic gauge fixing means a clever condition on

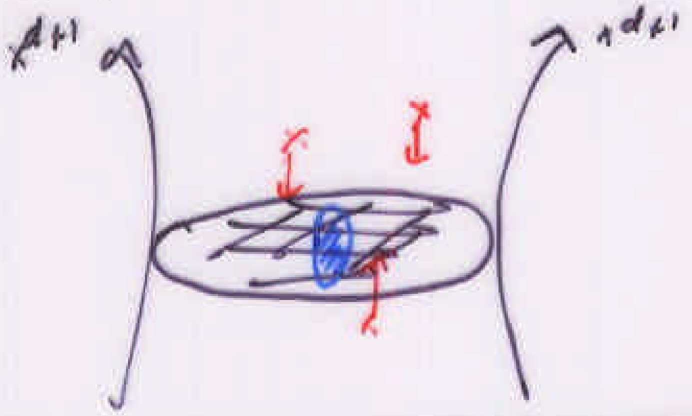
$$e^a_5 \text{ and } \omega^{ab}_5 .$$

We are thus seriously considering that:

$$\boxed{\int_{M_{D+1}} \mathrm{Tr}_{ISO(D)} \left(\hat{F} * \hat{F} \right) + \text{susy} + \text{gauge fixing},}$$

$\uparrow$ (non compact along the D+1 direction)

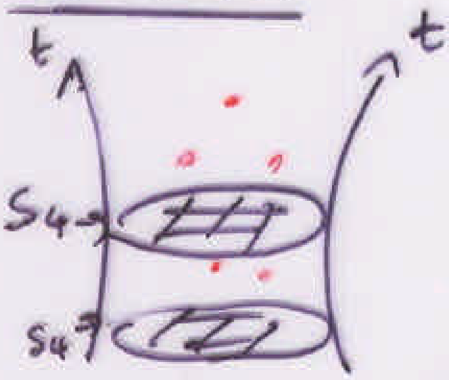
contains gravity as a theory defined in a slice!



A further step is to imagine that the
(a, [ab]) indices come from a contraction

$$SO(D+1),\ SO(D,1),\ SO(D-1,2) \longrightarrow ISO(D) \text{ or } ISO(D-1,1)$$

Conclusion



Using the trivial observation:

$$F_{\mu\nu}^2 = \int dt \frac{d}{dt} (F_{\mu\nu})^2 ,$$

$(QCD)_4$ has a $(TQFT)_5$ formulation

- Free of Gribov question.

- The relaxation to QCD_4 works provided no anomaly occurs (in a slice).

- Geometry is as beautiful in $D = 5$ as in $D = 4$. SUSY (twisted) enters the game.
- Chiral Fermions can aquire a mass $m(t)$ and look similar to the additional fermions of Kaplan + Neuberger.

[- Gravity in a slice ≡ Yang-Yang Mills in the bulk ??]