POWER CORRECTIONS IN SHORT-DISTANCE CROSS SECTIONS

(what is the field theory content?)

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- The classic:

OPE for inclusive 1-scale (e.g. DIS)

$$W(Q) = \sum_n |\langle i | J(0) | n \rangle|^2 \, \delta^4(p_n - Q)$$

$$= \int_x \langle i | J(0)\, J(x) | i \rangle \, e^{-iQ\cdot x}$$

$$\xrightarrow{\text{OPE}} \sum_K \frac{\langle i | \mathcal{O}_K | i \rangle}{Q^{d_K+4-2d_J}} \, C_K\!\left(\frac{Q}{\mu}\right)$$

connection to high orders in PT...

- What about

semi-inclusive 2-scale? (e.g. EW annih. near threshold)

$$\frac{d\sigma}{dm^2}(Q^2, m^2)_{a+b \to V(Q)+X}$$

$$m^2 \equiv 2p_X \cdot Q$$

$$1-\tau \equiv \frac{m^2}{Q^2}$$

phenomenology

Power corrections: $\left(\frac{\Lambda}{m}\right)^p, \left(\frac{\Lambda}{Q}\right)^{p'}$

- Factorization I:
  Effective H @ hard scattering
  R. Akhoury, M. Sotiropoulos, GS

$$\frac{d\sigma}{dm^2}(Q,m)_{a+b\to V+X} = \sum_n |\langle ab| J(0)|n\rangle|^2 \cdot \delta(m^2 - 2p_n\cdot Q)$$

$$= \sum_K \frac{C_K\left(\frac{Q}{\mu}\right)}{Q^{2d_K-2d_J}} \sum_n |\langle ab|\mathcal{O}_K(0)|n\rangle|^2 \cdot \delta(m^2 - 2p_n\cdot Q)$$

$$\mathcal{O}_K = \underbrace{\phi_a\phi_b}_{IS} \underbrace{\prod_{i\in K}\phi_i}_{FS}$$

- Factorization II: Incoherence of IS/FS

$$\frac{d\sigma}{dm^2}(Q,m) = \sum_K \frac{C_K}{Q^{2(d_K-d_J)}} \sum_{n_a n_b n_f} \delta(m^2 - 2(p_a+p_b+p_f)\cdot Q)$$

$$\cdot |\langle a|\phi_a|n_a\rangle|^2 |\langle b|\phi_b|n_b\rangle|^2$$

$$\cdot |\langle 0|\prod_{i\in K}\phi_i|n_f\rangle|^2$$

$$\xrightarrow[\text{+IR Safety}]{\text{PS}} (m^2)^{n_K} = [(1-\tau)Q^2]^{n_K}$$

- Hierarchy: n_K FS partons
- Effective theory for each n_K
- Fix n_K: study $\frac{\Lambda}{m}$ power corrections

...

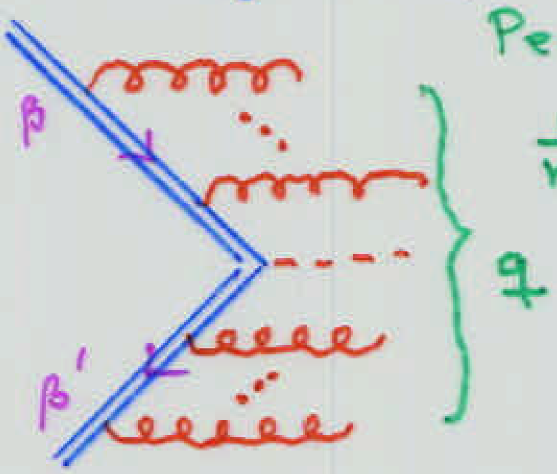
Transforms of the Eikonal Cross Section

FS fields

Field Theory for $n_R = 0$

Perturbative Resum

$\frac{1}{m^2}$ power correct.



$$\sigma_{ab}^{(\text{eik})}(q) = \int d^4x\; \mathrm{e}^{-iq\cdot x} \langle 0|W^\dagger(0)W(x)|0\rangle$$

$$W(0) = P\,\mathrm{e}^{ig\int_0^\infty d\lambda \beta\cdot A(\lambda\beta)}\;\left[P\,\mathrm{e}^{ig\int_0^\infty d\lambda \beta'\cdot A(\lambda\beta')}\right]^\dagger$$

$$\sigma_{ab}^{(\text{eik})}(N,\mathbf{b}) = \int dq^0 d^2\mathbf{q}\; \mathrm{e}^{-Nq_0}\; \mathrm{e}^{-i\mathbf{b}\cdot\mathbf{q}}\; \sigma_{ab}^{(\text{eik})}(q^0,\mathbf{q})$$

Energy Laplace; Transverse Momentum Fourier

$q_0 \simeq \frac{1}{2}(1-\tau)Q$

$m^2 = 2q_0 Q$

Large N – forces radiation to IR (threshold resummation)

Large $\mathbf{b}$ – low net transverse momentum (k_T-resummation)

q_T

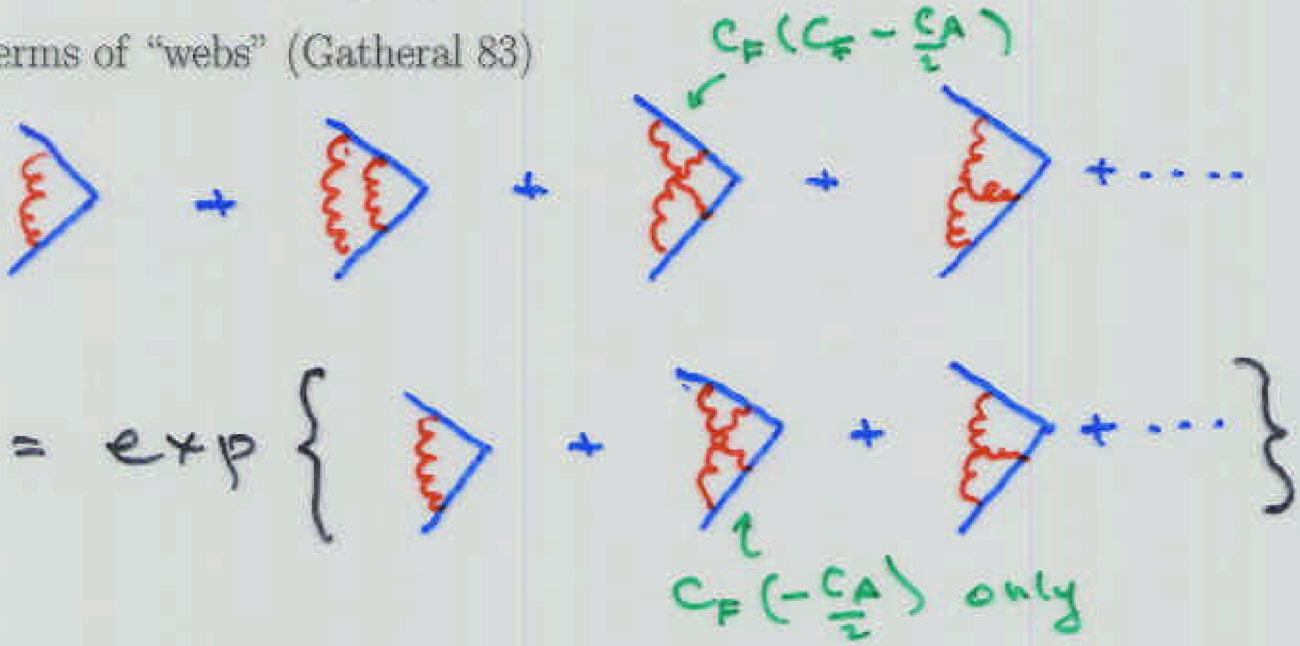
Exponentiation

$$\sigma_{ab}^{(\mathrm{eik})}(\beta,\beta',n,N,\mathbf{b}Q,\epsilon) \;=\; \exp\left\{2\int\frac{d^{4-2\epsilon}k}{\Omega_{1-2\epsilon}}\,\theta\left(\frac{Q}{\sqrt{2}}-k^{+}\right)\,\theta\left(\frac{Q}{\sqrt{2}}-k^{-}\right)\right.$$
$$\left.\times w_{ab}\left(k^2,\frac{k\cdot\beta k\cdot\beta'}{\beta\cdot\beta'},\mu^2,\alpha_s(\mu^2),\epsilon\right)\left(\mathrm{e}^{-N(k\cdot n/Q)+i\mathbf{k}\cdot\mathbf{b}}-1\right)\right\},$$

With normalization

$$\sigma_{ab}^{(\mathrm{eik})}(\beta,\beta',n,0,\mathbf{0},\epsilon)=1$$

In terms of "webs" (Gatheral 83)



Normalization of the webs:

$$w_{a\bar{a}}^{(1)(\mathrm{real})}(k) = \frac{2C_a\alpha_s}{\pi}\,\left(4\pi\mu^2 e^{-\gamma_E}\right)^\epsilon\,\frac{1}{k_T^2}\,\delta_+(k^2)\,.$$

At fixed k, webs are **boost invariant**

At *any* order web has **only one** overall collinear and IR divergence (k)

Overall renormalization:

$$\mu\frac{d}{d\mu}\,w_{ab}\left(k^2,\frac{k\cdot\beta k\cdot\beta'}{\beta\cdot\beta'},\mu^2,\alpha_s(\mu^2),\epsilon\right) = 0\,.$$

Use of boost invariance:

$$\begin{aligned}
&\sigma_{ab}^{(\mathrm{eik})}(\beta,\beta',n,N,\mathbf{b}Q,\epsilon)\\
&\quad = \exp\Big[\,2\int\frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}}\,\int_0^{Q^2-k_T^2} dk^2\,w_{ab}\left(k^2,k_T^2+k^2,\mu^2,\alpha_s(\mu^2),\epsilon\right)\\
&\qquad\times\left(\int_{(k_T^2+k^2)/\sqrt{2}Q}^{Q/\sqrt{2}}\frac{dk^+}{2k^+}\,e^{-N\sqrt{2}\left(\frac{k^+}{2Q}+\frac{k_T^2+k^2}{2Qk^+}\right)+i\mathbf{b}\cdot\mathbf{k}_T} - \ln\sqrt{\frac{Q^2}{k_T^2+k^2}}\right)\Big]\,,
\end{aligned}$$

Large-N limit:

$$
\begin{aligned}
&\sigma_{ab}^{(\mathrm{eik})}(\beta,\beta',n,N,\mathbf{b}Q,\epsilon) \\
&\quad = \exp\left\{ 2\int \frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}} \int_0^{Q^2-k_T^2} dk^2\, w_{ab}\left(k^2, k_T^2+k^2, \mu^2, \alpha_s(\mu^2), \epsilon\right) \right. \\
&\qquad \left. \times \left[\mathrm{e}^{-i\mathbf{b}\cdot\mathbf{k}_T} K_0\left(2N\sqrt{\frac{k_T^2+k^2}{Q^2}}\right) - \ln\sqrt{\frac{Q^2}{k_T^2+k^2}} \right] + \underline{\underline{\mathcal{O}\left(\mathrm{e}^{-N}\right)}} \right\}.
\end{aligned}
$$

Factorized eikonal cross section ($\hat{\sigma}_{ab}^{(\mathrm{eik})}$) – divide by moments of

$$
\tilde{\phi}_f^{(\mathrm{eik})}(N,\mu,\epsilon) = \exp\left[-\ln\bar{N} \int_0^{\mu^2} \frac{d\mu'^2}{\mu'^2} A_f\left(\alpha_s(\mu'^2)\right) \right],
$$

from P_{ff}

$$
\begin{aligned}
A_a^{(1)} &= C_a \\
A_a^{(2)} &= \frac{1}{2}C_a K \equiv \frac{1}{2}C_F\left[C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right) - \frac{10}{9}T_R N_f \right]
\end{aligned}
$$

The eikonal hard-scattering function

$$\hat{\sigma}_{ab}^{(\mathrm{eik})}(N,b,Q,\mu)$$
$$= \exp\Big[2\int \frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}}\Big\{\int_0^{Q^2-k_T^2} dk^2\, w_{ab}\left(k^2,k_T^2+k^2,\mu^2,\alpha_s(\mu^2),\epsilon\right)$$
$$\times\left[e^{-i\mathbf{b}\cdot\mathbf{k}_T}K_0\left(\frac{2Nk_T}{Q}\right)-\ln\left(\frac{k_T}{Q}\right)\right]+\frac{1}{(k_T^2)^{1-2\epsilon}}\ln\bar{N}\sum_{d=a,b}A_d\left(\alpha_s(k_T^2)\right)\Big\}\Big]$$
$$\times\exp\Big\{2\int\frac{d^{2-2\epsilon}k}{\Omega_{1-2\epsilon}}\int_0^{Q^2-k_T^2}dk^2\,w_{ab}\left(k^2,k_T^2+k^2,\mu^2,\alpha_s(\mu^2),\epsilon\right)$$
$$\times\left[e^{-i\mathbf{b}\cdot\mathbf{k}_T}\left\{K_0\left(2N\sqrt{\frac{k_T^2+k^2}{Q^2}}\right)-K_0\left(\frac{2Nk_T}{Q}\right)\right\}+\ln\left(\frac{\sqrt{k_T^2+k^2}}{k_T}\right)\right]\Big\}.$$

All $k_T^2, k^2 \to 0$ singularities cancel

Expand integrand in $\frac{k_T^2}{Q^2}, \frac{k^2}{Q^2}$

↳ structure of $\frac{1}{m^2}$ power correc.

Overall CO/IR behavior of web →

$$\int_0^{Q^2-k_T^2} dk^2\, w_{ab}\left(k^2,k_T^2+k^2,\mu^2,\alpha_s(\mu^2),\epsilon\right)$$
$$=\frac{A_a\left(\alpha_s(k_T^2)\right)+A_b\left(\alpha_s(k_T^2)\right)}{(k_T^2)^{1-2\epsilon}}+\mathcal{A}_{ab}\left(\alpha_s(k_T^2),k_T,Q\right),$$

↳ leading, NLL expansions

$$P_{aa} = \left[\frac{A(\alpha_s)}{1-x}\right]_+ + \cdots$$

Series expansion for K_0

$$K_0(z) = -\ln\frac{z}{2}\, I_0(z) + \sum_{k=0}^{\infty} \frac{z^{2k}}{2^{2k}(k!)^2}\,\psi(k+1)$$
$$I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$$

→ even powers in $\frac{N}{Q}$ only!

- full QCD (eikonal)
- all logs & constant terms
- full series in

$\left(\frac{1}{m^2}\right)^p \to \left(\frac{1}{(1-\tau)^2 Q^2}\right)^p$

$(bQ)^{2p}$

Contopanagos & GS
Beneke, Braun
Dokshitzer Marchesini Webber

→ Other applications
- event shapes (beyond naive universality
- relation to energy flow
- B decay
- 1PI...

→ Steps toward "theory" of power corrections