Chamseddine, Fröhlich, Grandjean Metric and Localized Higgs Coupling on Noncommutative Geometry

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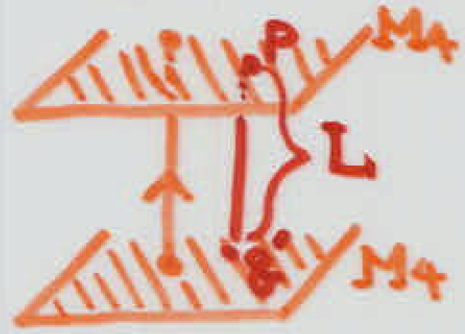
Field theory

— • Introduction • —

1990

A. Connes and J. Lott write the standard model in terms of the noncommutative geometry.

* Higgs boson = Gauge field corresponding to a finite difference



$M_4 \times Z_2$

* $\sqrt{d_s(P,q')^2 + L^2} \le d(P,q') \le d_s(P,q') + L$

where
$L = ((M^\dagger M))^{-\frac{1}{2}}$

* Distance M_4–M_4 $= \dfrac{1}{\text{Fermion Mass}}$

Can we define a gravitational theory in the language of the noncommutative geometry?

1995

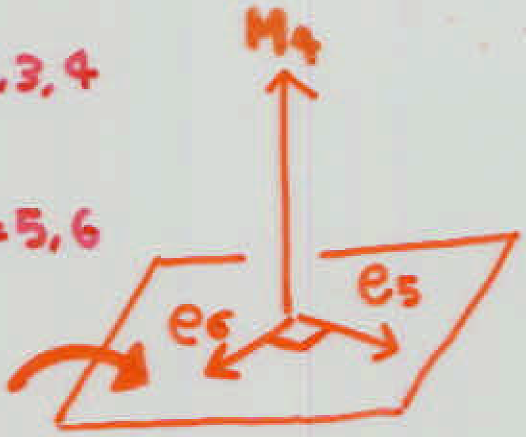
Chamseddine, Fröhlich, Grandjean (CFG) introduce the gravitational sector in the Connes and Lott formulation of the standard model.

CFG metric = Generators of $\underset{\text{1-form}}{\Omega^1_D(\mathcal{A})}$

$$E^a = \gamma^a \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 1 \end{pmatrix}, \quad a = 1,2,3,4$$

$$E^r = \gamma^5 \begin{pmatrix} \mathbb{O}_2 & k\otimes e_r \\ -k^*\otimes e_r^T & 0 \end{pmatrix}, \quad r = 5,6$$

k: family mixing



SU(2) doublet isospace

$$\mathcal{A} = \{ M_2(\mathbb{C}) \oplus \mathbb{C} \} \otimes C^\infty(M_4)$$

$$D = \left(\begin{array}{c:c} \not\nabla_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_3 & \gamma^5 \otimes \begin{pmatrix} \alpha(x) \\ \beta(x) \end{pmatrix} \otimes k \\ \hdashline \gamma^5 \otimes (\alpha(x), \beta(x)) \otimes k^* & \not\nabla_2 \otimes 1 \otimes \mathbb{1}_3 \end{array}\right)$$

$$\not\nabla_i = e_{i\,a}^{\mu}(x)\, \gamma^a (\partial_\mu + i\,\omega_{i\mu})$$

$V_1 = C^2$

$V_2 = C$

$$\mathcal{H} = L^2(S_1, dv_1) \oplus L^2(S_2, dv_2)$$

S_i = Dirac spinor bundle $\otimes V_i$

They also introduce Levi-Civita connections in $M_4 \times Z_2$

$$\nabla \mathbf{E}^A = -\Omega^A{}_B \otimes \mathbf{E}^B$$

$$\Omega^A{}_B = \begin{pmatrix} \gamma^\mu \omega_{1\mu}{}^A{}_B & k\gamma^5 e^{-\phi} \begin{pmatrix} \omega_1{}^A{}_B \\ \omega_2{}^A{}_B \end{pmatrix} \\ \overset{*}{k}\gamma^5 e^{-\phi} \left(\hat{\omega}_1{}^A{}_B \ \tilde{\omega}_2{}^A{}_B \right) & \gamma^\mu \omega_{2\mu}{}^A{}_B \end{pmatrix}$$

$A, B = 1, 2, 3, 4, 5, 6$

$\begin{pmatrix} \alpha(x) \\ \beta(x) \end{pmatrix} \xrightarrow[\text{C.C. of differential geometry}]{} \delta(x)$

⇩

- Unitarity conditions

$$d\langle E^A, E^B\rangle_D = -\Omega^A{}_d \langle E^d, E^B\rangle_D + \langle E^A, E^d\rangle_D (\Omega^A{}_d)^*$$

Symmetric metric

$$\langle E^A, E^B\rangle_D = -\frac{1}{2}\{\gamma(A), \gamma(B)\}_+ \check{E}^A \check{E}^B$$

(-Riemannian geometry-

metric covariant condition $\nabla g_{\alpha\beta} = 0$

$$g_{\alpha\beta} = \underbrace{\eta^{ab}}_{\text{symmetric}} e_{\alpha a} e_{\beta b}$$

)

- Weaker Torsion less condition

$$\mathrm{Tr}\, \mathbf{T}^A = 0$$

$$\mathbf{T}^A = d\mathbf{E}^A + \Omega^A{}_B \mathbf{E}^B$$

Finally.
$$\int d^4x \left\{ -\frac{1}{2}(3C_s + 4C_\ell) R + \alpha(\nabla_a \delta)^2 + \beta e^{-2\delta} \right\}$$

—• Our motivation •—

$\boxed{\text{In future}}$

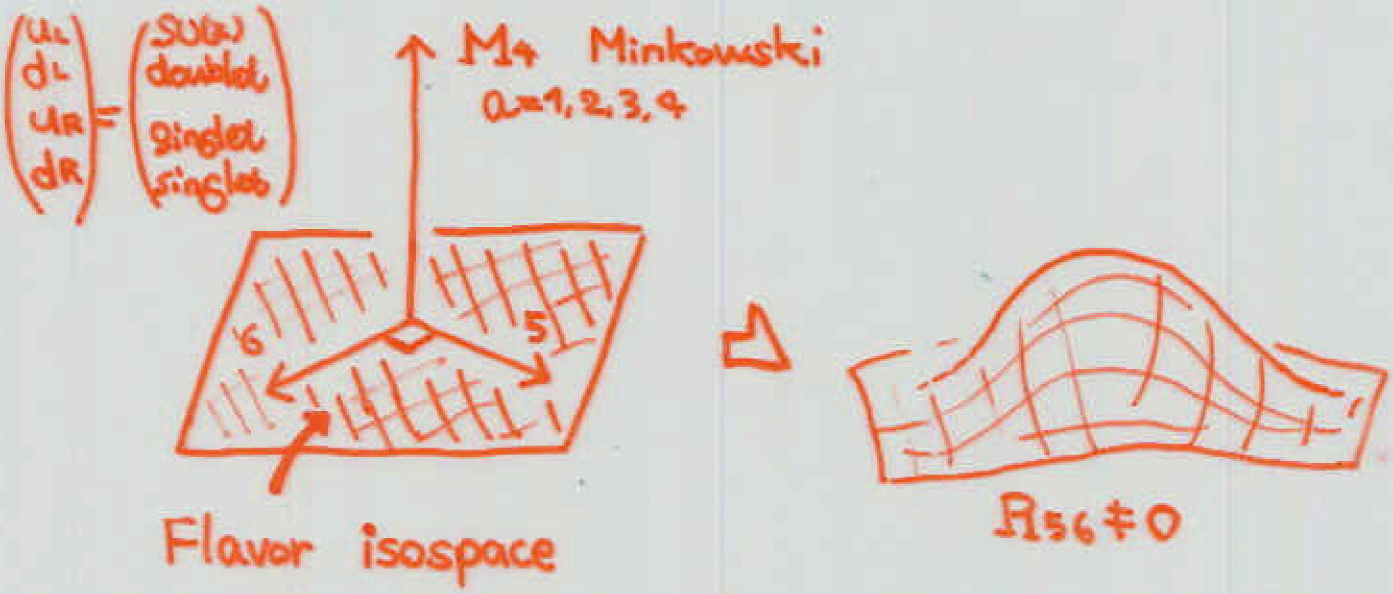
We would like to determine masses of quarks based on some noncommutative geometry.

⇩

Geometrized Higgs mechanism

Mass of quarks

$\sim$ Geometrical quantity



— Our idea —

* In noncommutative geometry
  → Distance $= \dfrac{1}{\text{Fermion Mass}}$

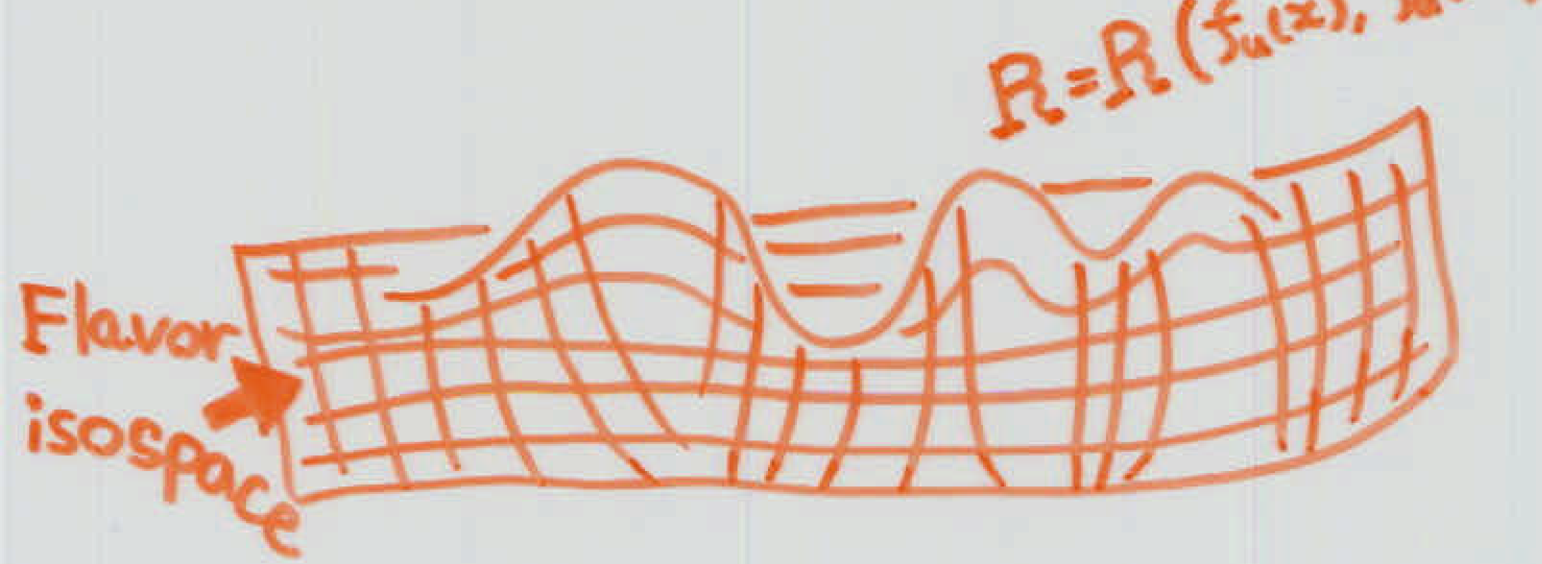
  → A geometrical interpretation for the Higgs field

* In the standard model
  $\langle 0 | \Phi_{\text{Higgs}} | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

  → f_i (coupling constant) $= \frac{\sqrt{2}}{v} m_i$ (mass)

We elevate the Higgs (Yukawa) couplings to vielbein-like fields in a discrete space.

$f_u, f_d \rightarrow f_u(x), f_d(x)$



— Our vielbein and localized Higgs couplings —

Our algebra

$$\mathcal{A} = \{ \mathbb{C} \oplus \mathbb{C} \oplus M_2(\mathbb{C}) \} \otimes C^{\infty}(M_4)$$

Our Hilbert space

$$\mathcal{H} = \begin{pmatrix} \left.\begin{matrix} u_R \\ d_R \end{matrix}\right] SU(2)\ \text{singlet} \\ \left.\begin{matrix} u_L \\ d_L \end{matrix}\right\} SU(2)\ \text{doublet} \end{pmatrix} \leftarrow \text{1-family quark}$$

Our Dirac operator

$$D = \left(\begin{array}{cc|c} i\nabla_R & 0 & \gamma^5 M^* \\ 0 & i\nabla_R & \\ \hline \gamma^5 M & & i\nabla_L \end{array}\right)$$

$$M^* = \begin{pmatrix} f^*(x)\,(\phi^{0*}, \phi^+) \\ \tilde{f}^*(x)\,(-\phi^-, \phi^0) \end{pmatrix}, \quad M = \left(f(x) \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \ \tilde{f}(x) \begin{pmatrix} -\phi^+ \\ \phi^{0*} \end{pmatrix} \right)$$

$$i\nabla_R = \gamma^\mu \left(i\partial_\mu - \frac{g'}{2} Y B_\mu \right)$$

$$i\nabla_L = \gamma^\mu \left(i\partial_\mu - \frac{g}{2} \underset{\substack{\uparrow \\ SU(2)}}{\tau_a A_{a\mu}} - \frac{g'}{2} Y \underset{\substack{\uparrow \\ U(1)}}{B_\mu} \right)$$

As our basis

"Flat basis"

$$\mathcal{E}^{a} = \gamma^{a}\,\mathbb{1}_4$$

$$\mathcal{E}^{\bar{u}} = \gamma^{5}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{\multirow{2}{*}{$\mathbb{0}_2$}} & v/\sqrt{2} & 0 \\ & & 0 & 0 \\ \hline -v/\sqrt{2} & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbb{0}_2$}} \\ 0 & 0 & & \end{array}\right)$$

$$\mathcal{E}^{\bar{d}} = \gamma^{5}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{\multirow{2}{*}{$\mathbb{0}_2$}} & 0 & 0 \\ & & 0 & v/\sqrt{2} \\ \hline 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbb{0}_2$}} \\ 0 & -v/\sqrt{2} & & \end{array}\right)$$

"Curved basis"

$$\mathcal{E}^{\mu} = \gamma^{\mu}\,\mathbb{1}_4\,, \quad \gamma^{\mu} = e^{\mu}_{a}(x)\,\gamma^{a}$$

$$\mathcal{E}^{u} = \gamma^{5}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{\multirow{2}{*}{$\mathbb{0}_2$}} & \frac{v}{\sqrt{2}} f^{*}(x) & 0 \\ & & 0 & 0 \\ \hline -\frac{v}{\sqrt{2}} f(x) & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbb{0}_2$}} \\ 0 & 0 & & \end{array}\right)$$

$$\mathcal{E}^{d} = \gamma^{5}\left(\begin{array}{cc|cc} \multicolumn{2}{c|}{\multirow{2}{*}{$\mathbb{0}_2$}} & 0 & 0 \\ & & 0 & \frac{v}{\sqrt{2}} f^{*}(x) \\ \hline 0 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbb{0}_2$}} \\ 0 & -\frac{v}{\sqrt{2}} f(x) & & \end{array}\right)$$

As our connection coefficients

$$
\Omega^{A}{}_{B} = \left(\begin{array}{c|c}
\gamma^{\mu}\,\omega_{1\mu}{}^{A}{}_{B,\ ij} & \gamma^{5} e^{-\delta}\dfrac{\upsilon}{\sqrt{2}}\begin{pmatrix} 0 & \underset{\sim}{f}(x)\,\omega'_{2}{}^{A}{}_{B} \\ \underset{\sim}{\tilde{f}}(x)\,\omega'_{2}{}^{A}{}_{B} & 0 \end{pmatrix} \\ \hline
\dfrac{\upsilon}{\sqrt{2}}\begin{pmatrix} 0 & \underset{\sim}{\tilde{f}}^{*}\,\omega'_{1}{}^{A}{}_{B} \\ \underset{\sim}{f}^{*}\,\omega_{1}{}^{A}{}_{B} & 0 \end{pmatrix}\gamma^{5} e^{-\delta} & \gamma^{\mu}\,\omega_{2\mu}{}^{A}{}_{B,\ ij}
\end{array}\right)
$$

$i, j = 1, 2$

A, B = 1, 2, 3, 4, u, d

$\Downarrow$

$$\left.\begin{array}{l}\text{Unitarity condition} \\ \text{Torsion less condition}\end{array}\right\}\ \text{following GFG}$$

Results

1. From the unitarity condition and the torsion less condition :

$$|\tilde{f}|^2(x) \propto \exp i \int^x dy^\nu \Big(2\,\overset{\overset{\omega_{2\nu}{}^{u}{}_{u,11}}{\updownarrow}}{\omega_{1\nu}{}^{u}{}_{u,11}} - \text{combination of } A^a_\mu, B_\nu \Big)$$

$$|f|^2(x) \propto \exp i \int^x dy^\nu \Big(2\,\underset{\underset{\omega_{2\nu}{}^{d}{}_{d,22}}{\updownarrow}}{\omega_{1\nu}{}^{d}{}_{d,22}} - \text{combination of } A^a_\mu, B_\nu \Big)$$

$\omega_{\substack{2\nu\\1}}{}^{u}{}_{u,11}$, $\omega_{\substack{2\nu\\1}}{}^{d}{}_{d,22}$ are connection coefficients which are introduced in the flavor isospace.

$\boxed{\text{Localized Higgs couplings} \propto \text{Geometrical objects in the flavor isospace}}$

2. To obtain the Einstein equation,

$$\sqrt{g}\, R \sim \text{No } \partial_a f(x),\ \partial_a \tilde{f}(x)$$

$$\sqrt{g}\, R^2 \sim \partial_a f(x),\ \partial_a \tilde{f}(x),\ \partial^2 f(x),\ \partial^2 \tilde{f}(x)$$

where

$$\left.\begin{array}{l}\displaystyle\int d^4x \sqrt{g}\, R = \underset{\sim}{\mathrm{Tr}}\left((E^A E^B)^*, R^A{}_B\right) \\ \qquad\qquad \nearrow \\ \qquad\qquad \text{in the Hilbert space} \\ \qquad\qquad\qquad \text{and} \\ \qquad\qquad \text{in the 4 dimensional} \\ \qquad\qquad \text{gamma matrix}\end{array}\right\}\ \text{CFG}$$

$$\int d^4x \sqrt{g}\, R^2 = \mathrm{Tr}\left((E^A E^B)^* R^A{}_B,\ (E^a E^D)^* R^a{}_D\right)$$