MEASURE FIELDS, THE COSMOLOGICAL CONSTANT AND SCALE INVARIANCE

by

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THE ACTION OF G.RELATIVISTIC THEORY IS TAKEN

$$S_1 = \int \underbrace{\sqrt{-g}\, d^4x}_{\text{Volume element}} \mathcal{L}_1$$

CAN TAKE INSTEAD (OR IN ADDITION)

$$S_2 = \int \Phi\, d^4x\, \mathcal{L}_2,$$ Φ A DENSITY

WHERE Φ IS BUILT OUT OF DEGREES OF FREEDOM INDEPENDENT OF $g_{\mu\nu}$, LIKE FOR EXAMPLE 4 SCALARS φ_a $(a = 1, 2, 3, 4)$

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd}\, \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d$$

WE TAKE $\mathcal{L}_1$ and $\mathcal{L}_2$ to be independent of THE MEASURE FIELDS φ_a

THAT IS $S = S_1 + S_2$

WHICH IS INVARIANT UP TO A TOTAL DIVERGENCE UNDER

i) $\mathcal{L}_2 \to \mathcal{L}_2 + \text{const}$

$$\Big(S \to S + \underbrace{\text{const} \int \Phi \, d^4x}_{= \int \partial_\mu \Omega^\mu d^4x}$$

since $\Phi = \partial_\mu \left(\varepsilon^{\mu\nu\alpha\beta} \epsilon_{abcd} \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \right)$

ii) If $\mathcal{L}_1, \mathcal{L}_2$ φ_a indep.

under $\varphi_a \to \varphi_a + f_a(\mathcal{L}_2)$

WE HAVE A SYMMETRY (UP TO TOTAL DIV.)

TWO MEASURE THEORY APPEARS ALSO NATURALLY IN BRANE WORLD SCENARIO

ONE MEASURE FOR BULK CONTRIBUTION ANOTHER, INDEPENDENT FOR THE BRANE CONTRIBUTION.

CONSIDER NOW A SCALAR FIELD ϕ (NOTATION ϕ - SCALAR FIELD, Φ - MEASURE)

S AS ABOVE $S = S_1 + S_2$

$$\mathcal{L}_1 = U(\phi)$$

$$\mathcal{L}_2 = -\frac{1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), \quad R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda}$$

$$R^\lambda_{\mu\nu\sigma} = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma}\Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\alpha_{\mu\sigma}$$

$g_{\mu\nu}$, $\Gamma^\lambda_{\mu\nu}$, ϕ AND φ_a INDEPENDENT VARIABLES

SUCH THEORY HAS GLOBAL SCALE INVARIANCE IF $V(\phi) = f_1 e^{\alpha\phi}$

$U(\phi) = f_2 e^{2\alpha\phi}$: $g_{\mu\nu} \to e^\theta g_{\mu\nu}$

$\varphi_a \to \lambda_a \varphi_a$, $\Phi \to (\prod_a \lambda_a) \Phi$ and

$\prod_a \lambda_a = e^\theta$, $\phi \to \phi - \theta/\alpha$

SPONTANEOUS BREAKING OF SCALE INVARIANCE AT CLASSICAL LEVEL

VARIATION WITH RESPECT TO φ_a

$$A^\mu_a \partial_\mu \mathcal{L}_2 = 0, \quad A^\mu_a = \varepsilon^{\mu\nu\kappa\lambda}\varepsilon_{abcd}\partial_\nu\varphi_b\partial_\kappa\varphi_c\partial_\lambda\varphi_d$$

$$\det(A^\mu_a) = \frac{4^{-4}}{4!}\Phi^3 \neq 0, \text{ if } \Phi \neq 0$$

$$\Rightarrow \partial_\mu \mathcal{L}_2 = 0 \Rightarrow \mathcal{L}_2 = M = \text{constant}$$

M SPONTANEOUSLY BREAKS SCALE INV.

CONSIDERING VARIATION W/R TO $g_{\mu\nu}$ AND USING $\mathcal{L}_2 = M$ ALLOWS TO SOLVE $\chi = \frac{\Phi}{\sqrt{-g}}$

$$\boxed{\chi = \frac{2U(\phi)}{M + V(\phi)} \equiv \frac{\Phi}{\sqrt{-g}}}$$

THEORY HAS NON RIEMANNIAN CONTR. FOR $\Gamma^\lambda_{\alpha\beta}$, $\Gamma^\lambda_{\alpha\beta} = \left\{{}^{\lambda}_{\alpha\beta}\right\} + \Sigma^\lambda_{\alpha\beta}$

BUT NON RIEMANNIAN CONTRIBUTION IS ELIMINATED BY CONFORMAL ~~CONT~~ TRANSFORMATION $\bar{g}_{\mu\nu} = \chi g_{\mu\nu}$

$$\Gamma^\lambda_{\mu\nu} = \left\{{}^{\lambda}_{\mu\nu}\right\}\Big|_{\bar{g}_{\mu\nu}}$$

IN TERMS OF $\bar{g}_{\mu\nu}$ EQS. HAVE THE EINSTEIN FORM

$$R_{\mu\nu}(\bar{g}_{\alpha\beta}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu}(\phi)$$

($R_{\mu\nu}(\bar{g}_{\alpha\beta})$ usual Ricci tensor in terms of $\bar{g}_{\alpha\beta}$)

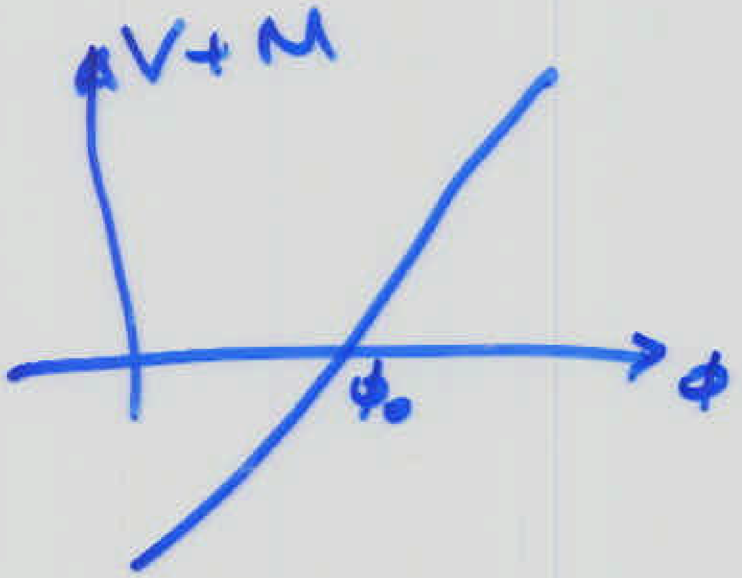
$$T^{eff}_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta} + \bar{g}_{\mu\nu}V_{eff}(\phi)$$

$$V_{eff}(\phi) = \frac{1}{4U(\phi)}(V+M)^2$$

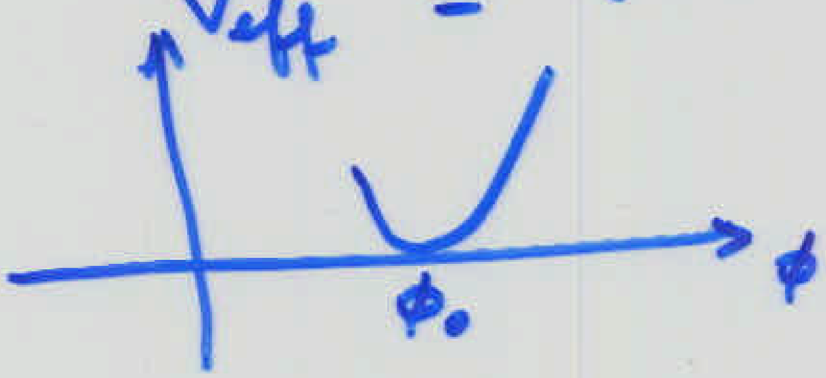
SIMILAR RESULTS ARE OBTAINED IN "NAIVE" (NON STABILIZED EXTRA DIMENSIONS) BRANE WORLD SCENARIOS.
($H \propto \rho$ instead of $H \propto \sqrt{\rho}$)

If $V + M = 0$ for $\phi = \phi_0$ 6

And If $U(\phi_0) > 0$



$$V_{eff} = (V + M)^2 / 4\,U(\phi)$$



zero of V_{eff} and $V'_{eff} = 0$ at the same time without fine tuning!

FOR THE SCALE INVARIANT CASE

$V(\phi) = f_1 e^{\alpha\phi}$, $U(\phi) = f_2 e^{2\alpha\phi}$ $\Rightarrow$

$$V_{eff}(\phi) = \frac{1}{4f_2}\left(f_1 + M e^{-\alpha\phi}\right)^2$$ [MORSE POTENTIAL!

If $\frac{f_1}{M} < 0$, $V_{eff} = 0$ at $\phi = \phi_{min} = \frac{-1}{\alpha}\ln\left|\frac{f_1}{M}\right|$

IF WE MAKE THE SHIFT $\phi \to \phi + \Delta$
THIS IS EQUIVALENT TO $M \to M e^{-\alpha\Delta}$,
SO THE CHOICE OF INTEGRATION
CONSTANT M DETERMINES THE POSITION
OF ϕ_{min}. BUT NOT THE SHAPE OF
THE POTENTIAL V_{eff}.

INFINITE FLAT REGION WITH $V_{eff} > 0$
IS DESIRABLE FOR NEW INFLATION
AS A MODEL FOR THE LATE UNIVERSE

$V_{eff} \to f_1^2/4f_2$ CAN BE VERY SMALL IF
$f_2 >> f_1$, i.e. TROUGH A SEE-SAW
MECHANISM

In terms of $\bar{g}_{\mu\nu} = \chi g_{\mu\nu}$

THE SCALE INVARIANCE AFFECTS ONLY ϕ

$$\bar{g}_{\mu\nu} \to \bar{g}_{\mu\nu}, \quad \phi \to \phi - \frac{\theta}{\alpha}$$

Notice that M spontaneously breaks scale invariance, which in terms of $\bar{g}_{\mu\nu}$, ϕ is just a translation invariance in ϕ.

THIS IS LIKE THE BREAKING OF TRANSLATION INVARIANCE WHEN A SOLITON SOLUTION APPEARS: THE SOLITON HAS TO BE CENTERED AT SOME POINT

IN THIS CASE THE APPEARENCE OF THE NON-TRIVIAL-NON CONSTANT $V_{eff}(\phi)$ IS THE ANALOG OF THE SOLITON.

CORRESPONDENCE WITH

Induced Gravity Model of Zee.

$$S = \int \sqrt{-g}\,\Big(-\tfrac{1}{2}\epsilon \varphi^2 R + \tfrac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu} - \tfrac{\lambda}{8}(\varphi^2 - \eta^2)^2 \Big) d^4x$$

where:

$R = R(g) =$ usual Riemannian scalar curvature defined in terms of $g_{\mu\nu}$. If $\eta = 0$ model is invariant under global scale invariance

$$g_{\mu\nu} \to e^{\theta} g_{\mu\nu}$$

$$\varphi \to e^{-\theta/2} \varphi$$

However we look at $\eta \neq 0$.

Define $\bar{g}_{\mu\nu} = k^2 \epsilon \varphi^2 g_{\mu\nu}$ $\left(\begin{smallmatrix}2k^2 \\ =\kappa\end{smallmatrix}\right)$
and the scalar field
$\phi = \frac{1}{k}\sqrt{6 + \frac{1}{\epsilon}}\, \ln \varphi$. Then

Zee's model is equivalent to GR coupled minimally to the scalar field ϕ which has a potential

$$V_{eff} = \frac{\lambda}{8k^4\epsilon^2}\left(1 - \eta^2 e^{-2\sqrt{\frac{\epsilon}{1+6\epsilon}}\, k\phi}\right)^2$$

which is to be compared with the form obtained here

$$V_{eff} = \frac{1}{4f_2}\left(f_1 + M e^{-\alpha\phi}\right)^2$$

$\Rightarrow \quad \alpha = 2\sqrt{\frac{\epsilon}{1+6\epsilon}}\, k$ (correspondence to Zee's model)

$M \propto -\eta^2$, etc.

As in Zee's model, no scale is really introduced if $\eta = 0$, also if $M = 0$ in our case. The introduction of κ is a choice to keep $\Phi\, d^4x$ having dimensions of L^4, but by rescaling φ_a in

$$\Phi\left(-\frac{1}{\kappa}R + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \cdots\right)$$

can set $\kappa = 1$ (rescaling of ϕ mantains the $\frac{1}{2}\partial_\mu\phi'\partial^\mu\phi'$ form for ϕ). In that case V_{eff}, effective masses in the flat region and in high density approx. become dimensionless quantities!

(see fermion masses next).

CONSIDER NOW A FIELD THEORY DESCRIPTION OF MATTER, LIKE A DIRAC FIELD AND TAKE KINETIC PART TO BE PART OF $\mathcal{L}_1$

$$S_{fk} = \int \mathcal{L}_{fk}\, \Phi\, d^4x$$

$$\mathcal{L}_{fk} = \frac{i}{2} \bar{\Psi} \Big(\gamma^a V_a^\mu \big(\vec{\partial}_\mu + \tfrac{1}{2} \omega_\mu^{cd} \sigma_{cd} \big) - \big(\overleftarrow{\partial}_\mu + \tfrac{1}{2} \omega_\mu^{cd} \sigma_{cd} \big) \gamma^a V_a^\mu \Big) \Psi$$

V_a^μ is the vierbein $\left(g^{\mu\nu} \equiv \eta^{ab} V_a^\mu V_b^\nu \right)$, $\sigma_{cd} = \frac{1}{2} [\gamma_c, \gamma_d]$

ω_μ^{ab} spin connection, $R = V^{\mu a} V^{b\nu} R_{\mu\nu ab}$

$$R_{\mu\nu ab}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \left(\omega_{\mu a}^{c} \omega_{\nu cb} - \omega_{\nu a}^{c} \omega_{\mu cb} \right)$$

GLOBAL SCALE INVARIANCE IS THERE IF Ψ TRANSFORMS ALSO

$$\Psi \longrightarrow \lambda^{-1/4} \Psi$$

SCALE INVARIANT MASS TERM

$$S_{fm} = m_1 \int \bar{\Psi} \psi \, e^{\alpha\phi/2} \, \Phi \, d^4x$$
$$+ m_2 \int \bar{\Psi} \psi \, e^{3\alpha\phi/2} \sqrt{-g} \, d^4x$$

CONSTRAINT IN HIGH DENSITY OF FERMION FIELD ($m_1 e^{\alpha\phi/2} \bar{\Psi}\psi$ or $m_2 e^{3\alpha\phi/2} \bar{\Psi}\psi \gg M+V$) THE CONSTRAINT IS

$$\left(3m_2 e^{3\alpha\phi/2} + m_1 e^{\alpha\phi/2} \chi\right) \bar{\Psi}\psi = 0$$

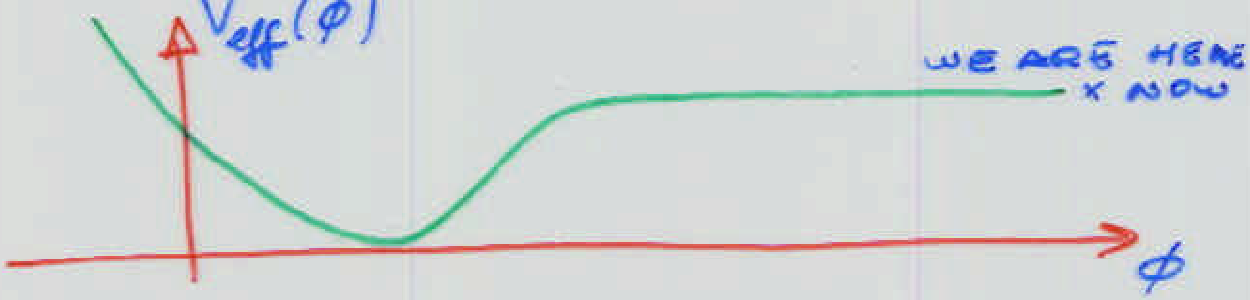
$\rightarrow \chi = -\dfrac{3m_2}{m_1} e^{\alpha\phi}$ AND MASS TERM IN EINSTEIN FRAME ($\bar{g}_{\mu\nu} = \chi g_{\mu\nu}$) IS

$$S_{fm} = -2m_2 \left(\frac{|m_1|}{3|m_2|}\right)^{3/2} \int \sqrt{-\bar{g}} \, \bar{\Psi}' \psi' \, d^4x$$

(in order to get Einstein - Cartan eq. ψ has to be transformed also $\psi' = \chi^{-1/4} \psi$)

NOTICE THAT IN EINSTEIN FRAME COUPLING TO ϕ GOES AWAY! AGAIN ORDINARY DYNAMICS! WHEN EXPRESSED IN TERMS OF SCALE INVARIANT METRIC ($\bar{g}_{\mu\nu}$) AND FERMI FIELD ($\psi' \rightarrow \psi'$ IN SCALE TRANSF.)

LOOK AT LOW DENSITY ~~APPROX~~ APPROXIMATION FOR ψ FIELD ψ IN THE FLAT REGION OF THE POTENTIAL ($\phi \to \infty$).



THEN IF ψ REPRESENTS A SMALL ENERGY PERTURBATION AS COMPARED TO $V(\phi)$, $U(\phi)$ AND IF $\phi \to \infty$, M CAN BE NEGLECTED ALSO $\Rightarrow$

$$\chi = \frac{2f_2}{f_1} e^{\alpha\phi}$$

$$\to S_{fm} = m \int \sqrt{-g}\, \bar{\psi}' \psi' \, d^4x$$

$$m = m_1 \left(\frac{f_1}{2f_2}\right)^{1/2} + m_2 \left(\frac{f_1}{2f_2}\right)^{3/2}$$

THEN TAKING $\frac{f_1}{f_2} << 1$ AS MOTIVATED BEFORE, IF $m_1 \sim m_2$, THE MASS IN THE HIGH DENSITY OF FERMION FIELD $\sim m_2 \left(\frac{|m_1|}{3|m_2|}\right) \sim m_1 \sim m_2$ APPROPRIATE FOR $V+M \sim 0$ IS MUCH BIGGER THAN MASS IN FLAT REGION

SUMMARY:

i) ABSOLUTE VACUUM IN THE ABSENCE OF MATTER $V+M=0 \Leftrightarrow V_{eff}=0$
→ ZERO COSMOLOGICAL CONSTANT

ii) WITH MATTER V_{eff} IS MINIMIZED BY $V+M=0$ BUT MATTER ENERGY MAXIMIZED THERE. ALSO MATTER ENERGY MINIMIZED FOR MATTER IN FLAT REGION.

⇒ iii) TRUE VACUUM STATE HAS $|V+M|>0$ HOW MUCH ABOVE ZERO V_{eff} IS DEPENDS ON AMOUNT OF MATTER PRESENT.

⇒ iv) SINCE $V_{eff} \neq 0$, WE HAVE VACUUM ENERGY FOR THE LATE UNIVERSE.

MODIFIED MEASURE THEORIES OF EXTENDED OBJECTS (STRINGS, BRANES, SUPER " ")

THE POLYAKOV ACTION, FOLLOWS VERY MUCH THE STRUCTURE OF A GRAVITATIONAL THEORY:

FOR THE STRING IT IS

$$S_p[X^\mu] = -T\int d\sigma d\tau \sqrt{-\gamma}\,\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

γ_{ab} is the metric defined in the 1+1 world sheet of the string

$\gamma = \det(\gamma_{ab})$, $g_{\mu\nu}$ metric of embedding space.

POLYAKOV TREATS γ^{ab} AS AN INDEPENDENT VARIABLE AND IT IS FOUND THAT

$$\gamma_{ab} = \text{conformal factor} \times$$
$$\times \text{ induced metric}$$

$$= \Omega^2(x)\, \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

Ω^2 cannot be determined because of conformal invariance of theory.

LET US NOW CONSTRUCT A MODIFIED MEASURE FORMULATION OF THE STRING

MOST STRAIGHTFORWARD APPROACH FAILS, I.E. IF WE JUST $\sqrt{-\gamma}\, d^2x \equiv \sqrt{-\gamma}\, d\sigma d\tau$

$$\longrightarrow \Phi\, d\sigma d\tau \quad , \quad \Phi = \varepsilon^{ab} \varepsilon_{ij} \partial_a \varphi_i \partial_b \varphi_j$$

$$i = 1,2 \quad (\varphi_i = (\varphi_1, \varphi_2))$$

THEN ACTION

$$S = -\int d\tau\, d\sigma\; \Phi\, \gamma^{ab}\, \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

(notice a factor in front is irrelevant since it can be absorbed in definition of φ_j's if no boundary conditions are specified).

gives (variating w/r to γ^{ab})

$$\Phi\, \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} = 0$$

$\Rightarrow$ $\Phi = 0$ or induced metric $= 0$

not good dynamics.

TO MAKE FURTHER PROGRESS IT IS IMPORTANT TO NOTICE THAT IF $\int L \sqrt{-\gamma}\, d\sigma\, d\tau$ IS THE INTEGRAL OF A TOTAL DIVERGENCE, FOR THE SAME L

THEN $\int L\, \Phi\, d\sigma\, d\tau$

IS NOT

CONSIDER $\int \overset{L}{\overset{\shortparallel}{\left[\frac{\varepsilon^{ab}}{\sqrt{-\gamma}} F_{ab}\right]}} \sqrt{-\gamma}\, d\sigma\, d\tau$

$= \int \epsilon^{ab} F_{ab}\, d\sigma\, d\tau = \text{T.D.}$

$F_{ab} = \partial_a A_b - \partial_b A_a$

HERE $\int L \sqrt{-\gamma}\, d\sigma\, d\tau$

DOES NOT CONTRIBUTE TO EQUATIONS OF MOTION.

$\int L\, \Phi\, d\sigma\, d\tau$ WILL HOWEVER

$\left(= \int \frac{\varepsilon^{ab}}{\sqrt{-\gamma}} F_{ab}\, \Phi\, d\sigma\, d\tau \right)$

CONSISTENT MODIFIED MEASURE STRING THEORY (MMST)

$$S = -\int \Phi \left[\gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{\sqrt{-\gamma}} F_{ab} \right] d\sigma d\tau$$

VARIATION WITH RESPECT TO A_a GIVES

$$\epsilon^{ab} \partial_b \left(\frac{\Phi}{\sqrt{-\gamma}} \right) = 0$$

$$\Rightarrow \quad \Phi = c \sqrt{-\gamma}$$

c IS THE STRING TENSION, WHICH APPEARS AS A CONSTANT OF INTEGRATION!

THE MMST DOES NOT HAVE ORIGINALLY A SCALE IN THE ACTION, SCALE APPEARS SPONTANEOUSLY!

CONFORMAL SYMMETRY

$$\varphi_j \to \varphi_j' = \varphi_j'(\varphi_i) \qquad (*)$$

$$\Phi \to \Phi' = J\,\Phi \quad , \quad J = \text{Jacobian of } (*)$$

$$\gamma_{ab} \to \gamma_{ab}' = J\,\gamma_{ab}.$$

$$\left.\begin{array}{l} \gamma^{ab}\,\Phi \to \gamma'^{ab}\,\Phi' \\ \dfrac{\Phi}{\sqrt{-\gamma}} \to \dfrac{\Phi'}{\sqrt{-\gamma'}} = \dfrac{\Phi}{\sqrt{-\gamma}} \end{array}\right\}\ \text{inv.}$$

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variation of action w/r to φ_j

$$\varepsilon^{ab}\,\partial_b\varphi_j\,\partial_a\left[-\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\varepsilon^{cd}}{\sqrt{-\gamma}}F_{cd}\right] = 0$$

if $\det(\varepsilon^{ab}\partial_b\varphi_j) = \Phi \neq 0$

$$-\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} + \frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd} = M$$

WHERE $M =$ constant

VARY γ^{ab} AND GET

$$\Phi \left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2} \gamma_{ab} \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right)$$

$$= 0$$

$\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}$ solve from eq. involving $M \Rightarrow$

$$\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu g_{\mu\nu}$$

$$- \frac{1}{2} M \Phi \gamma_{ab} = 0$$

Trace $\Rightarrow$ $M = 0$

Eq. as in Polyakov's case!

FOR HIGHER P- BRANES

$$S = -\int d^{p+1}x \, \Phi \Big[\gamma^{ab} \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

$$- \frac{\epsilon^{a_1 \cdots a_{p+1}}}{\sqrt{-\gamma}} \partial_{[a_1} A_{a_2 \cdots a_{p+1}]} \Big]$$

NO COSMOLOGICAL CONSTANT IN BRANE NEEDED (MANDATORY IN USUAL TREATMENT).

SUPERSTRINGS WITH A MODIFIED MEASURE

standard approach

$$S = T \int L \sqrt{-\gamma}\, d^2x$$

UNDER SUPERSYMMETRY $L\sqrt{-\gamma}$ IS INVARIANT UP TO A TOTAL DIVERGENCE ONLY

BEFORE CHANGING MEASURE IT IS IMPORTANT TO DO SO IN A FORMULATION WHERE $L\sqrt{-\gamma}$ IS EXACTLY INVARIANT (REMEMBER DISCUSSION $\delta(L\sqrt{-\gamma})$ BEING A TOTAL DERIVATIVE USUALLY MEANS $\delta(\Phi L)$ NOT

AN INTERESTING THING HAPPENS IF WE INTRODUCE COMPENSATING FIELD SO AS TO MAKE L INVARIANT, THE GAUGE FIELDS ARE AUTOMATICALLY PROVIDED!

$$S = T \int L \sqrt{-\gamma}\, d^2x$$

$$L = -\frac{1}{2}\gamma^{ab} J^{\mu}_{a} J_{\mu b} - \frac{i\epsilon^{ab}}{\sqrt{-\gamma}} J^{\alpha}_{a} J_{\alpha b}$$

$$J^{\alpha}_{a} = \partial_a \theta^{\alpha}, \quad J^{\mu}_{a} = \partial_a X^{\mu} - i \partial_a \theta^{\alpha} \Gamma^{\mu}_{\alpha\beta} \theta^{\beta}$$

$$J_{\alpha a} = \partial_a \phi_{\alpha} - 2i(\partial_a X^{\mu}) \Gamma_{\mu\alpha\beta} \theta^{\beta} - \frac{2}{3}(\partial_a \theta^{\beta}) \Gamma^{\mu}_{\beta\delta}$$

$$\times\, \theta^{\delta} \Gamma_{\mu\alpha\epsilon} \theta^{\epsilon}$$

SUPERSYMMETRY IS ACHIEVED

$$\delta\theta^\alpha = \epsilon^\alpha \,, \quad \delta x^\mu = -i\epsilon^\alpha \Gamma^\mu_{\alpha\beta}\theta^\beta$$

$$\delta\phi_\alpha = 2i\epsilon^\beta \Gamma_{\mu\alpha\beta} X^\mu - \frac{2}{3}(\epsilon^\beta \Gamma^\mu_{\beta\sigma}\theta^\sigma) \times$$

$$\times \Gamma_{\mu\alpha\kappa}\theta^\kappa$$

ϕ_α : extra compensating fields

$$T\sqrt{-\gamma}\, d^2x \longrightarrow \Phi\, d^2x$$

$$S = \int L\, \Phi\, d^2x$$

identification of abelian gauge field $\quad -i\epsilon^{ab}\partial_a\theta^\alpha\partial_b\phi_\alpha = \epsilon^{ab}\partial_a A_b$

$$\boxed{A_b \equiv -i\theta^\alpha \partial_b \phi_\alpha}$$ extra gauge field

does not need to be introduced independently

THESE IDEAS CAN BE
GENERALIZED ALSO
TO SUPER BRANES.

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