Running Couplings in Extra Space-Time Dimensions

based on
J.Kubo (Kanazawa)
G.Zoupanos (NTUA)
H.T.
NPB574(2000)459

Topics:

1. Running couplings in large extra dimensions
   - Threshold corrections by Kaluza-Klein modes
   - Effective dimensions
2. Scheme Dependence and "Finite Renormalization"
3. Running couplings in warped extra dimensions

I. RG in large extra dimensions

Extra dimensions may explain hierarchy problems

- Gauge hierarchy
- Yukawa hierarchy
- Cosmological constant

Superstring theories

⇒ Possibility of large compact dimensions

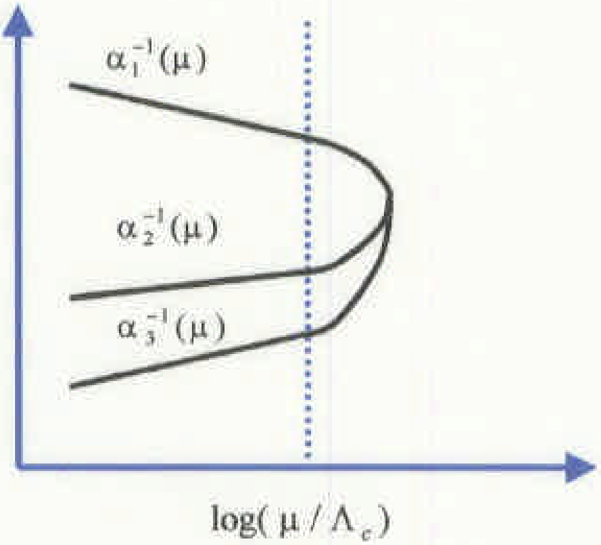
Compactification scale ≪ String scale

⇒ Effective F.T. with Kaluza-Klein modes

Gauge coupling unification

Dienes,Dudas,Ghergetta,PLB436(1998)
NPB537(1999)
Ghilencea,Ross PLB442(1998)

Power law running



Threshold corrections by Kaluza-Klein modes

⇒ Power law running

Formalism of RG

F.T. in large extra dimensions

= F.T. with infinitely many KK modes

: Nonrenormalizable

⇒ Cutoff theories (Wilsonian effective theories)

⇩

Wilson RG is a natural framework

Decoupling of the KK modes

Mass independent RG (e.g. MS scheme)

$$\beta_g = \mu \frac{dg}{d\mu} = \sum_{\mathrm{K-K}} b g^3 = \infty$$

Decoupling effect of the KK modes

⇒ Finite beta functions

∵ Only a finite number of KK modes contribute

Schemes

- Wilson RG with smooth momentum cutoff
- Proper time cutoff
- Momentum subtraction scheme

⇒ Scheme dependence in one loop level

⇩

RG predictions are reliable?

Scalar field in (4+1) dimensions

$$S^{(5)} = \int_0^{2\pi R} dy \int d^4x \left\{ \frac{1}{2}(\partial_y \tilde{\phi})^2 + \frac{1}{2}(\partial_\mu \tilde{\phi})^2 + \frac{\tilde{\lambda}}{4!}\tilde{\phi}^4 \right\}$$

Kaluza-Klein decomposition

$$\tilde{\phi}(x,y) = \sum_{n\in\mathbf{Z}} \tilde{\phi}_n(x)\exp(i\omega_n y), \qquad \omega_n = \frac{n}{R}$$

$$S^{(4)} = \int d^4x \left\{ \frac{1}{2}\sum_{n\in\mathbf{Z}} \phi_{-n}[-\partial_\mu^2 + \omega_n^2]\phi_n + \frac{\lambda}{4!}\sum_{n_i\in\mathbf{Z}} \delta_{n_1+n_2+n_3+n_4,0}\phi_{n_1}\phi_{n_2}\phi_{n_3}\phi_{n_4} \right\}$$

$$\tilde{\phi}_n \to \phi_n = \tilde{\phi}_n/\sqrt{2\pi R}, \qquad \tilde{\lambda} \to \lambda = 2\pi R\tilde{\lambda}$$

One loop correction

$\sum_{\mathrm{KK}}$ 

- Wilson (exact) RG
  Λ : internal momentum cutoff scale
- Momentum subtraction
  Λ : external momentum scale
- Proper time regularization
  Λ : proper time cutoff scale

⇒
$$\beta_\lambda = \Lambda\frac{d\lambda}{d\Lambda} = b\epsilon_k(R\Lambda)\lambda^2$$

$$\epsilon_k(R\Lambda) \to 1 \qquad \text{for } R\Lambda \ll 1$$

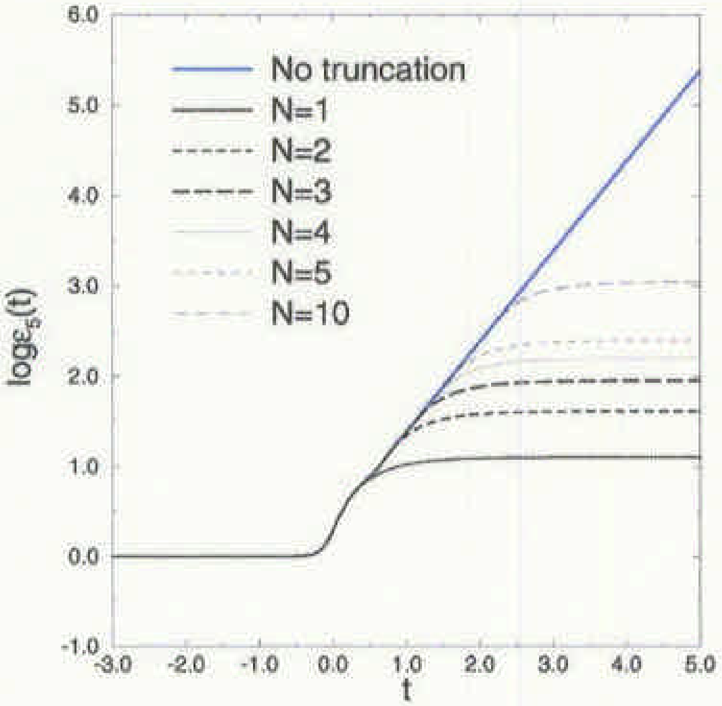
$$\to B(k)(R\Lambda) \qquad \text{for } R\Lambda \gg 1$$

B(k) : scheme dependent constant

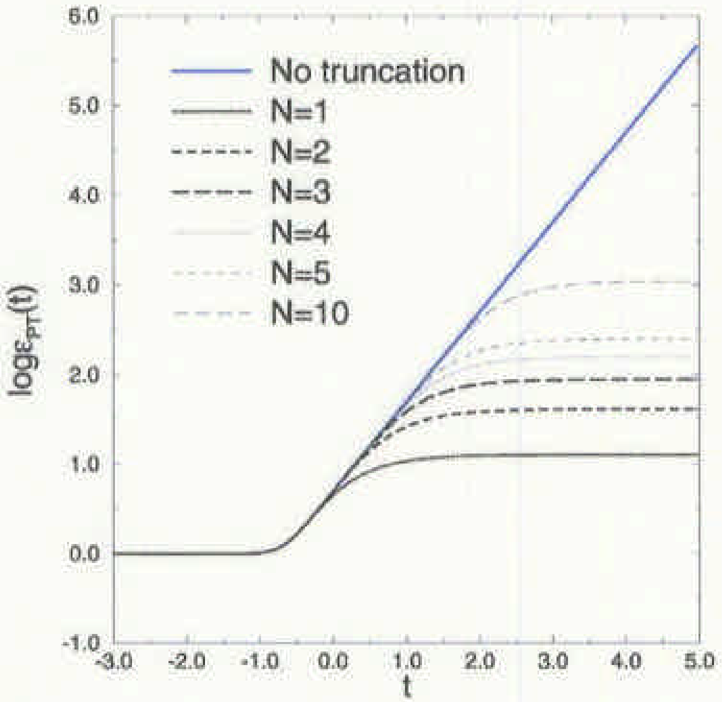
⇒ power law running in the high energy region

Threshold corrections of KK modes (t=ln(RΛ))

Power behavior in ERG (k=5)



Power behavior in proper time (r=1)



Successive threshold corrections of KK modes

⇒ Rapid shift to the power law running

Effective dimension

$$\beta_\lambda = \Lambda \frac{d\lambda}{d\Lambda} = b\epsilon_k(R\Lambda)\lambda^2$$

Note: The coefficient depends on the scale

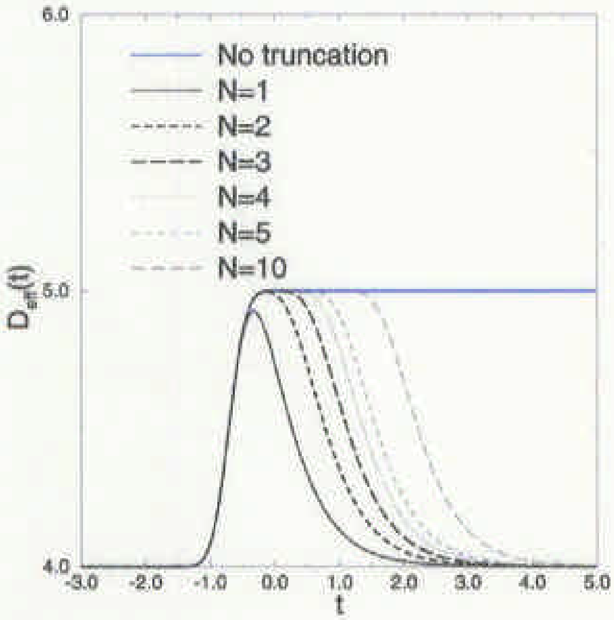
⇒ New coupling absorbing the scale dependence

$$h_k = \epsilon_k(R\Lambda)\lambda$$

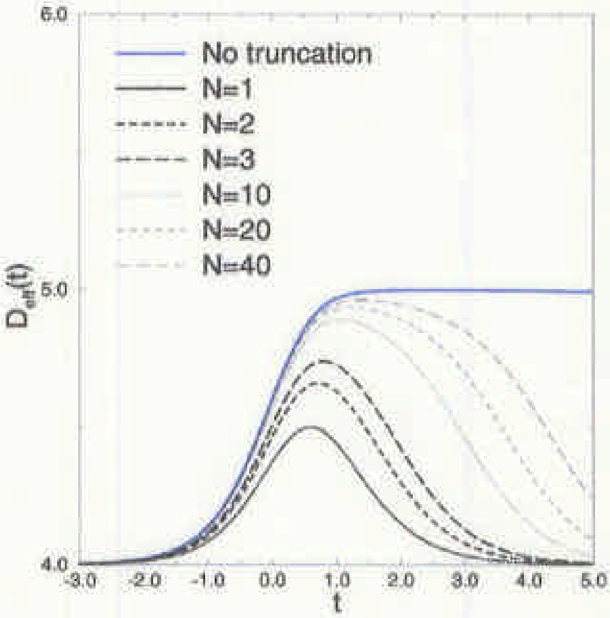
$$\begin{aligned}\beta_h &= \Lambda \frac{dh_k}{d\Lambda} = \frac{d\ln\epsilon_k}{d\ln(R\Lambda)} h_k + b h_k^2 \\ &= (D_{\rm eff} - 4)\, h_k + b h_k^2\end{aligned}$$

$$D_{\rm eff}(R\Lambda) = 4 + \frac{d\ln\epsilon_k}{d\ln(R\Lambda)}$$: Effective dimension

Effective dimension in proper time (r=1)



Effective dimension in momentum subtraction



II. Scheme dependence

Large scheme dependence in the RG coeff.

$$\beta_\lambda = B(R\Lambda)\lambda^2 + O(\lambda^2) \qquad \text{for} \quad R\Lambda \gg 1$$

$$\beta_{\lambda'} = B'(R\Lambda)\lambda'^2 + O(\lambda'^2)$$

Note: Low energy limit is scheme independent

∵ reducing to the 4D beta function

"Finite renormalization" by coupling redef.

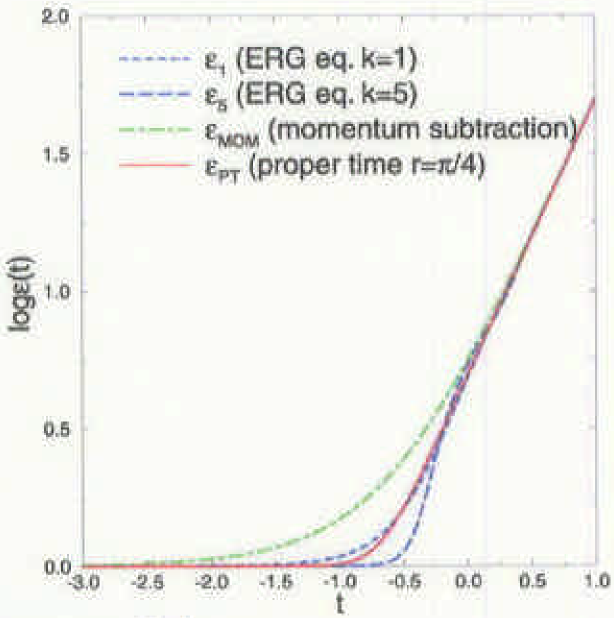
$$\lambda' = \lambda(1 + (B' - B)\lambda(R\Lambda) + \cdots)$$

⇒ Same beta function in the high energy region

After the "finite renormalization"

The scheme dependence remains in the K-K threshold corrections, however ...

Threshold corrections in various schemes



⇒ rather small !

∴ Prediction is almost scheme independent

III. RG in the warped extra dimensions

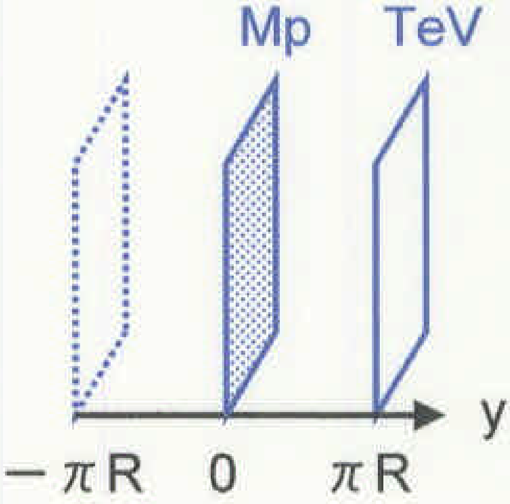
AdS5 geometry with boundaries

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$-\pi R \le y \le \pi R$$

Scales at the boundaries

$$\begin{cases} k & \sim \; M_p \\ ke^{-k\pi R} & \sim \; \text{TeV} \quad (R \sim 11/k) \end{cases}$$



Scalar field in the bulk

$$S = \frac{1}{2}\int d^4x \int_{-\pi R}^{\pi R} dy \left[e^{-2k|y|}(\partial_\mu \Phi)^2 + \frac{1}{R^2}\Phi\partial_y\left(e^{-4k|y|}\partial_y\Phi\right) - m^2 e^{-4k|y|}\Phi^2 \right]$$

Kaluza-Klein decomposition

$$\Phi(x, y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}(x) f_n(y)$$

$f_n(y)$: eigenfunction on AdS5

Mass spectra

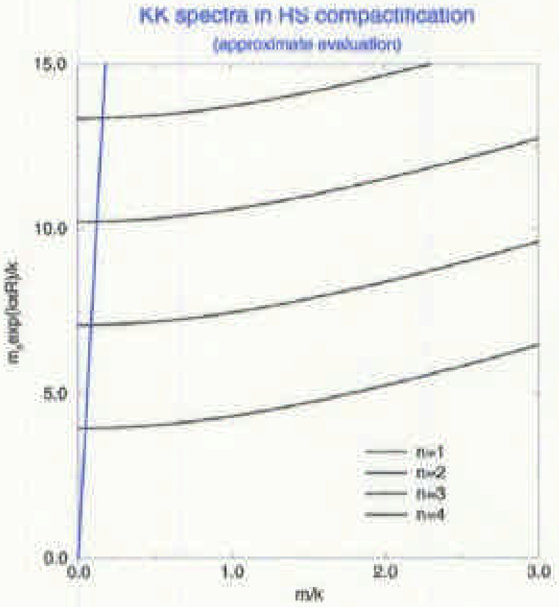
$$\begin{cases} m_0 \; \sim \; m/\sqrt{2} \\ m_n \; \sim \; \left(n + \frac{\nu}{2} - \frac{3}{4}\right) m_{\text{KK}}, \end{cases}$$

$$(n = 1, 2, 3, \cdots)$$

$$m_{\text{KK}} \; = \; \pi k e^{-k\pi R} \sim \text{O(TeV)}$$

$$\nu \; = \; \sqrt{4 + \frac{m^2}{k^2}}$$

⇒ KK mass ~ O(TeV),
even if m ~ Mp



One loop corrections by KK modes

 $\sim \; g^2\Pi_{\mu\nu}(q) = g^2(q^2\eta_{\mu\nu} - q_\mu q_\nu)\Pi(q)$

(1) PV regularization

Pomarol,hep-ph/0005293

Corrections by KK modes are cancelled out by the PV field

⇒ 4D running ∵zero modes are dominant

$$\Pi(0) \simeq \frac{b_0}{8\pi^2}\ln\frac{\mu}{M}$$

M: PV mass

⇒ Gauge unification occurs at 10^(-16) Gev

Note: PV regulators appear at TeV scale !

(2) Proper time cutoff

$$\Pi(0) = \frac{b_0}{16\pi^2}\int_{\Lambda^{-2}}^{\mu^{-2}}\frac{d\tau}{\tau}\left\{1 + \sum_{n=1}^{\infty}\left(e^{-\tau\left(n+\frac{1}{4}\right)^2 m_{\rm KK}^2} - e^{-\tau\left(n+\frac{\nu}{2}-\frac{3}{4}\right)^2 m_{\rm KK}^2}\right)\right\}$$

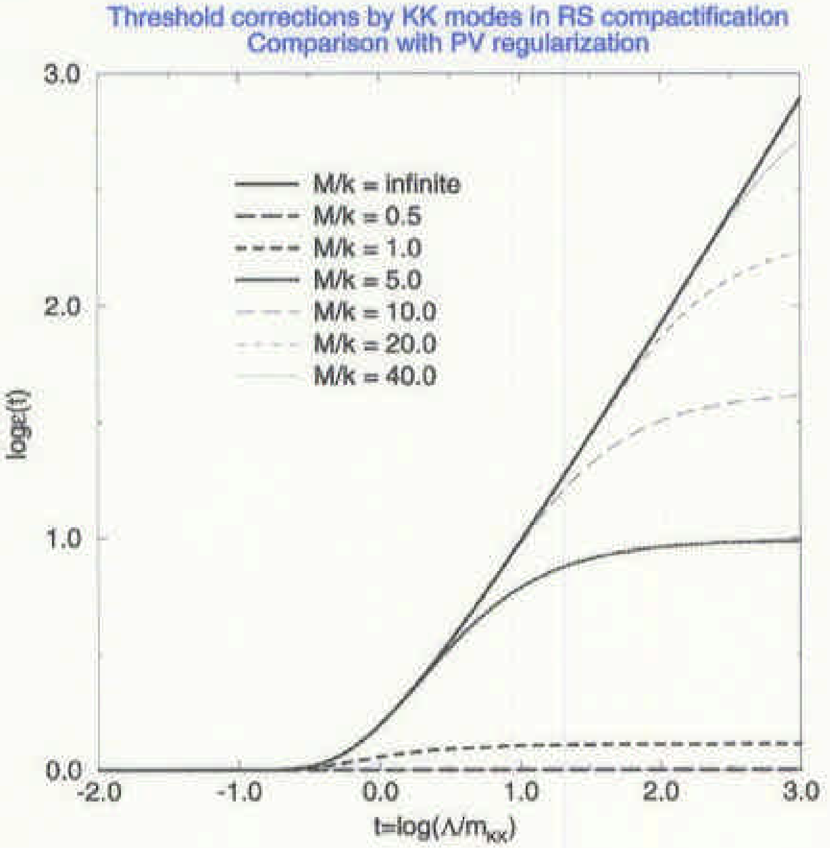
⇒ Beta function

$$\beta_g = \Lambda\frac{dg}{d\Lambda} = \frac{b_0}{16\pi^2}\epsilon\left(\frac{\Lambda}{m_{\rm KK}}\right)g^3$$

Power law running

For M < k ~ Mp ⇒ 4D running

For M ≫ k ⇒ Power law running



Running couplings above TeV scale is regularization dependent.

⇩

Need to study string corrections in AdS5

IV. Conclusions

1. The origin of the power law behavior is the successive threshold corrections of KK modes.
2. The scheme dependence of the KK threshold corrections remaining after “finite renormalization” is small.
3. The TeV scale KK modes in the Randall-Sundrum compactification may generate power law running. However,

   the running couplings above TeV scale are dependent on the Plank scale (string) physics.

Thank you !