Unifeid Field Theory Induced on the Brane World

by K. Akama & T. Hattori

gravitational
gauge field theories
matter

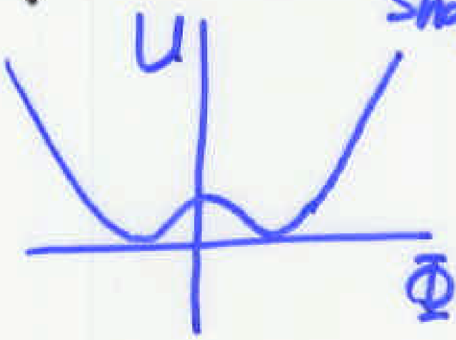
Brane World Fronsdal '59
K.A. '82

3-dim domain wall in 4+1 spacetime Rabakov & Shaposhnikov '83

$$L = \frac{1}{2}(\partial_\mu \Phi)^2 - U(\Phi)$$

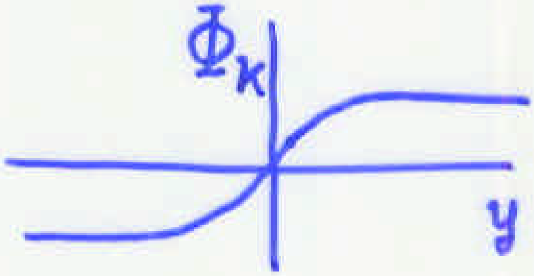
$$U(\Phi) = \frac{1}{4}\lambda\left(\Phi^2 - \frac{m^2}{\lambda}\right)^2$$

double wall potential

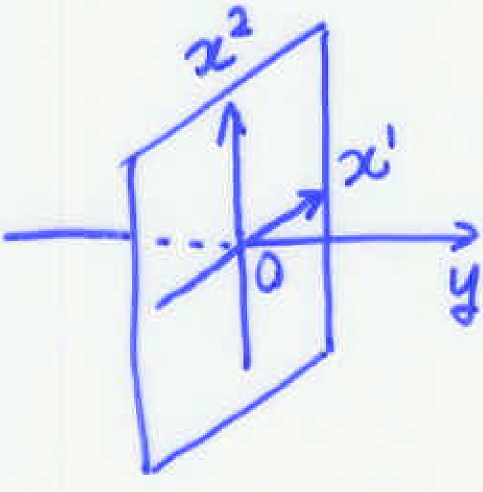


kink solution

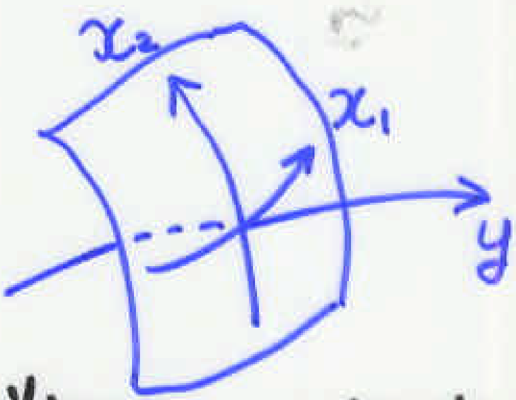
$$\Phi = \Phi_K(y) \equiv \frac{m}{\sqrt{\lambda}} \tanh \frac{my}{\sqrt{2}}$$



flat domain wall



curved domain wall



curvilinear coordinate

$y = 0$ at the brane

$\Phi_K(y)$: approximate solution

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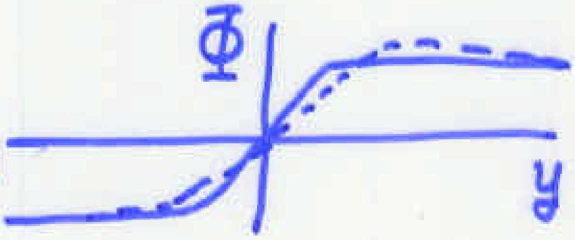
Equation of motion $\partial_M E G^{MN} \partial_N \Phi = -E\, U'(\Phi)$

$E = \det E_{kM}$, E_{kM}: vielbein, $E_{kM} E^k{}_N = G_{MN}$: metric

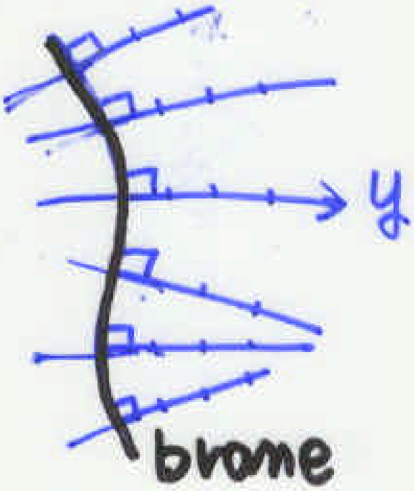
The solution is distorted from the kink function $\Phi_k(y)$ depending on the metric.



Assume the whole spacetime is flat, for simplicity, and y is taken along the straight normal line

Bulk vielbein

$$\begin{pmatrix} E_{k\mu} & E_{k4} \\ E_{4\mu} & E_{44} \end{pmatrix} = \begin{pmatrix} e_{k\mu} - y b_{k\mu} & 0 \\ 0 & -1 \end{pmatrix}$$



$k, \mu = \underset{\text{time}}{0}, \underbrace{1, 2, 3}_{\text{brane}}, \underset{\text{extra dim.}}{4}$ $\quad x^4 = y$

$e_{k\mu}$: brane vielbein $\quad b_{\mu\nu}$: extrinsic curvature

Gauss eq. $\quad {}^{(4)}R_{ab\mu\nu} - b_{a[\mu} b_{b\nu]} = {}^{(5)}R_{ab\mu\nu}$

Codazzi eq. $\quad b_{a[\mu,\nu]} + \omega_{ac[\mu} b^c{}_{\nu]} = {}^{(5)}R_{a4\mu\nu}$

${}^{(4)}R_{ab\mu\nu}$: brane curvature $\quad \omega_{ab\mu}$: brane connection

${}^{(5)}R_{ABMN}$: bulk curvature

Solve the equation of motion by rewriting Φ
to $\Phi = \Phi_K(y) + \chi(x^\mu, y)\,\Phi_K'(y)$ with $\Phi_K(y) = \frac{m}{\sqrt{\lambda}} \tanh \frac{my}{\sqrt{2}}$

Expand $\chi(x^\mu, y) = \chi_0(x^\mu) + \chi_1(x^\mu)\, y + \chi_2(x^\mu)\, y^2 + \cdots$

eq. of motion $\Rightarrow$ recursion formulae for $\chi_i(x^\mu)$

determin $\chi_i(x^\mu)$ one by one

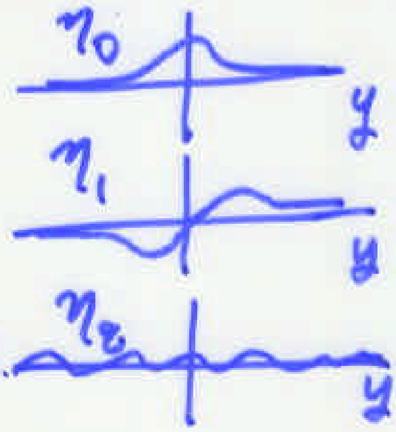
Solution $\Phi = \Phi_{br} \equiv \Phi_K + \{\frac{1}{2} b y^2 + \frac{1}{6}(b_2 + b^2) y^3 + \cdots\} \Phi_K'$
+ (terms depending on χ_0 & χ_1)

$$b = b_{\mu\nu} g^{\mu\nu},\ b_2 = b_{\mu\nu} b_{\rho\sigma} g^{\nu\rho} g^{\mu\sigma}$$

Quantum fluctuations around Φ_{br}

Expand $\Phi = \Phi_{br} + \varphi_0 \eta_0 + \varphi_1 \eta_1 + \int \varphi_q \eta_q \, dq$

complete set of the small fluctuation modes of kink Φ_K



$\eta_0 = 1/\cosh^2 z \quad (z = my/\sqrt{2})$

$\eta_1 = \sinh z / \cosh^2 z$

$\eta_q = e^{iqz}(3\tanh^2 z - 1 - q^2 - 3iq \tanh z)$

Omit η_0 $\because$ only translate the position of the brane.

: should be described by other $e_{k\mu}$ & $b_{\mu\nu}$.

The dynamical degrees of η_0
are transferred to $e_{k\mu}$ & $b_{\mu\nu}$

Substitute $\Phi = \Phi_{br} + \varphi_1\eta_1 + \int \varphi_q \eta_q \, dq$

to the action $S = \int L\, d^5x = \int E\left(\frac{1}{2} G^{MN}\partial_M\Phi\,\partial_N\Phi - U(\Phi)\right)d^5x$

Rearrange terms with repect to $e_{\mu\mu}$, $b_{\mu\nu}$, φ_1, & φ_q

The terms with $e_{\mu\mu}$, $b_{\mu\nu}$ & φ_1 are localized around the brane

trapped

$$S = \int E\Big[\frac{1}{2} G^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1\,\eta_1^2 - \frac{1}{2}\varphi_1^2\eta_1'^2 + \frac{1}{4}\lambda(\Phi_K + \chi\Phi_K')^4$$
$$- \frac{3}{2}\lambda(\Phi_K + \chi\Phi_K')\varphi_1^2\eta_1^2 + \frac{1}{2}m^2\varphi_1^2\eta^2 - \frac{m^4}{4\lambda}\Big]dy\, d^4x$$

$$(\chi = \frac{1}{2}by^2 + \frac{1}{6}(b_2 + b^2)y^3 + \dots)$$

Perform y-integration in the localized sector

$$\int\left(\Phi_K^4 - \frac{m^4}{\lambda^2}\right)dy = -\frac{8\sqrt{2}m^3}{3\lambda^2}, \quad \int\left(\Phi_K^4 - \frac{m^2}{\lambda^2}\right)y^2 dy = -\frac{4\sqrt{2}m}{\lambda^2}\left(1 - \frac{\pi^2}{12}\right)$$

$$\int \eta_1^2 dy = \frac{2\sqrt{2}}{3m}, \quad \int y^2\eta_1^2 dy = \frac{8\sqrt{2}}{3m^2}\left(1 + \frac{\pi^2}{12}\right), \quad \int \eta_1'^2 dy = \frac{14m}{15\sqrt{2}}$$

$$\int \Phi_K^2\eta_1^2 dy = \frac{2\sqrt{2}m}{5\lambda}, \quad \int y^2\Phi_K^2\eta_1^2 dy = \frac{2\sqrt{2}}{\lambda m}\left(\frac{\pi^2}{30} + \frac{2}{3}\right)$$

$$\int y^2\Phi_K\Phi_K'\eta_1^2 dy = \frac{2\sqrt{2}}{\lambda m}\left(\frac{7\pi^2}{180} - \frac{1}{6}\right) \qquad \int y^2\eta_1'^2 dy = \frac{7\sqrt{2}}{90m}\pi^2$$

$$\int y^4\Phi_K'^2\eta_1^2 dy = \frac{2\sqrt{2}}{\lambda m}\left(-\frac{6}{5} + \frac{16\pi^2}{15} - \frac{2\pi^4}{75}\right)$$

$$\int y^4\Phi_K^2\Phi_K'^2 dy = \frac{2\sqrt{2}m}{\lambda^3}\left(\frac{7\pi^4}{800} - \frac{2}{5}\right)$$

Effective action for $e_{k\mu}$, $b_{\mu\nu}$ & φ_1 on the brane

$$S=\int e\Big[-\frac{2\sqrt{2}\,m^3}{3\lambda}$$

(cosmological term)

$$+\frac{\sqrt{2}\,m}{\lambda}\left(1-\frac{\pi^2}{12}\right)b_2+\frac{\sqrt{2}\,m}{\lambda}\left(\frac{7\pi^4}{1200}+\frac{\pi^2}{8}-\frac{9}{10}\right)b^2$$

(b-mass term)

$$+\frac{\sqrt{2}}{3m}\left[g^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1-\frac{3}{2}m^2\varphi_1^2\right]$$

(φ_1 kinetic & mass term)

$$+\frac{2\sqrt{2}}{3m^3}\left(1-\frac{\pi^2}{12}\right)\left\{6b^\mu{}_\lambda b^{\lambda\nu}-4b\,b^{\mu\nu}-(b_2-b^2)g^{\mu\nu}\right\}\partial_\mu\varphi_1\partial_\nu\varphi_1$$

$$+\frac{\sqrt{2}}{m}\left\{\left(\frac{1}{6}-\frac{\pi^2}{40}\right)b_2+\left(\frac{9}{10}+\frac{11\pi^2}{120}+\frac{\pi^4}{50}\right)b^2\right\}\varphi_1^2\Big]d^4x$$

(e-b-φ_1 interaction terms)

Masses & coupling constants are calculated.

If bsonic & fermionic fields exist in the whole space and are coupled to Φ

their low-lying modes are trapped around the brane and described by similar effective actions as above.

No kinetic terms of $e_{k\mu}$ & $b_{\mu\nu}$

because we assumed that the whole space is flat

If the whole spacetime is curved,
the E_{KM} involves the derivatives of $e_{k\mu}$ & $b_{\mu\nu}$
which give rise to their kinetic terms

brane-Einstein gravity action --- trapped graviton

extrinsic curvature fields --- dynamical in the brane
under the constraints of Gauss & Codazzi equations.

Other effects which contribute to the kinetic terms
are the quantum fluctuations of the trapped matters,
which are calculated through
the quantum loop diagrams.

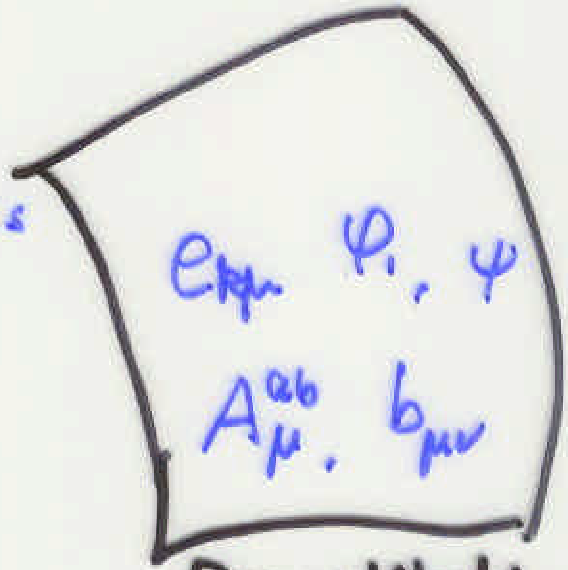
The kinetic terms due to the quantum effects
can be induced even if the whole spacetime is flat.

So far we assumed 3 brane in a 4+1 dimensions.
If the number of the extra dimensions $N > 1$
the normal connections are the gauge fields of $O(N)$.
If we interpret them as the physical gauge fields
like photon, gluon, and weak bosons,
interesting phenomenological consequences may emerge,
since they are constrained by Gauss-Codazzi-Ricci eqs.

Conclusion

On the dynamically localized brane world,

(i) low lying small fluctuation modes are trapped with proper field theoretical actions



Brane World

(ii) the gravitational field $e_{k\mu}$, the extrinsic curvature $b_{\mu\nu}$, and the gauge fields A^{ab}_μ are induced on the brane

(iii) masses and coupling constants are calculated

(iv) if the whole spacetime is curved, the kinetic terms of $e_{k\mu}$, $b_{\mu\nu}$, & A^{ab}_μ are induced

(v) the kinetic terms of $e_{k\mu}$, $b_{\mu\nu}$, & A^{ab}_μ are induced also through quantum fluctuations, and they are induced even if the whole space is flat.

(vi) $e_{k\mu}$, $b_{\mu\nu}$, & A^{ab}_μ should obey the Gauss-Codazzi-Ricci equations in addition to the equations of motion.