

①

ICHEP 2000

Osaka

7/29/00

NEUTRALINO-PROTON CROSS SECTIONS

R. ARNOWITT

B. DUTTA

Y. SANTOSO

Texas A&M University

1. INTRODUCTION

Gravity mediated supergravity models with R-parity invariance automatically possess a dark matter candidate, the lightest supersymmetric particle (LSP), which in most models is the neutralino, $\tilde{\chi}_i^0$. Experiments to directly test the existence of the $\tilde{\chi}_i^0$ in the halo of the Milky Way look $\tilde{\chi}_i^0$ scattering by terrestrial nuclear targets. For heavy nuclei, the spin independent scattering dominates, allowing one to determine the $\tilde{\chi}_i^0$ -proton cross section $\sigma_{\tilde{\chi}_i^0-p}$. Current experiments are sensitive to halo $\tilde{\chi}_i^0$ of (DAMA, CDMS):

$$\sigma_{\tilde{\chi}_i^0-p} \gtrsim 1 \times 10^{-6} \text{ pb}$$

and future detectors plan to achieve a sensitivity of (GENIUS, Cryoarray):

$$\sigma_{\tilde{\chi}_i^0-p} \gtrsim (10^{-9} - 10^{-10}) \text{ pb}$$

We consider here two questions:

- * What is the range of SUSY parameters current detectors are sensitive to?
- * What is the smallest values of $\tilde{\chi}_{i,0-p}$ theory is predicting?

The answer to these questions depends in part on what models one considers:

- * We examine here models based on grand unification at $M_G \cong 2 \times 10^{16}$ GeV with radiative breaking of $SU(2) \times U(1)$ at the electroweak scale

and also in part on the choice of input parameters and range of SUSY parameters (i.e. assumptions on "naturalness" conditions)

- * Range of input parameters can produce factors ≈ 3 in $\tilde{\chi}_{i,0-p}$

3

There are a number of models based on grand unification of the gauge coupling constants at $M_G \cong 2 \times 10^{16}$ GeV:

Minimal Supergravity GUT (mSUGRA)
Universal soft breaking at M_G

Nonuniversal Soft Breaking Models
Nonuniversal scalar masses in Higgs and third generation at M_G

There are "string inspired" models

D-brane Models (IIB Orientifolds)
Nonuniversal gaugino masses at M_G
 $SU(2)_L$ doublet masses different from singlet scalar masses at M_G

Horava-Witten M-Theory (Calabi-Yau)
Universal gaugino masses at M_G
Generation nonuniversality of scalar masses at M_G

Theory allows one to calculate both the detection cross section, $\sigma_{\tilde{\chi}_i^0-p}$, and the relic density $\Omega_{\tilde{\chi}_i^0} h^2$:

$$\Omega_{\tilde{\chi}_i^0} h^2 = 2.48 \times 10^{-11} \left(\frac{T_{\tilde{\chi}_i^0}}{T_r} \right)^3 \left(\frac{T_r}{2.73} \right)^3 \frac{N_f^{1/2}}{\int_0^{k_s} dx \langle \sigma_{\text{ann}} v \rangle}$$

where

$$\Omega_{\tilde{\chi}_i^0} = \rho_{\tilde{\chi}_i^0} / \rho_c; \quad \rho_c = 3H_0^2 / 8\pi G; \quad h = H_0 / 100$$

and

σ_{ann} = early universe annihilation cross section

Astronomical data gives:

$$H_0 = 70 \pm 7 \text{ km/s Mpc} \quad [\text{Friedman}]$$

$$\Omega_m \cong 0.3 \pm 0.1 \quad [\text{Dodelson, Knox; Mohr et al., Lineweaver; Efsthiva}]$$

and using $\Omega_b \cong 0.05$ one has

$$\Omega_{\tilde{\chi}_i^0} \cong 0.25 \pm 0.10$$

Combining errors in quadrature gives

$$\Omega_{\tilde{\chi}_i^0} h^2 = 0.12 \pm 0.05$$

We assume here a $\approx 2\sigma$ range

$$0.02 \leq \Omega_{\tilde{\chi}_i^0} h^2 \leq 0.25$$

2. CALCULATIONAL DETAILS

In order to get reasonably accurate theoretical answers, necessary to include a number of corrections and constraints. Summarize some of these now:

Parameter space (mSUGRA)

$$m_0 \leq 1 \text{ TeV}$$

$$m_{1/2} \leq 600 \text{ GeV} \quad (m_g \leq 1.5 \text{ TeV}; m_{\tilde{\chi}_i^0} \leq 240 \text{ GeV})$$

$$2 \leq \tan\beta \leq 50 \quad ; \quad \tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$$

($\sigma_{\tilde{\chi}_i^0-p}$ increases with $\tan\beta$; SO(10) $\tan\beta \approx 40$)

$$|A_0/m_0| \leq 5 \quad (\text{default})$$

Theoretical analysis

(6)

- (i) Run 1-loop Yukawa and 2-loop gauge RGE from $M_G = 2 \times 10^{16}$ GeV down to electroweak scale, iterating to get a consistent SUSY spectrum
- (ii) QCD RGE corrections included for contributions dominated by light quarks
- (iii) 1-loop, 2-loop and pole mass corrections to Higgs mass m_h .
- (iv) L-R mixing in sfermion mass matrices (Important for large $\tan\beta$ in 3rd generation)
- (v) 1-loop correction to m_b, m_τ (Important for large $\tan\beta$)
- (vi) LO and (approximate) NLO corrections to $b \rightarrow s + \gamma$ decay.

We do not impose $b-\bar{c}$ (or $b-\bar{c}-t$) Yukawa unification as this depends sensitively on unknown **post-GUT** physics. Thus this does not naturally occur in string models where $SU(5)$ (or $SO(10)$) are broken by **Wilson lines**.

Accelerator bounds

LEP: * $m_h > 104 \text{ GeV}$; $\tan\beta = 3$

* $m_h > 100 \text{ GeV}$; $\tan\beta = 5$

$m_{\chi_i^\pm} > 102 \text{ GeV}$

Tevatron: $m_{\tilde{g}} \gtrsim 270 \text{ GeV}$ ($m_{\tilde{t}} \equiv m_{\tilde{b}}$)

CLEO: * $1.8 \times 10^{-4} \leq \mathcal{B}(B \rightarrow \chi_0 \gamma) \leq 4.5 \times 10^{-4}$

Properties of proton

Theory gives naturally the $\tilde{\chi}_i^0$ -quark cross section and one must convert this to $\tilde{\chi}_i^0$ -proton scattering to compare with experiment. One need the quantities [Ellis, Flores; Drees, Nojiri]:

$$\sigma_{\pi N} = \frac{1}{2}(m_u + m_d) \langle p | \bar{u}u + \bar{d}d | p \rangle$$

$$\sigma_0 = \frac{1}{2}(m_u + m_d) \langle p | \bar{u}u + \bar{d}d - 2S_S | p \rangle$$

$$r = \frac{m_S}{\frac{1}{2}(m_u + m_d)}$$

We use in the following

$$\sigma_{\pi N} = 65 \text{ MeV} \text{ [Ollson; Paven et al.]}$$

$$\sigma_0 = 30 \text{ MeV} \text{ [Botino et al.]}$$

$$r = 24.4 \pm 1.5 \text{ [Leutwyler]}$$

Other choices can change $\sigma_{\tilde{\chi}_i^0-p}$ by about a factor ≈ 3 .

3. mSUGRA MODEL

The mSUGRA model depends on 4 parameters and one sign. Since there is now a considerable amount of constraints from data, it has become relatively predictive. The parameters are:

m_0 : universal scalar mass at M_G

$m_{1/2}$: universal gaugino mass at M_G
(alternately: $m_{\tilde{\chi}_i^0} \cong 0.4 m_{1/2}$; $m_{\tilde{g}} \cong 2.8 m_{1/2}$)

A_0 : universal cubic soft breaking mass

$\tan\beta$: $\cong \langle H_2 \rangle / \langle H_1 \rangle$

$\frac{\mu}{|\mu|}$: sign of Higgs mixing parameter ($W_\mu = \mu H_1 H_2$)

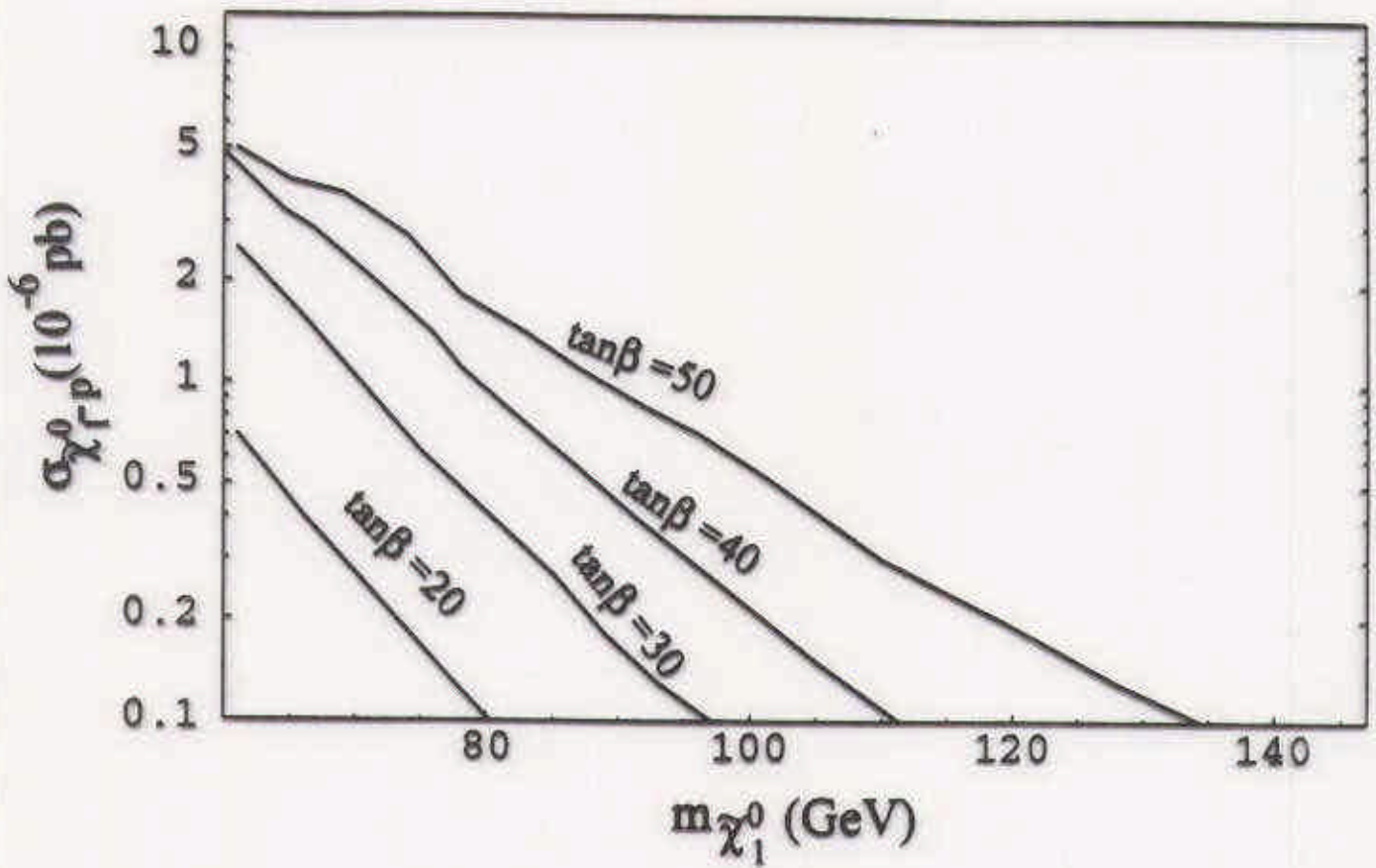
Maximum cross section:

$\sigma_{\tilde{\chi}_i^0 - p}$ is an **increasing** function of $\tan\beta$
and a **decreasing** function of $m_{1/2}, m_0$

Thus maximum cross sections should occur at large $\tan\beta$ and small $m_{\tilde{\chi}_i^0}$.

Fig. $(\sigma_{\tilde{\chi}_i^0 - p})_{\max}$ obtained by varying A_0 over parameter space for $\tan\beta = 20, 30, 40, 50$.

Maximum $\sigma_{\tilde{\chi}_1^0-p}$ for $\tan\beta = 20, 30, 40, 50$ (mSUGRA)



One sees that current sensitivity of $\sigma_{\tilde{\chi}_i^0 - p} \gtrsim 1 \times 10^{-6} \text{ pb}$ is sampling mSUGRA for

$$\tan\beta \gtrsim 25 ; m_{\tilde{\chi}_i^0} \lesssim 90 \text{ GeV}$$

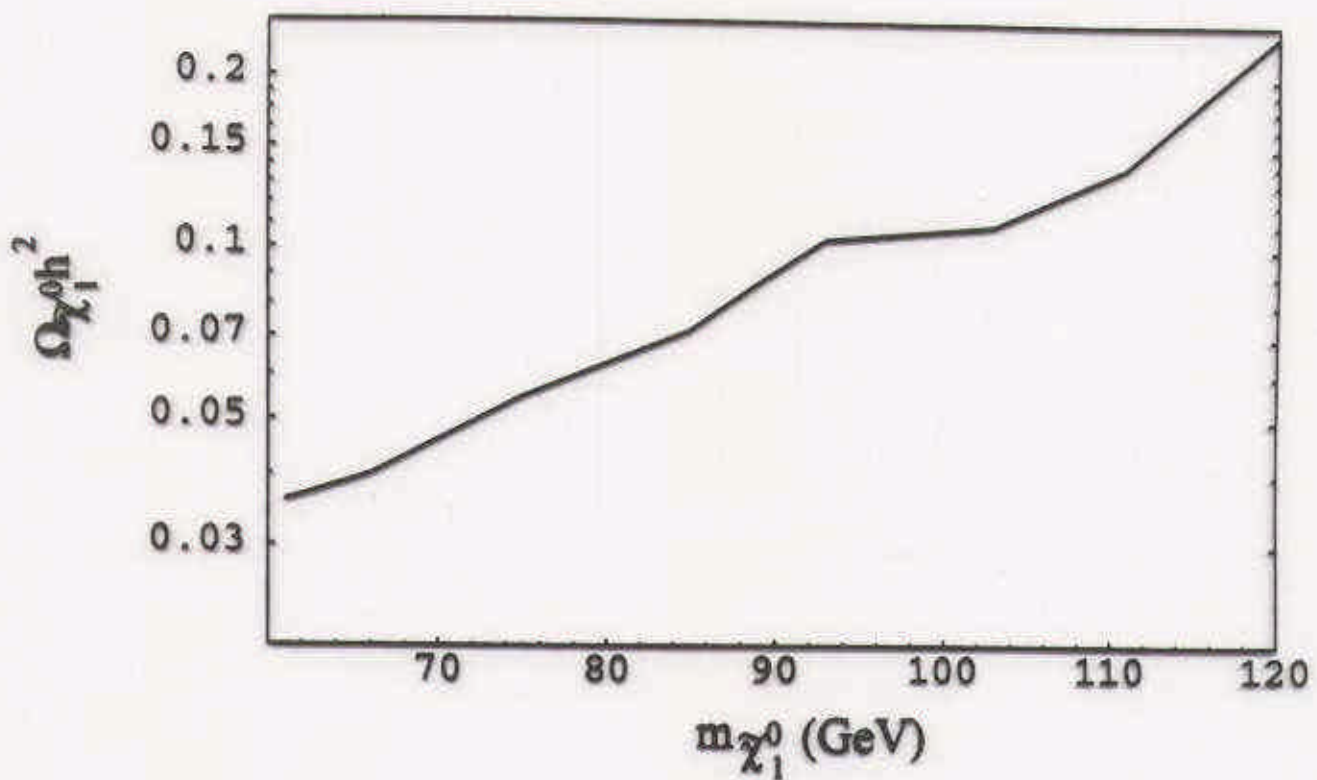
The early universe σ_{ann} decreases with $m_{\tilde{\chi}_i^0}$ and hence $\Omega_{\tilde{\chi}_i^0} h^2$ increases with $m_{\tilde{\chi}_i^0}$

Fig $\Omega_{\tilde{\chi}_i^0} h^2$ as function of $m_{\tilde{\chi}_i^0}$ for $\tan\beta = 30$ when $\sigma_{\tilde{\chi}_i^0 - p}$ is at maximum.

Thus current sensitivity is sampling low $\Omega_{\tilde{\chi}_i^0} h^2$ i.e.

$$\Omega_{\tilde{\chi}_i^0} h^2 \lesssim 0.1$$

$\Omega_{\tilde{\chi}_1^0} h^2$ for $(\sigma_{\tilde{\chi}_1^0-p})_{\max}$; $\tan\beta=30$



(11)

Minimum cross section ($m_{\tilde{\chi}_1^0} \lesssim 150 \text{ GeV}$):

In this region there is no coannihilation, and the lowest cross sections occur at the smallest $\tan\beta$.

Fig. Minimum $\sigma_{\tilde{\chi}_1^0-p}$ as function of $m_{\tilde{\chi}_1^0}$ for $\tan\beta=3$.

One sees that for this domain

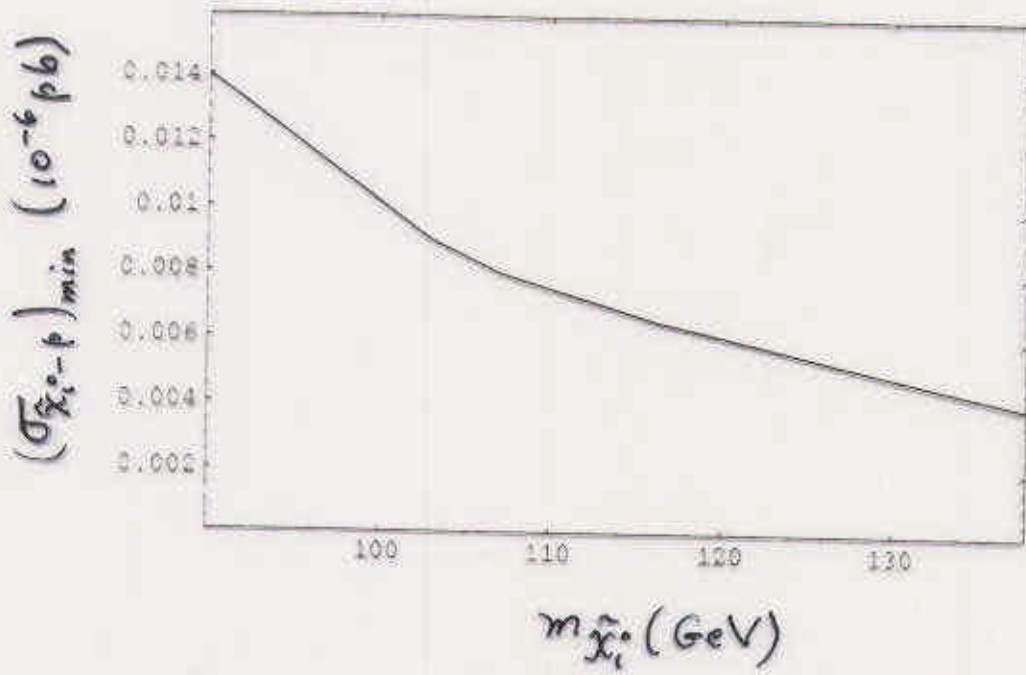
$$\sigma_{\tilde{\chi}_1^0-p} \gtrsim 4 \times 10^{-9} \text{ pb} ; m_{\tilde{\chi}_1^0} \lesssim 140 \text{ GeV}$$

which would be accessible to planned detectors (e.g. GENIUS).

Coannihilation ($m_{\tilde{\chi}_1^0} \gtrsim 150 \text{ GeV}$):

Coannihilation in the early universe occurs when a second SUSY particle becomes nearly degenerate with the LSP (i.e. $\tilde{\chi}_1^0$). The effect is significant in mSUGRA due to two "peculiarities":

mSUGRA: $(\sigma_{\tilde{\chi}_1^0 \rightarrow p})_{\min}$; $\tan\beta = 3$
 $m_{1/2} < 345 \text{ GeV}$



(1) $\tilde{\chi}_i^0$ is Majorana spinor and so its early universe annihilation cross section $\sigma_{\tilde{\chi}_i^0 \tilde{\chi}_i^0}$ is suppressed relative to sleptons (\tilde{l}_R):

$$\sigma_{\tilde{\chi}_i^0 \tilde{\chi}_i^0} \approx \frac{1}{10} (\sigma_{\tilde{\chi}_i^0 \tilde{l}_R}, \sigma_{\tilde{l}_R \tilde{l}_R})$$

(2) Accidental near degeneracy between \tilde{l}_R and $\tilde{\chi}_i^0$ in a small region of parameter space. Thus low $\tan\beta$:

$$m_{\tilde{e}_R}^2 = m_0^2 + \frac{6}{5} \frac{\alpha_0}{4\pi} f_1 m_{1/2}^2 - 5m^2 E_W M_W \cos 2\beta$$
$$f_1 = \frac{1}{\beta_1} \left[1 - \frac{1}{(1+\beta_1)^2} \right]$$

$$m_{\tilde{\chi}_i^0}^2 \approx \frac{\alpha_i}{\alpha_0} m_{1/2}^2$$

or numerically:

$$m_{\tilde{e}_R}^2 \approx m_0^2 + \underline{0.15} m_{1/2}^2 + (40 \text{ GeV})^2$$

$$m_{\tilde{\chi}_i^0}^2 \approx \underline{0.16} m_{1/2}^2$$

Thus by choosing m_0 one can make $m_{e_R} > m_{\tilde{\chi}_i^0}$ but nearly degenerate in a narrow "chimney" in $m_0 - m_{1/2}$ space

Fig $m_0 - m_{1/2}$ allowed region for $A_0 = m_{1/2}, \mu < 0$
 $\tan\beta = 3, 5, 10, 20$ [Ellis, Falk, Gais, Olive].

$$A = m_{1/2}; \mu < 0$$

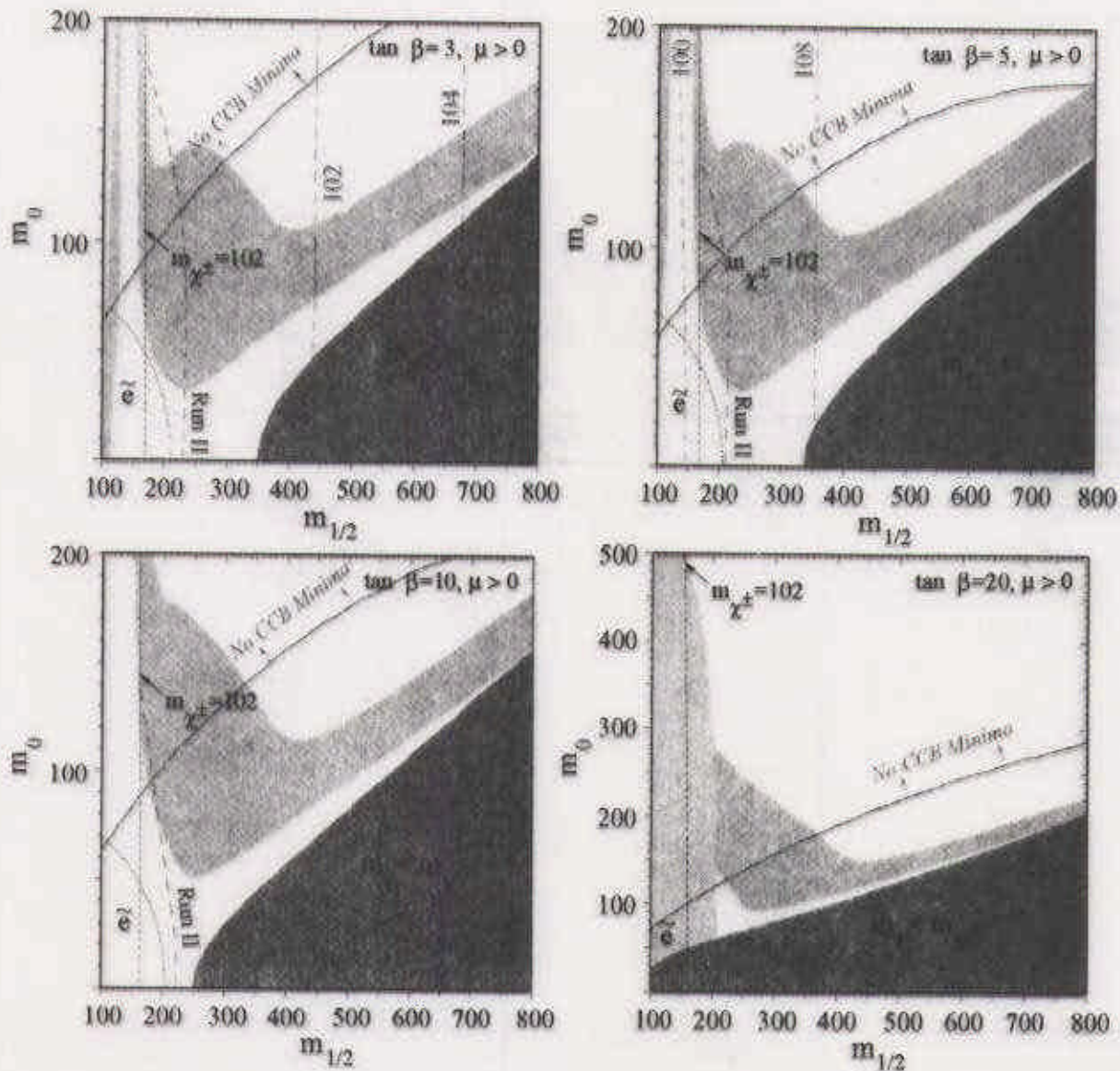


Figure 9: The $m_{1/2}, m_0$ plane for $\mu > 0$, $A = -m_{1/2}$ and $\tan \beta = (a) 3, (b) 5, (c) 10$ and $(d) 20$. The significances of the curves and shadings are the same as in Fig. 8. The light-shaded region in panel (d) is excluded by the $b \rightarrow s\gamma$ constraint. The long dashed curves in panels (a), (b) and (c) represent the anticipated limits from trilepton searches at Run II of the Tevatron [2].

[Ellis, Falk, Gais, Olive]

One sees that coannihilation begins at:

(12)

$$m_{1/2} \geq 400 \text{ GeV} \quad (m_{\tilde{\chi}_1^0} = 160 \text{ GeV})$$

The region of coannihilation is narrow, i.e.

$$\Delta m_0 \approx 25 \text{ GeV}$$

It is interesting to see what happens at high $\tan\beta$. We consider first: $\mu < 0$.

Fig. Allowed region in $m_0 - m_{1/2}$ plane for $\tan\beta = 40$, $\mu < 0$,
 $A_0 = m_{1/2}, 2m_{1/2}, 4m_{1/2}$ (bottom to top).

For large $\tan\beta$, the \tilde{t}_1 is lightest slepton and due to large L-R mixing is considerably lighter than other sleptons. One has

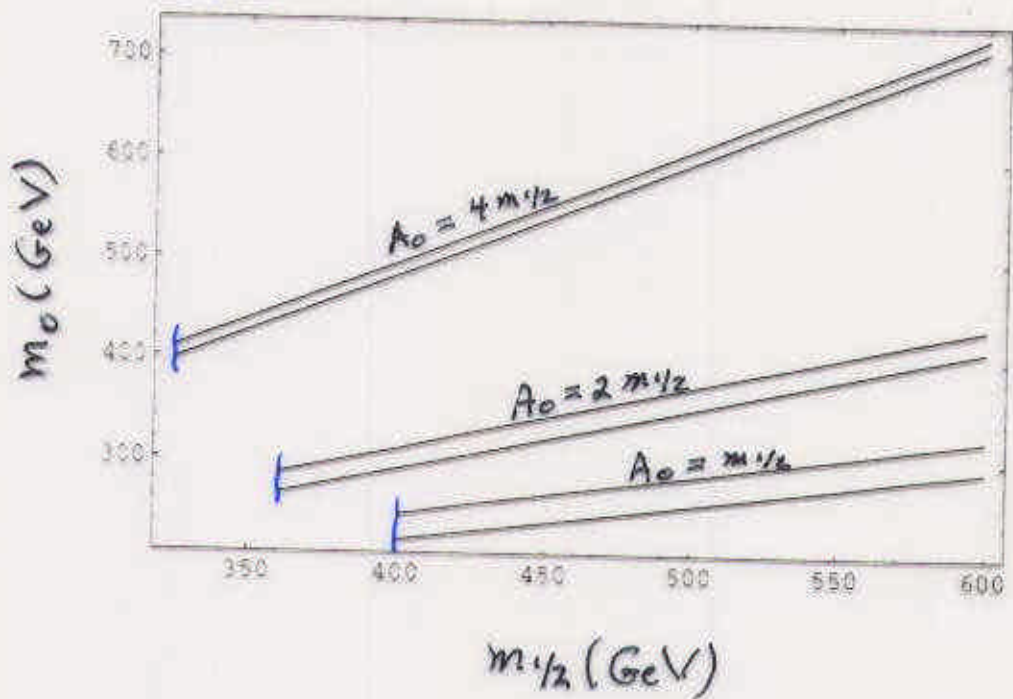
Only coannihilation region left and allowed region sensitive to A_0

Region to left of minimum $m_{1/2}$ excluded by $b \rightarrow s\tau\tau$ constraint

Minimum $m_{1/2}$ decreases with increasing A_0

Thickness of coannihilation chimney decreases with increasing A_0 .

mSUGRA: $\tan\beta = 40$; $A_0 = m_{1/2}, 2m_{1/2}, 4m_{1/2}$
 $m_0 - m_{1/2}$ allowed regions
 $\mu < 0$



| : low $m_{1/2}$ bound due to $b \rightarrow s\gamma$ constraint

The minimum $\sigma_{\tilde{\chi}_i^0-p}$ still arises from smallest allowed $\tan\beta$. However $\sigma_{\tilde{\chi}_i^0-p}$ decreases with increasing A_0 for large $\tan\beta$:

Fig. $\sigma_{\tilde{\chi}_i^0-p}$ as a function of $m_{1/2}$ for $A_0 = 4m_{1/2}$ (lower), $A_0 = 2m_{1/2}$ (upper), $\tan\beta = 40$, $\mu < 0$

and for large $m_{1/2}$, small and large $\tan\beta$ curves approach each other (in coannihilation region):

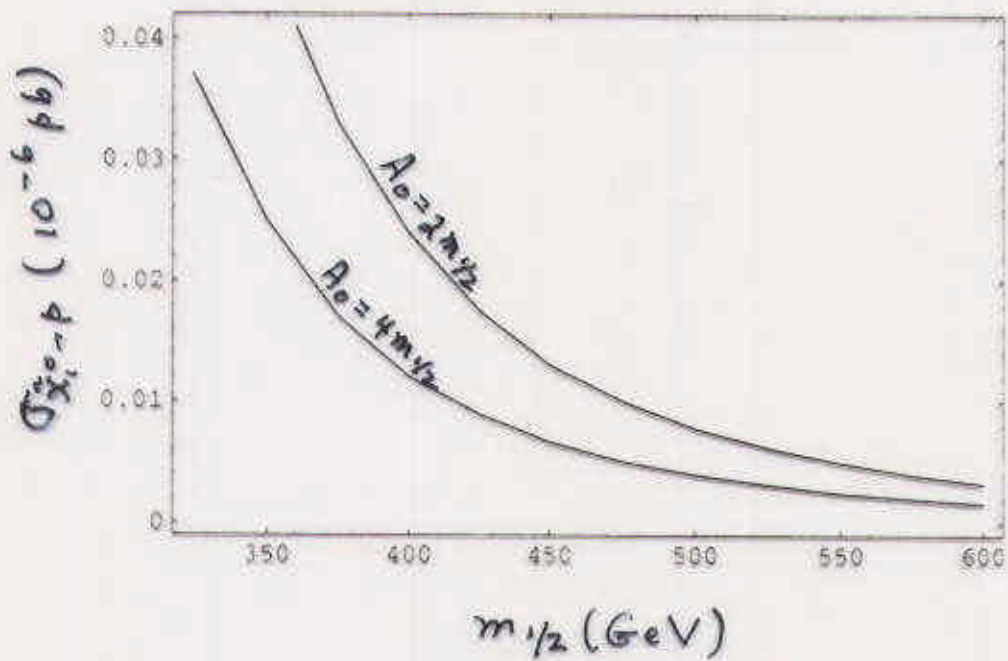
Fig. $\sigma_{\tilde{\chi}_i^0-p}$ as a function of $m_{1/2}$ for $A_0 = 4m_{1/2}$, $\tan\beta = 40$ (upper), $\tan\beta = 3$ (lower), $\mu < 0$.

Thus:

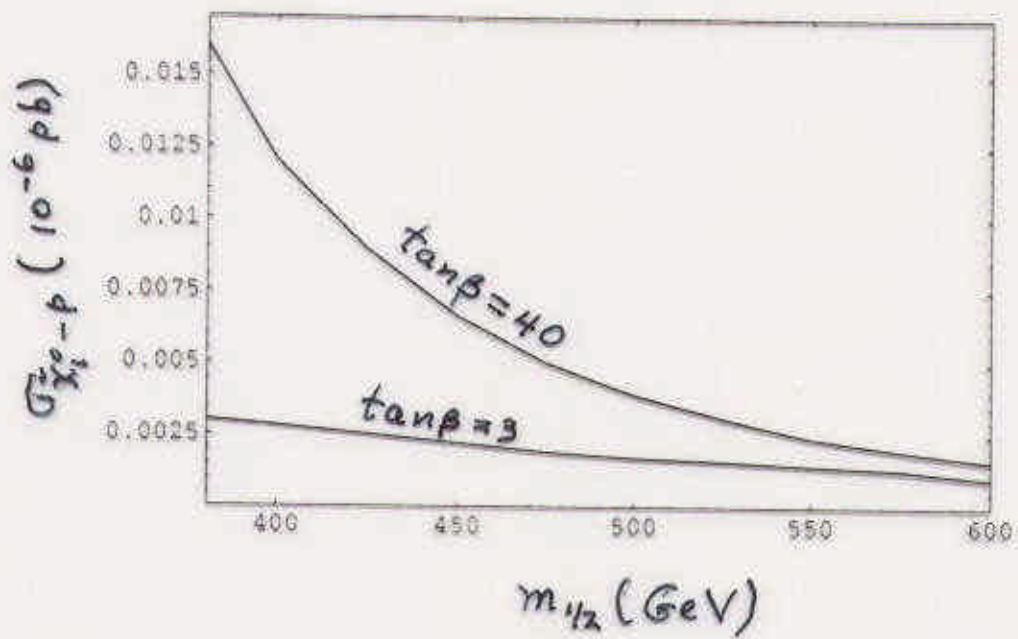
$$\sigma_{\tilde{\chi}_i^0-p} \gtrsim 1 \times 10^{-9} \text{ pb} ; m_{1/2} < 600 \text{ GeV} \\ (m_{\tilde{\chi}_i^0} < 240 \text{ GeV}) \\ \mu < 0$$

and the region should be accessible to proposed detectors (e.g. GENIUS).

mSUGRA: $\tan\beta = 40$, $A_0 = 2m_{1/2}, 4m_{1/2}$
 $\mu < 0$



mSUGRA: $A_0 = 4 m_{1/2}$;
 $\tan\beta = 40,3$
 $\mu < 0$



We consider next $\mu > 0$: As pointed out by Ellis, Ferstl and Olive, an "accidental" cancellation can occur in part of the parameter space in the coannihilation region which can greatly reduce $\sigma_{\tilde{\chi}_i^0-p}$. This can be seen in following figures:

Fig. $\sigma_{\tilde{\chi}_i^0-p}$ for $\tan\beta = 5, 10, 20$; $\mu > 0, m_{1/2} > 300 \text{ GeV}$

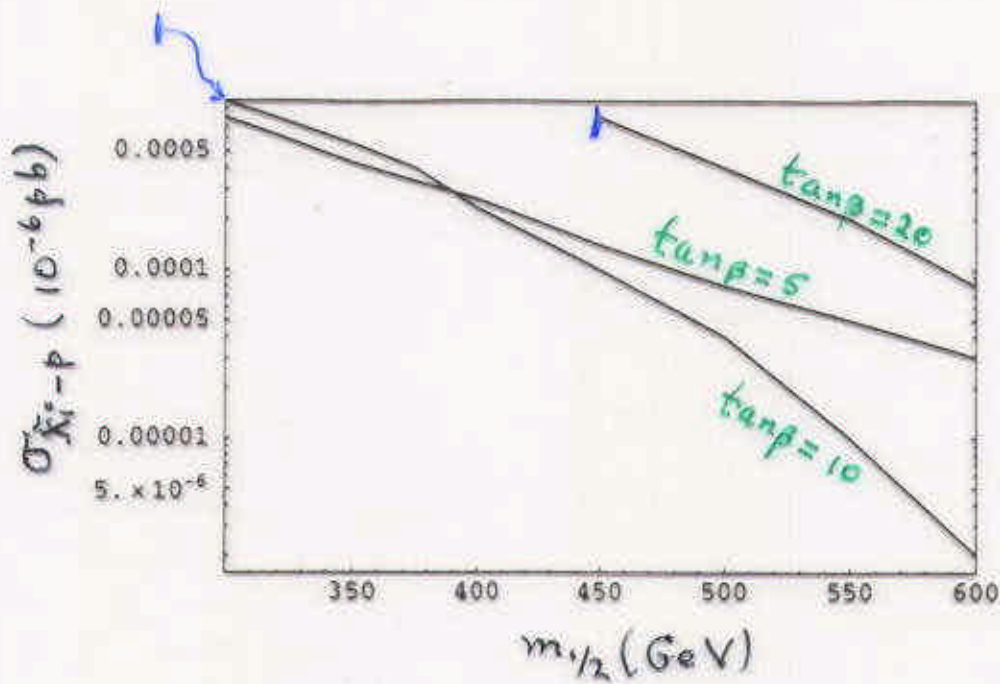
One sees:

- * $\sigma_{\tilde{\chi}_i^0-p} < 10^{-10} \text{ pb}$ for $4 \lesssim \tan\beta < 20$; $\mu > 0$
 $m_{1/2} \gtrsim 450 \text{ GeV} (m_g > 1.1 \text{ TeV})$
- * $(\sigma_{\tilde{\chi}_i^0-p})_{\min} \approx 1 \times 10^{-12} \text{ pb}$; $\tan\beta = 10, m_{1/2} \leq 600 \text{ GeV}$
 $\mu > 0$

In this region, proposed detectors would **not** be able to detect halo $\tilde{\chi}_i^0$ wimps. However (assuming mSUGRA is a valid theory), the absence of detection of wimps would imply that the squarks and gluino lie $\gtrsim 1 \text{ TeV}$, which could be verified at the LHC.

$\sigma_{\tilde{\chi}_1^0 \rightarrow p}$ for $\tan\beta = 5, 10, 20$; $\mu > 0$

$m_{1/2} > 300 \text{ GeV}$



\downarrow = lower bound due to $b \rightarrow s\gamma$ constraint

4. NONUNIVERSAL SUGRA MODELS

In most discussions of nonuniversal SUGRA models, universality of first two generations of squark/slepton masses at M_G is maintained (to suppress FCNC) allowing Higgs and third generation masses to become nonuniversal.

One can parameterize this situation as follows:

$$m_{H_1}^2 = m_0^2 (1 + \delta_1); \quad m_{H_2}^2 = m_0^2 (1 + \delta_2)$$

$$m_{q_L}^2 = m_0^2 (1 + \delta_3); \quad m_{t_R}^2 = m_0^2 (1 + \delta_4); \quad m_{\tau_R}^2 = m_0^2 (1 + \delta_5)$$

$$m_{b_R}^2 = m_0^2 (1 + \delta_6); \quad m_{\mu_L}^2 = m_0^2 (1 + \delta_7) \quad ; \quad Q = M_G$$

[If one imposes $SU(5)$ or $SO(10)$ symmetry then $\delta_3 = \delta_4 = \delta_5$ and $\delta_6 = \delta_7$.]

In following we limit the δ_i :

$$-1 \leq \delta_i \leq +1$$

and maintain gauge and gaugino unification at M_G .

(15)

While there are a large number of new parameters, one can get an understanding of what effects they produce from the following:

The $\tilde{\chi}_1^0$ is a mixture of **gaugino** (mostly \tilde{B}) and **higgsino** parts:

$$\tilde{\chi}_1^0 = \alpha \tilde{W}_3 + \beta \tilde{B} + \gamma \tilde{H}_1 + \delta \tilde{H}_2$$

The dominant spin independent $\tilde{\chi}_1^0$ -p cross section is proportional to the interference between the gaugino and higgsino amplitudes and this interference is governed by μ^2 :

As μ^2 decreases interference and hence $\sigma_{\tilde{\chi}_1^0-p}$ increases

Now radiative breaking of $SU(2) \times U(1)$ determines μ^2 at the electroweak scale. For low $\tan\beta$ an analytic form exists for μ^2 (with similar structure holding at high $\tan\beta$):

$$\mu^2 = \frac{t^2}{t^2-1} \left[\left(\frac{1-3D_0}{2} - \frac{1}{t^2} \right) + \left(\frac{1-D_0(\delta_3+\delta_4)}{2} - \frac{1+D_0}{2} \delta_2 + \frac{\delta_1}{t^2} \right) \right] m_0^2$$

+ universal parts + loop corrections

where

$$t \equiv \tan\beta; \quad D_0 \equiv 1 - (m_t/200 \text{ smp})^2 \leq 0.2$$

Hence

μ^2 is reduced, $\sigma_{\tilde{\chi}_i^0-p}$ is increased, for $\delta_3, \delta_4, \delta_1 < 0, \delta_2 > 0$.

μ^2 is increased, $\sigma_{\tilde{\chi}_i^0-p}$ is reduced, for $\delta_3, \delta_4, \delta_1 > 0, \delta_2 < 0$.

The Higgs and squark nonuniversalities enter coherently, roughly in combination: $\delta_3 + \delta_4 - \delta_2$

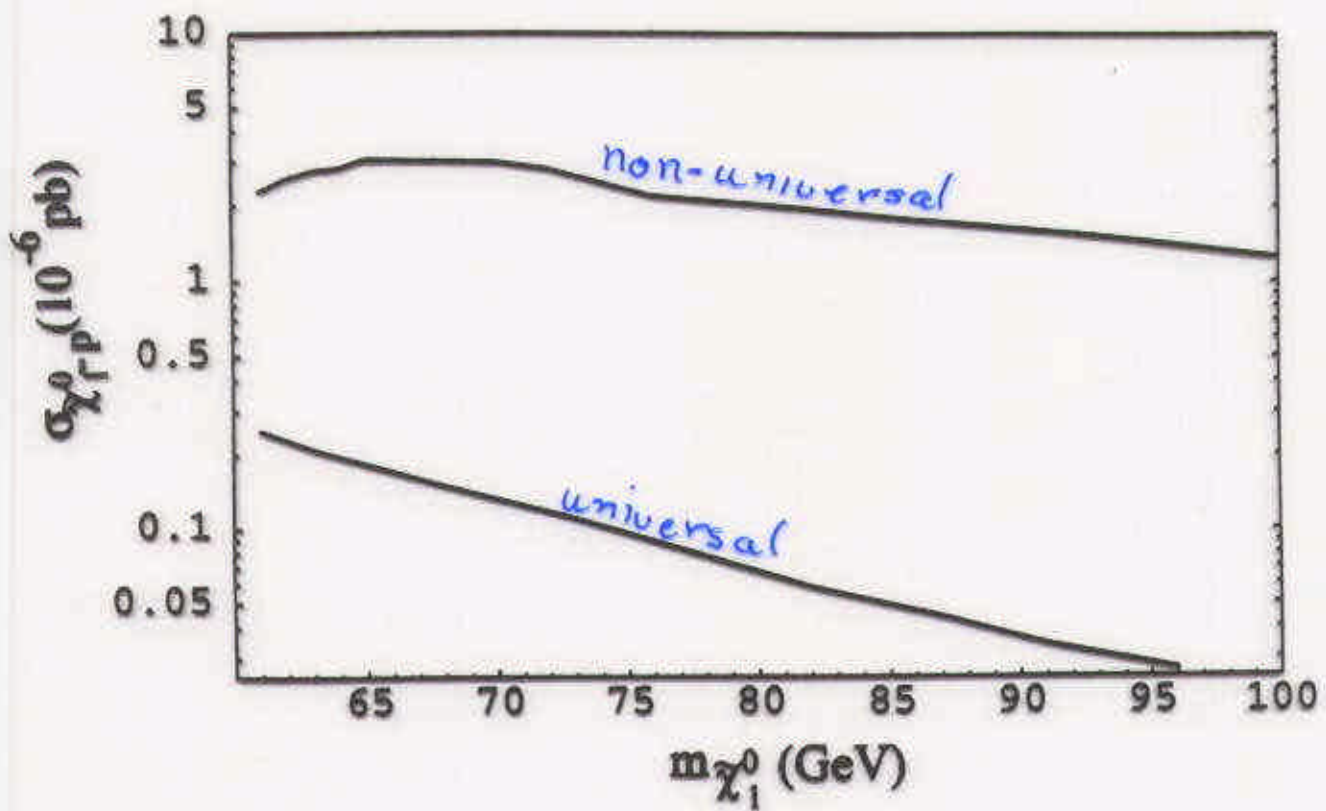
Thus one can get significantly larger cross sections in nonuniversal models:

Fig. Maximum $\sigma_{\tilde{\chi}_i^0-p}$ for $\tan\beta=7$, universal (lower) nonuniversal (upper) curve.

$\sigma_{\tilde{\chi}_i^0-p}$ can increase by factor of 10-100 due to nonuniversal soft breaking.

Maximum $\sigma_{\chi_1^0-p}$; $\tan\beta=7$, $\mu < 0$

$\delta_3, \delta_4, \delta_1 < 0$; $\delta_2 > 0$



(17)

As a consequence, current detectors can probe nonuniversal models to lower $\tan\beta$:

Fig. Maximum $\sigma_{\tilde{\chi}_i^0-p}$ as a function of $m_{\tilde{\chi}_i^0}$ for $\tan\beta = 7,5$ for nonuniversal soft breaking

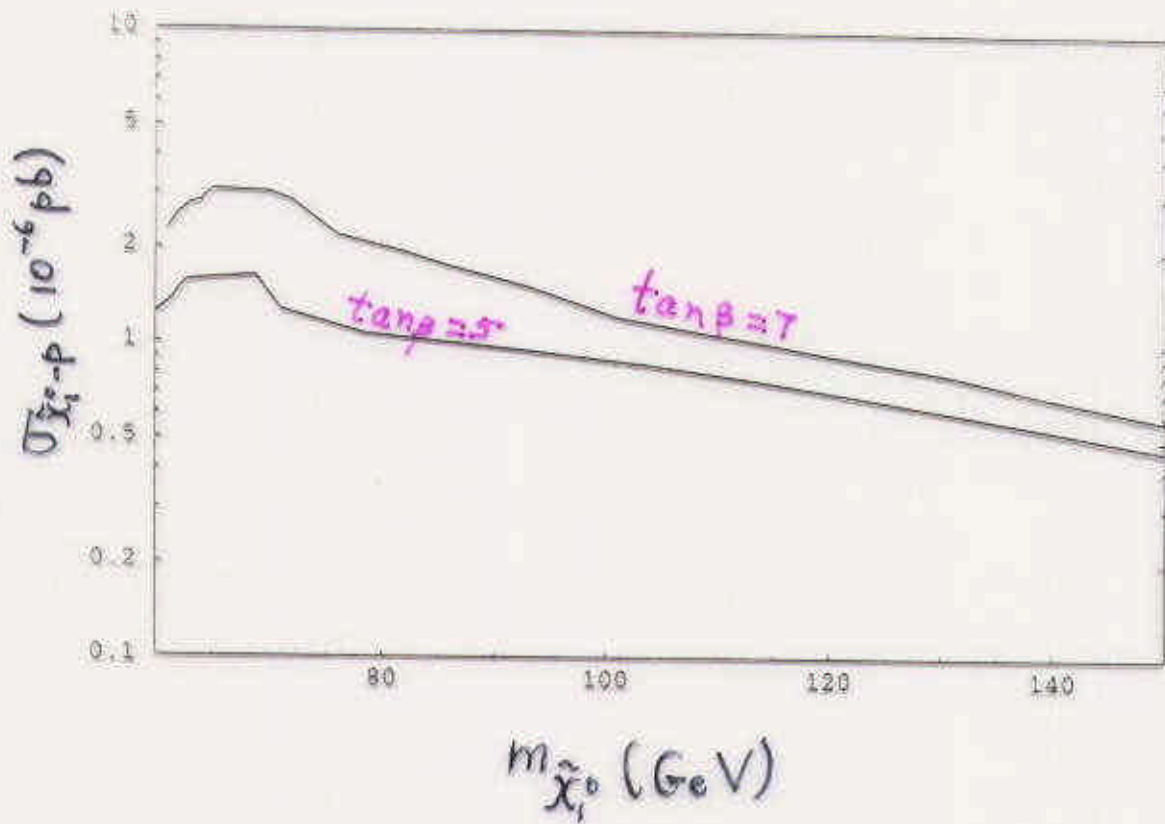
Thus current detectors can probe part of the parameter space for $\tan\beta \gtrsim 4$; nonuniversal models

The minimum $\sigma_{\tilde{\chi}_i^0-p}$ occurs (as before) for lowest $\tan\beta$:

Fig. $(\sigma_{\tilde{\chi}_i^0-p})_{\min}$ as a function of $m_{\tilde{\chi}_i^0}$ for $\tan\beta = 3$ as other parameters are varied across parameter space in region below coannihilation.

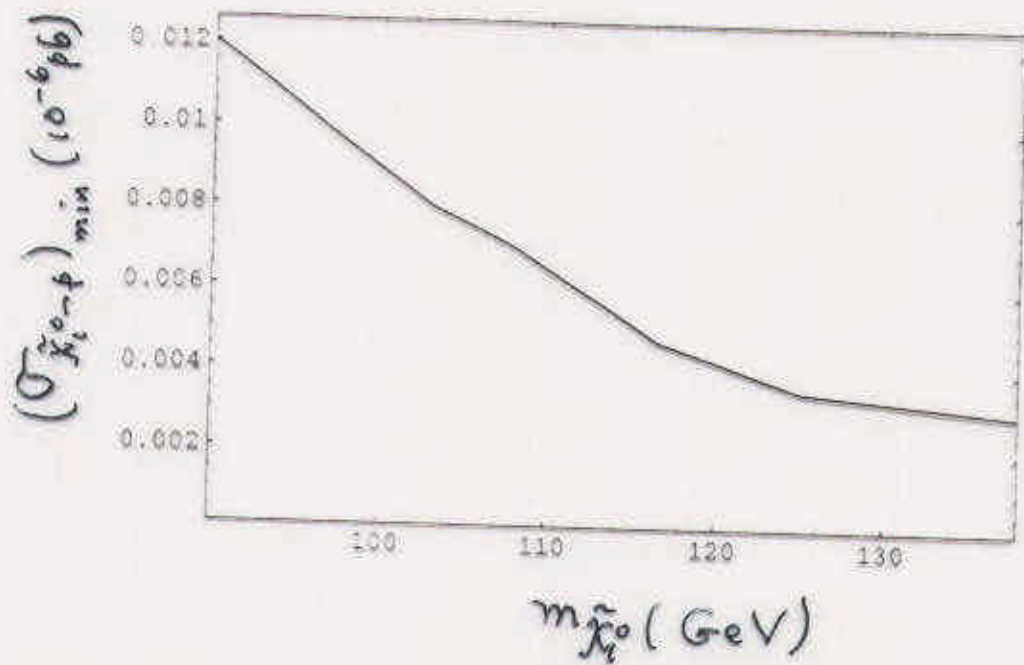
Nonuniversal soft breaking

$$(\sigma_{\tilde{\chi}_i^0-p})_{\max} \text{ vs. } m_{\tilde{\chi}_i^0}$$



Nonuniversal Soft Breaking

$$(\sigma_{\tilde{\chi}_i^0-p})_{\min}; \tan\beta=3$$



The coannihilation region for the nonuniversal models has not been fully examined ($m_{\tilde{e}_R}$ can be made independently light) but over much of parameter space one finds results similar to mSUGRA i.e.

$$\sigma_{\tilde{\chi}_1^0 - p} \gtrsim 10^{-9} \text{ pb} ; m_{1/2} \leq 600 \text{ GeV} ; \mu < 0$$

which again would be accessible to proposed detectors.

CONCLUSIONS

In all of the SUGRA-type models considered, there are regions of parameter space with $\tilde{\chi}_i^0 - p$ cross sections of size that could be observed with current detectors. Thus current detectors sample part of the parameter space of

$$\tan\beta \gtrsim 25; m_{\tilde{\chi}_i^0} \lesssim 90 \text{ GeV}; \Omega_{\tilde{\chi}_i^0} h^2 \lesssim 0.1; m\text{SUGRA}$$

$$\tan\beta \gtrsim 4; \text{nonuniversal soft breaking}$$

$$\tan\beta \gtrsim 15; \text{D-brane models}$$

nonuniversality allowing significantly larger cross sections than mSUGRA.

The experimental bounds on m_h (for low $\tan\beta$) and from $b \rightarrow s\gamma$ (particularly for high $\tan\beta$) are playing an important role in eliminating parameter space, and coannihilation has extended the allowed parameter space to larger $m_0, m_{1/2}$.

For $\mu < 0$, $b \rightarrow s\gamma$ constraint is less powerful unless $\tan\beta$ is large. Then it leaves only the coannihilation bands (for $A > 0$) and these bands are sensitive to the value of A_0 and become increasingly narrow as A_0 increases. One finds generally

$$\sigma_{\tilde{\chi}_1^0 - p} \gtrsim 10^{-9} \text{ pb}; m_{1/2} \leq 600 \text{ GeV}; \mu < 0$$

$$(m_g \lesssim 1.5 \text{ TeV}, m_{\tilde{\chi}_1^0} \lesssim 240 \text{ GeV})$$

This is three orders of magnitude below current detector sensitivity, but within the range planned in future detectors (such as GENIUS).

For $\mu > 0$, $b \rightarrow s\gamma$ constraint eliminates more of the parameter space, particularly at high $\tan\beta$. For mSUGRA, an accidental cancellation occurs which allows $\sigma_{\tilde{\chi}_1^0 - p}$ to become quite small:

$$\sigma_{\tilde{\chi}_1^0 - p} < 10^{-10} \text{ pb for } 4 \lesssim \tan\beta \lesssim 20; \mu > 0$$

$$m_{1/2} \gtrsim 450 \text{ GeV}$$

$$(m_g \gtrsim 1.1 \text{ TeV})$$

and can get as small as

$$\sigma_{\tilde{\chi}_1^0 - p} \cong 1 \times 10^{-12} \text{ pb}; \tan\beta \cong 10, m_{1/2} = 600 \text{ GeV}$$

$$\mu > 0$$