PQCD approach to B decays

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Keum, Sanda, Li

Lu, Ukai, Yang

- Introduction
- Factorization Theorem
- $B \to K\pi, \pi\pi$
  — determination of ϕ_3 (γ)
- Comparision to Factorization Approach,
  Beneke - Buchalla - Neubert - Sachrajda Approach
- $B \to KK$
  — test of nonfactorizable, annihilation, FSI effects

PQCD factorization theorem

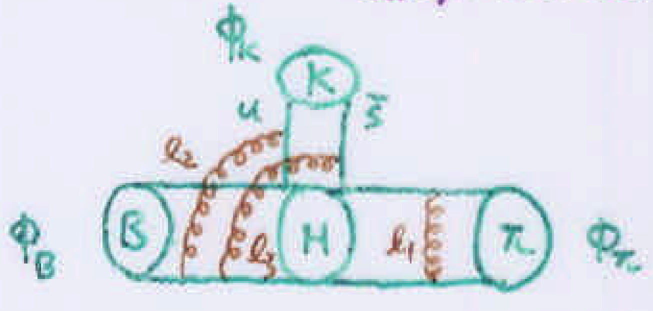
QCD has asymptotic freedom $\lim_{Q\to\infty} \alpha_s(Q) \to 0$ (perturbative)

and confinement $\lim_{Q\to\Lambda_{QCD}} \alpha_s(Q) \to \infty$ (nonperturbative)

high energy QCD processes can be factorized into convolution of hard parts with hadron wave functions

hard parts (scale $\sim Q$) are calculable in pert. theory

wave functions (scale $\sim \Lambda_{QCD}$) not calculable but universal



soft pole from $\ell_1 \sim 0$ absorbed into $\Phi_\pi(\alpha_s(\Lambda_{QCD}))$

finite piece into $H(\alpha_s(M_B))$

soft divergences from $\ell_2 \sim \ell_3 \sim 0$ cancel at leading power

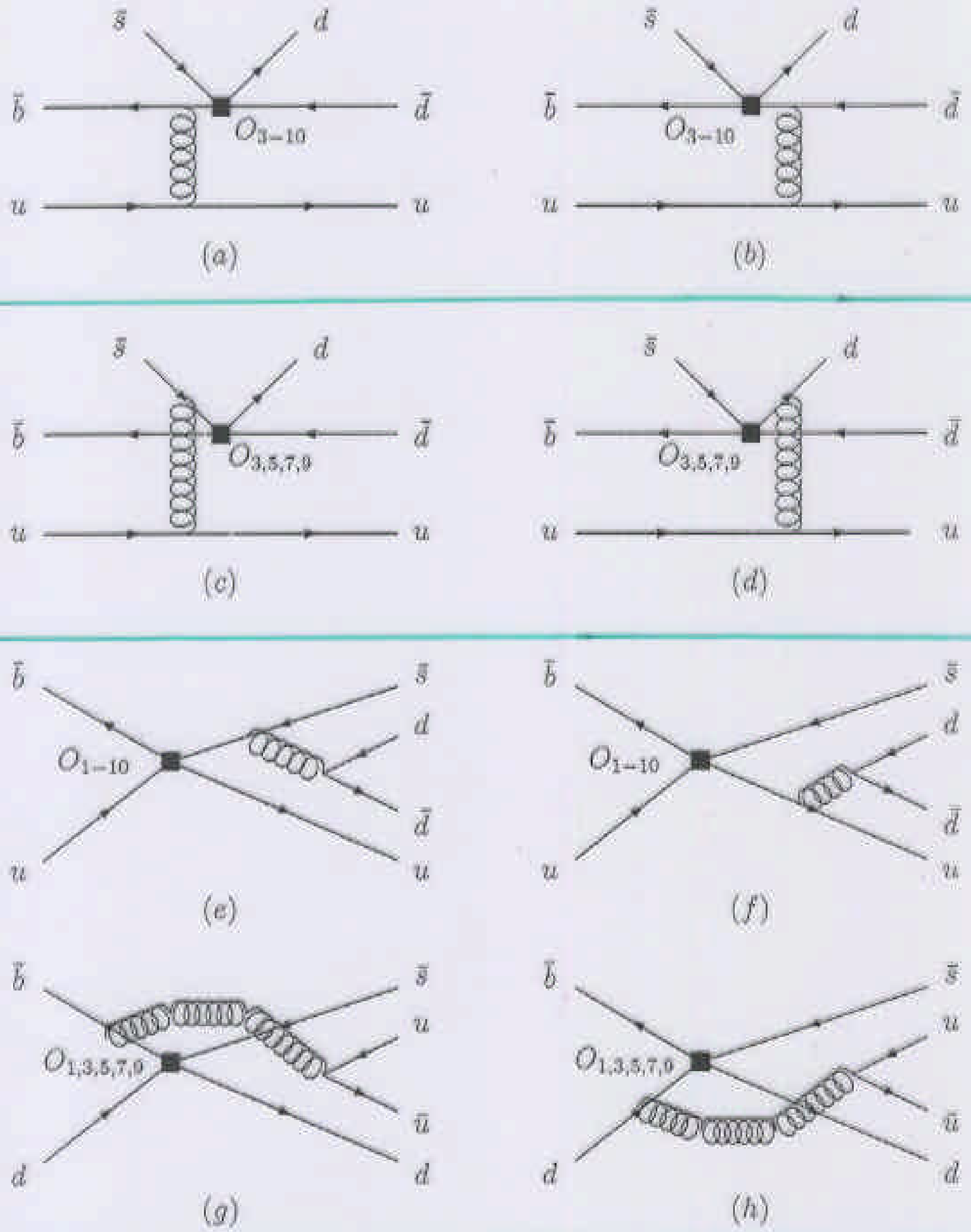
reason: soft gluons (global in space-time) do not interact with the color-singlet object $K = \bar{s}u$

nonfactorizable gluons are infrared finite

⇒ wave functions are independent of the other parts i.e. universal.

⇒ model independence

$B^{\pm} \to K^0 \pi^{\pm}$



factorization approach

nonfactorizable

↑ BBNS

factorizable annihilation

nonfactorizable annihilation

↑ PQCD

Figure 2: *Feynman diagrams for $B^{\pm} \to \bar{K}^0 \pi^{\pm}$ decay*

W.S. Hou et al. (99) factorization approach

$m_0 \sim 4.0$ GeV

FIGURES

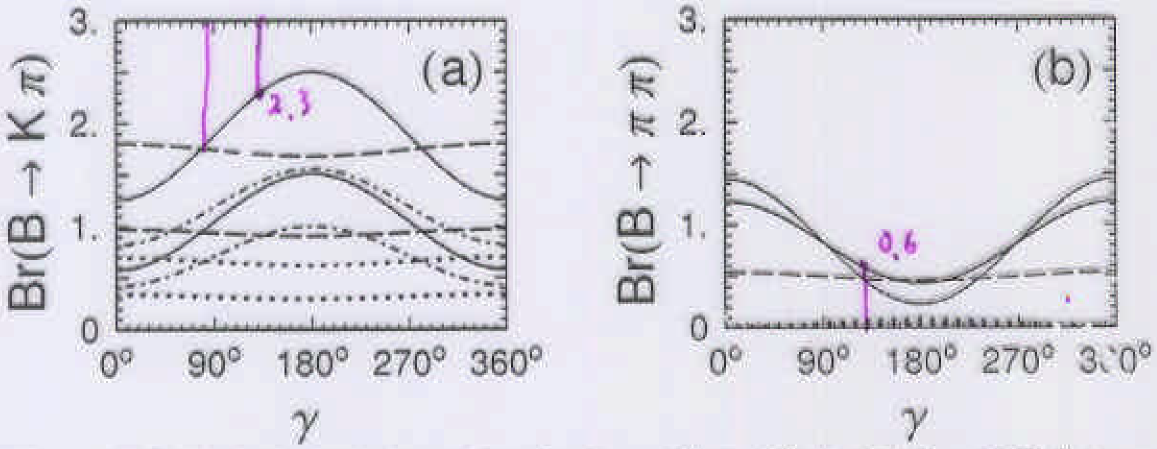


FIG. 1. (a) Solid, dash, dotdash and dots for $B \to K^+\pi^-$, $K^0\pi^+$, $K^+\pi^0$ and $K^0\pi^0$, for $m_s =$ 105 (upper curves) and 200 MeV. (b) Solid, dash and dots for $B \to \pi^+\pi^-$, $\pi^+\pi^0$ and $\pi^0\pi^0$ for $m_d = 2m_u = 3$ and 6.4 MeV, where the lower (upper) curve at $\gamma = 180°$ for $\pi^+\pi^-$ ($\pi^0\pi^0$) is for lower $m_{u,d}$. In all figures Brs are in units of 10^{-5}, and $|V_{ub}/V_{cb}| = 0.08$.

CLEO data (central values) PQCD ($\phi_3 \sim 90°$)

$B(B^\pm \to K^0\pi^\pm) \sim 18.2 \times 10^{-6}$ (20×10^{-6}) Keum, Li, Sanda

$B(B^0 \to K^\pm\pi^\mp) \sim 17.2 \times 10^{-6}$ (19×10^{-6})

$B(B^0 \to \pi^\pm\pi^\mp) \sim 4.3 \times 10^{-6}$ (4.6×10^{-6}) Lu, Ukai, Yang

(6.3×10^{-6}, Belle) (9.3×10^{-6}, BaBar)

$$R = \frac{B(B^0 \to K^\pm\pi^\mp)}{B(B^\pm \to K^0\pi^\pm)} \sim 1 \quad \Rightarrow \quad \phi_3 \sim 90°$$

$$R_\pi = \frac{B(B^0 \to K^\pm\pi^\mp)}{B(B^0 \to \pi^\pm\pi^\mp)} \sim 4 \quad \Rightarrow \quad \phi_3 \sim 130°$$

(3)

in factorization approach (neglect δ)

$B \to K\pi$

overall, dropped

$\gamma_5\gamma_\mu$ axial, γ_5 pseudoscalar components of the kaon wave fn.

$$P \propto f_K \underline{V_{ts}} V_{tb}^*(a_4 + 2r_K a_6) F^{B\pi} \propto -A\lambda^2 (a_4 + 2r_K a_6)$$

$$T \propto V_{us} V_{ub}^* a_1 = A\lambda^4 (\rho + i\eta) a_1$$

$$r_K = \frac{M_0^2(K)}{M_b(M_s + M_d)} \qquad M_0(K) = \frac{M_K^2}{M_s + M_d}$$

$$P \gtrsim T \qquad (a_1 \gtrsim 10\,(a_4 + 2r_K a_6))$$

$B \to \pi\pi$

$$P \propto \underline{V_{td}} V_{tb}^* (a_4 + 2r_\pi a_6) = A\lambda^3 (1 - \rho - i\eta)(a_4 + 2r_\pi a_6)$$

$$T \propto V_{ud} V_{ub}^* a_1 = A\lambda^3 (\rho + i\eta) a_1$$

$$r_\pi = \frac{M_0^2(\pi)}{M_b(M_u + M_d)} \qquad M_0(\pi) = \frac{M_\pi^2}{M_u + M_d}$$

chiral symmetry breaking scale ~ 1.4 GeV

$$T \gg P$$

$$a_1 = C_2 + C_1/N$$

$$a_4 = C_4 + \frac{C_3}{N} + \frac{3}{2} e_q \left(C_{10} + \frac{C_9}{N} \right)$$

$$a_6 = C_6 + \frac{C_5}{N} + \frac{3}{2} e_q \left(C_8 + \frac{C_7}{N} \right)$$

- $\mathcal{B}(B^0 \to K^\pm \pi^\mp) = |T - P|^2$

$$= a_1^2 A^2 \lambda^4 \left| \frac{a_4 + 2r_K a_6}{a_1} + \lambda^2 \rho + i\lambda^2 \eta \right|^2$$

$$= a_1^2 A^2 \lambda^4 \left[(a_K + \lambda^2 \rho)^2 + \lambda^4 \eta^2 \right], \qquad a_K \equiv \frac{a_4 + 2r_K a_6}{a_1} \propto \frac{-|P|}{|T|} < 0$$

$$\simeq a_1^2 A^2 \lambda^4 (a_K^2 + 2\lambda^2 \rho a_K) + O(\lambda^8)$$

- $B(B^\pm \to K^0 \pi^\pm) = |P|^2 = a_1^2 A^2 \lambda^4 a_K^2$

- $B(B^0 \to \pi^\pm \pi^\mp) = |T - P|^2$

$$= a_1^2 A^2 \lambda^6 \left| (1 - \rho - i\eta) a_\pi - \rho - i\eta \right|^2, \qquad a_\pi \equiv \frac{a_4 + 2r_\pi a_6}{a_1} < 0$$

$$= a_1^2 A^2 \lambda^6 \left[(a_\pi (1-\rho) - \rho)^2 + (a_\pi + 1)^2 \eta^2 \right]$$

$$= a_1^2 A^2 \lambda^6 \left[\rho^2 + \eta^2 - 2\rho(1-\rho) a_\pi + 2\eta^2 a_\pi + O(a_\pi^2) \right]$$

$$\simeq a_1^2 A^2 \lambda^6 \left[R_b^2 - 2(\rho - R_b^2) a_\pi \right], \qquad R_b^2 = \rho^2 + \eta^2$$

$$R = \frac{a_K^2 + 2\lambda^2 R_b \cos\phi_3 \, a_K}{a_K^2}, \qquad \rho = R_b \cos\phi_3$$

$$O(2\lambda^2 R_b) = O(a_\pi)$$

$$R_\pi = \frac{a_K^2 + 2\lambda^2 R_b \cos\phi_3 \, a_K}{\lambda^2 R_b \left[R_b - 2(\cos\phi_3 - R_b) a_\pi \right]}$$

$R \sim 1 \Rightarrow \phi_3 \sim 90^\circ$ indep of a_K, λ, R_b

$\uparrow$ model-dependent

R is a good quantity

R_π dep on a_K, a_π, λ, R_b

R_π is not a good quantity.

if we believe $\phi_3 \lesssim 90^\circ$, can we get larger R_π?

chiral symmetry $a_K \sim a_\pi \sim a$

if choosing $M_0 \sim 2.0$ GeV, $a \sim -\dfrac{0.03 + 2\frac{2.4}{4.8}\cdot 0.04}{1.0} \sim -0.06$

$\lambda \sim 0.2$, $R_b \sim 0.4 \Rightarrow R_\pi \sim 1.$ (if BaBar is right, fine!)

in factorization approach

must require large $M_0 \sim 3.8$ GeV, large $\phi_3 \sim 130^\circ$
$(m_u = 1.5$ MeV$)$

$\Rightarrow a \sim -0.1$. $R_\pi \sim 4.$

in PQCD, using $M_0 \sim 1.5$ GeV, $\phi_3 \sim 90^\circ$, $|a|$ enhanced
we can!

$P = F_{e4} + F_{e6}$

$F_{e4} \propto a_4^{(t)}[(1+x_3)\phi_B\phi_\pi h_e(x_1,x_3) + 2r_\pi \phi_B \phi_\pi' h_e(x_3,x_1)]$

$F_{e6} \propto 2r_K a_6^{(t)}[\phi_B\phi_\pi + (2+x_3)r_\pi \phi_B\phi_\pi']\, h_e(x_1,x_3)$

$\quad + 2r_K a_6^{(t)}[x_1\phi_B\phi_\pi + 2(1-x_1)r_\pi \phi_B\phi_\pi']\, h_e(x_3,x_1)$

different forms $\neq$ factorization approach

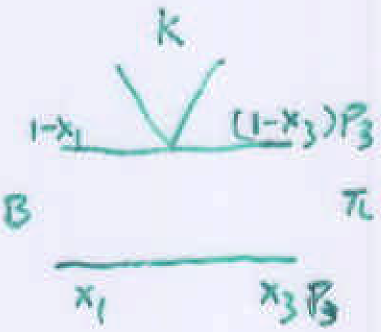
hard functions

$h_e(x_1,x_3) = \dfrac{1}{x_1 x_3^2} \qquad h_e(x_3,x_1) = \dfrac{1}{x_1^2 x_3}$

$T = F_e = a_1[(1+x_3)\phi_B\phi_\pi h_e(x_1,x_3) + 2r_\pi \phi_B\phi_\pi h_e(x_3,x_1)]$

as $r_\pi \to 0$, $x_1, x_3 \to 0$

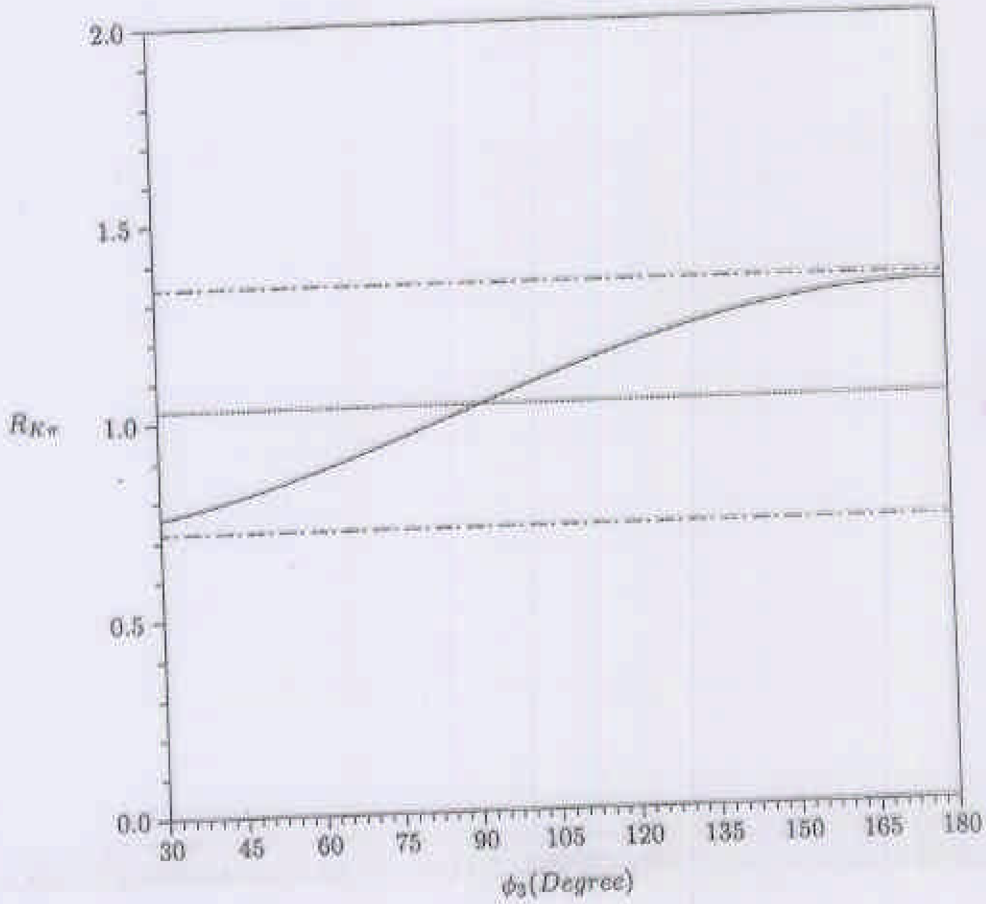
$\dfrac{P}{T} \propto \dfrac{a_4 + 2r_K a_6}{a_1}$



same as in the factorization approach

penguin enhancement in PQCD

Keum, Li, Sanda



data uncertainty

Figure 6

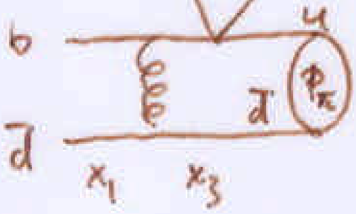
need more precise data

	PQCD	BBNS	FA
calculable	form factors nonfactorizable annihilation	nonfactorizable (for $B\to\pi\pi$ not for $B\to D\pi$)	
not calculable	FSI	form factors FSI	form factors nonfactorizable FSI
neglected	FSI	annihilation FSI	annihilation FSI
input	wave functions	form factors wave functions	form factors
strong phase	annihilation nonfactorizable	BSS nonfactorizable	BSS

BBNS approach

- factorizable contribution (form factor) is not calculable in BBNS but is PQCD. Sudakov logs are irrelevant in BBNS

  k_T not considered in BBNS but is in PQCD



$\Phi_\pi(x_3) \propto x_3(1-x_3)$

(BBNS's conclusion dep on models of Φ_π)

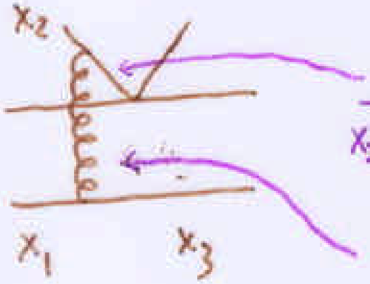
$\propto \dfrac{\Phi_\pi(x_3)}{x_1 x_3^2}$ diverges in BBNS

$\propto \dfrac{\Phi_\pi(x_3)}{[x_1x_3 + (k_{1T}-k_{3T})^2/M_B^2]\,[x_3 + k_3^2]}$ finite in PQCD

$\Rightarrow \ln k_T \Rightarrow k_T$ resummation is relevant

- nonfactorizable contribution is calculable in BBNS, PQCD

  but real in BBNS, complex in PQCD



$\dfrac{1}{x_2x_3 - x_1(x_2-x_3) + i\epsilon} \xrightarrow{x_1 \to 0} \dfrac{1}{x_2x_3}$ in BBNS

$\dfrac{1}{x_1x_3} \xrightarrow{x_1 \to 0}$ soft gluon

contains $-i\pi\,\delta(x_2x_3 - x_1(x_2-x_3))$ in PQCD

- Penguin enhancement does not exist in BBNS, but does in PQCD.

  ϕ_3 determination from $B \to \pi\pi, K K$

  PQCD $\phi_3 \sim 90^\circ$

  BBNS $\phi_3 > 90^\circ$ (Du et al., Muta et. al.)

Amplitudes	Left-handed column	Right-handed column	PQCD	BBNS
$Re(f_\pi F^T)$	(a) $4.39 \cdot 10^{-2}$	(b) $2.95 \cdot 10^{-2}$	$7.34 \cdot 10^{-2}$	$7.57 \cdot 10^{-2}$
$Im(f_\pi F^T)$	–	–	–	$1.13 \cdot 10^{-3}$
$Re(f_\pi F^P)$	(a) $-3.54 \cdot 10^{-3}$	(b) $-2.33 \cdot 10^{-3}$	$-5.87 \cdot 10^{-3}$	$-3.04 \cdot 10^{-3}$
$Im(f_\pi F^P)$	–	–	–	$-1.27 \cdot 10^{-3}$
$Re(f_B F_a^P)$	(e) $5.05 \cdot 10^{-4}$	(f) $-1.94 \cdot 10^{-3}$	$-1.42 \cdot 10^{-3}$	–
$Im(f_B F_a^P)$	(e) $2.19 \cdot 10^{-3}$	(f) $3.72 \cdot 10^{-3}$	$5.91 \cdot 10^{-3}$	–
$Re(M^T)$	(c) $5.02 \cdot 10^{-3}$	(d) $-6.55 \cdot 10^{-3}$	$-1.53 \cdot 10^{-3}$	$-7.71 \cdot 10^{-4}$
$Im(M^T)$	(c) $-3.83 \cdot 10^{-3}$	(d) $7.03 \cdot 10^{-3}$	$3.20 \cdot 10^{-3}$	–
$Re(M^P)$	(c) $-2.29 \cdot 10^{-4}$	(d) $2.75 \cdot 10^{-4}$	$4.66 \cdot 10^{-5}$	$4.17 \cdot 10^{-3}$
$Im(M^P)$	(c) $1.95 \cdot 10^{-4}$	(d) $-3.08 \cdot 10^{-4}$	$-1.13 \cdot 10^{-3}$	–
$Re(M_a^P)$	(g) $1.14 \cdot 10^{-5}$	(h) $-1.48 \cdot 10^{-4}$	$-1.37 \cdot 10^{-4}$	–
$Im(M_a^P)$	(g) $-9.12 \cdot 10^{-6}$	(h) $-1.27 \cdot 10^{-4}$	$-1.36 \cdot 10^{-4}$	–

varied independently in BBNS

Table 1: Amplitudes for the $B_d^0 \to \pi^+\pi^-$ decay from Fig. 3, where F (M) denotes factorizable (nonfactorizable) contributions, P (T) denotes the penguin (tree) contributions, and a denotes the annihilation contributions. Here we adopted $\phi_3 = 90^0$, $R_b = 0.38$, and $\alpha_s(m_b) = 0.2552$ in the numerical analysis for the BBNS approach.

factorizable contributions are close. (BBNS can use our results as inputs)

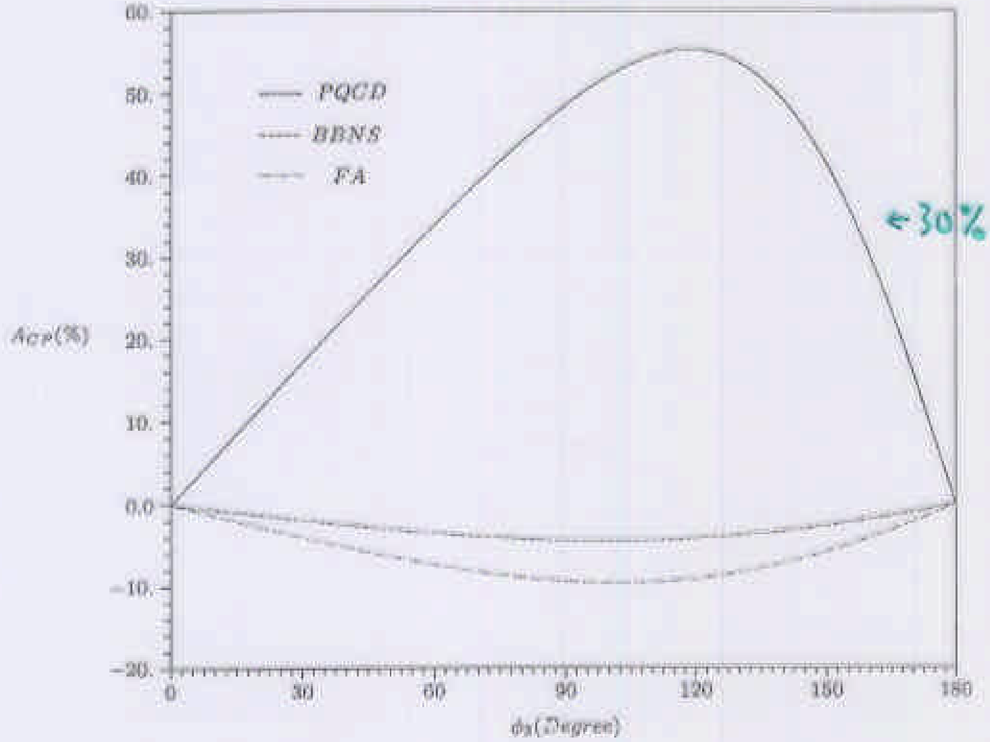
numerical difference between branching ratios are small.

nonfactorizable contributions are small in $B \to \pi\pi$, $K\pi$

but important in $B \to D\pi$

annihilation contributions make the distinction annihilation diagrams give large strong phases in PQCD

$\Rightarrow$ large CP asymmetry



← 30% uncertainty

Figure 5

ϕ_3 determination from $B \to \pi\pi$, $K\pi$

PQCD $\quad \phi_3 \sim 90°$

BBNS $\quad \phi_3 > 90°$ (Du et al. , Muta et al.)