PREDICTIONS FOR THE DECAYS OF RADIALLY-EXCITED BARYONS

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SUBJECT: STRONG DECAYS OF RADIALLY EXCITED BARYONS.

VIEWPOINT & TECHNIQUE: FUNDAMENTAL QCD d.o.f.
ARE QUARKS & GLUONS, BUT MUST DEAL WITH
OBSERVED STATES WHICH ARE BARYONS & MESONS.

* USE EFFECTIVE FIELD THEORY
FIND ALL OPERATORS CONSISTENT WITH QCD SYMM.

* USE LARGE N_c
PROVIDES SIZE ESTIMATE FOR EACH OPERATOR.

OUTLINE:

1. PRESENT Y2K DERIVATION OF OLD MASS FORMULA.
NEED MASS FORMULA — NOT ALL RELEVANT STATES FOUND.
ILLUSTRATES EFT AND LARGE N_c TECHNIQUES.

2. ACTUAL RADIALLY EXCITED DECAYS

NOTE: HAVE EARLIER WORKED ON MASSES AND DECAYS (STRONG & RADIATIVE) OF ORBITALLY EXCITED BARYONS [NEGATIVE PARITY, P-WAVE, 70-PLET]. WILL ALLUDE TO BUT NOT DISCUSS.

MASS FORMULA

METHOD: ENUMERATE, DISCARD, REORGANIZE

(1) STATES: TOTALLY SYMMETRIC SPIN-FLAVORSTATES MADE FROM 3 OR N_c QUARKS

- BARYONS HAVE N_c QUARKS
- SPIN $\uparrow, \downarrow$ AND THREE FLAVORS: 6 OBJECTS
- FOR $N_c = 3$, $\exists$ 56 TOTALLY SYMMETRIC STATES FROM $u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow$
- GROUND STATE 56, RADIALLY EXCITED 56'.
  STATES: $N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$.

(2) MAKE MASS OPERATORS FROM

SPIN $\quad S^i = \sum_{\text{QUARK } \alpha} \frac{1}{2}\sigma_\alpha{}^i$

FLAVOR $\quad T^a = \sum_\alpha \frac{1}{2}\tau_\alpha^a \quad [\tau^a = 3\times 3]$

SU(6) $\quad G^{ia} = \sum_\alpha \frac{1}{2}\sigma_\alpha{}^i \cdot \frac{1}{2}\tau_\alpha^a$

IF ROTATION AND FLAVOR SYMMETRIC

$$\boxed{H_{\text{eff}} = \ldots \mathbb{1} + \ldots S^2 + \mathcal{O}(S^4)}$$

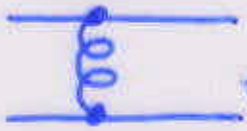
[NOTE: FOR SYMMETRIC STATES, T^2 AND G^2 NOT INDEPENDENT OF S^2 AND $\mathbb{1}$.]

③ LARGE N_c.

$$\frac{g^2(\mu^2)}{(4\pi)^2} = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$$

$\therefore$ $N_c \to \infty$, $g^2 \propto 1/N_c$

S^2 COEFF. COMES FROM  $+\ldots \propto 1/N_c$

[NICE THEOREM: ALL DIAGRAMS FALL IN N_c AT SAME RATE (OR FASTER) AS LO.]

$\therefore$ $H_{eff} = a_1 \mathbb{1} + \frac{a_2}{N_c} S^2 + \ldots$

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④ ALLOW FLAVOR SYMMETRY BREAKING.

TERM LIKE $\bar{q} \begin{pmatrix} 0 & & \\ & 0 & \\ & & m_s \end{pmatrix} q$ IN $\mathcal{L}_{QCD}$.

RECALL $T^8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$: ABOVE IS LIN. COMB. OF T^8 & $\mathbb{1}$

ALLOW FLAVOR INDICES "8" IN EFFECTIVE OPERATOR:

$$H_{eff} = {}_m 1 + {}_m S^2 + {}_{\cdots} T^8 + {}_{\cdots} S^i q^{i8} + {}_m S^2 T^8$$
$$+ {}_m T^8 T^8 + {}_{\cdots} T^8 S^i q^{i8} + {}_{\cdots} T^8 T^8 T^8$$

[$\leq$ 3-BODY]

$$H_{\text{eff}} = a_1 \mathbb{1} + \frac{a_2}{N_c} S^2 + \varepsilon a_3 T^8 + \frac{\varepsilon}{N_c} a_4 S^i G^{i8} + \frac{\varepsilon}{N_c^2} a_5 S^2 T^8 + \frac{\varepsilon^2}{N_c} a_6 T^8 T^8 + \frac{\varepsilon^2}{N_c^2} a_7 T^8 S^i G^{i8} + \frac{\varepsilon^3}{N_c^2} T^8 T^8 T^8$$

ε MEASURES FLAVOR SYMM. BREAKING. $\varepsilon \sim \frac{1}{8} \sim \frac{1}{N_c}$.

ALL $a_i = \mathcal{O}(N_c^0)$.

(5) CAST AWAY $a_5 \ldots a_8$ (3^{RD} ORDER & BEYOND)

USE/DERIVE M.E.

$$\langle \mathbb{1} \rangle = N_c$$

$$\langle S^2 \rangle = S(S+1)$$

$$\langle T^8 \rangle = (N_c - 3N_s)/\sqrt{12}$$

$$\langle S^i G^{i8} \rangle = \frac{1}{4\sqrt{3}} \left[3I(I+1) - S(S+1) - \frac{3N_s(N_s+2)}{4} \right]$$

REORGANIZE TO GET

$$\boxed{M = A + BN_s + C\left[I(I+1) - \tfrac{1}{4}N_s^2\right] + D\,S(S+1)}$$

[GÜRSEY-RADICATI, 1964]

SUBSUMES

* GELL-MANN OKUBO MASS FORMULA.
* EQUAL SPACING RULE FOR $S = \frac{3}{2}$ STATES (DECUPLET).

FOR 56', PREDICT MASSES OF 4 UNDISCOVERED STATES FROM THE 4 KNOWN,

$N'(1440)$, $\Delta'(1600)$, $\Lambda'(1600)$, $\Sigma'(1660)$

STRONG DECAYS OF 56'

$[(D, L_N^P) = (56, 0_2^+)]$

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* ANTICIPATE NEW RESULTS FROM CLAS AT CEBAF
  4 STATES UNDISCOVERED / UNCONFIRMED
  EXISTING MEASUREMENTS HAVE LARGE UNCERTAINTY

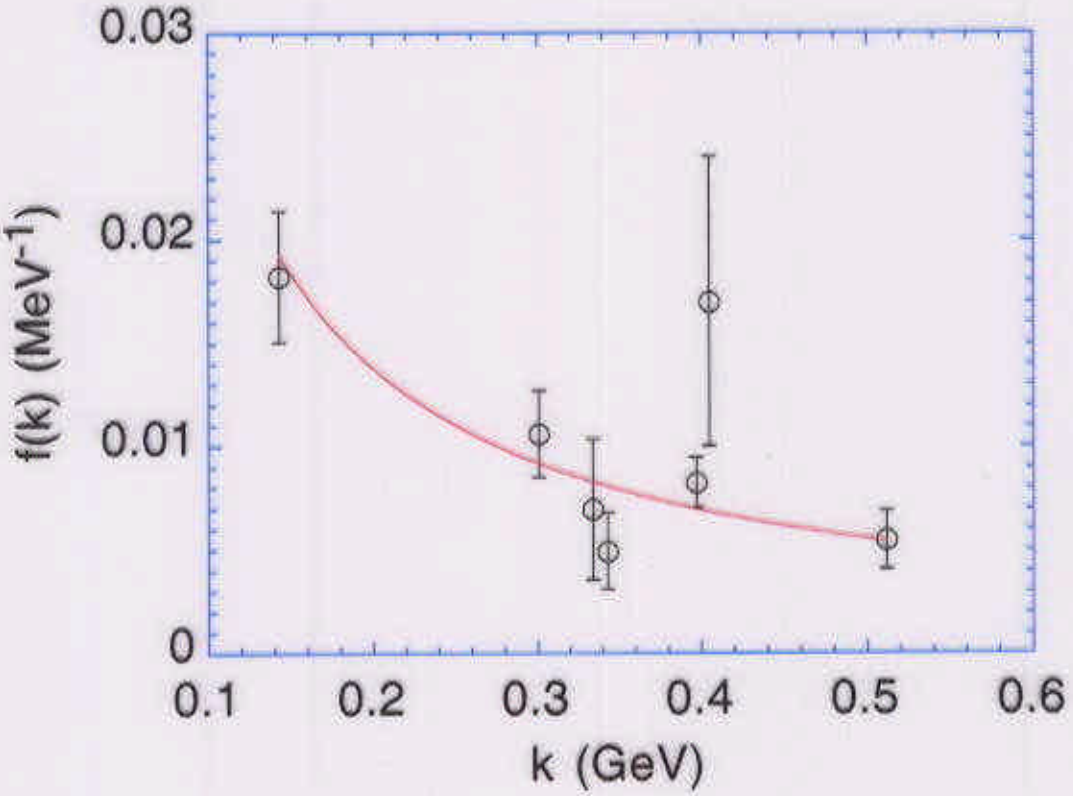
* ELUCIDATE ROPER, N(1440), $J^P = \frac{1}{2}^+$.
  - 3q RADIAL EXCITATION ?
    AGAINST: MASS TOO HIGH IN OGE (SPIN-COLOR) MODELS.
    RECENT: MASS FINE W/ SPIN-FLAVOR INTERACTIONS
  - HYBRID STATE $(q^3 \bar{q} q)$?
  - NON-RESONANT X-SCTN ENHANCEMENT ?

ASSUME
* SINGLE-QUARK OPERATORS
  STUDIED ORBITALLY EXCITED DECAYS
  $70 \to 56 + \pi$ (1994, CARONE ET AL.)
  $70 \to 56 + \gamma$ (US, + HEROES OF 60's & 70's.)
  INCLUDED POSSIBLE TWO-QUARK OPERATORS
  FOUND: OBSERVED DECAY PATTERN DISFAVORED THEM.

∴ ONLY OPERATOR HERE $\boxed{H_{eff} \propto G_{1*}^{ia} k^i \pi^a}$

k^i = MESON 3-MOMENTUM
π^a = MESON FIELD OPERATOR

* NO CONFIGURATION MIXING [E.G. ~~$[70, 0_2^+]$~~]



THE FUNCTION $f(k)$. THE BEST FIT CORRESPONDS TO $f(k) = 2.8/k$, WITH $\chi^2/\text{d.o.f.} = 1.1$.

Decay	M_i(MeV)	k (MeV)	$\|\mathcal{G}\|^2$	Γ (MeV)
$N' \to N\pi$ †	1450	397	25/16	164
$N' \to \Delta\pi$ †	1450	143	2	99
$\Delta' \to N\pi$ †	1625	512	1/2	60
$\Delta' \to \Delta\pi$ †	1625	301	25/16	145
$\Lambda' \to N\bar{K}$ †	1630	343	9/8	91
$\Lambda' \to \Sigma\pi$ †	1630	336	3/4	75
$\Lambda' \to \Sigma^*\pi$	1630	188	3/2	97
$\Sigma' \to N\bar{K}$ †	1660	405	1/24	4
$\Sigma' \to \Lambda\pi$	1660	439	1/4	30
$\Sigma' \to \Sigma\pi$	1660	385	2/3	75
$\Sigma' \to \Sigma^*\pi$	1660	218	1/3	25
$\Xi' \to \Lambda\bar{K}$	1825	403	1/16	6
$\Xi' \to \Sigma\bar{K}$	1825	323	25/16	134
$\Xi' \to \Xi\pi$	1825	420	1/16	8
$\Xi' \to \Xi^*\pi$	1825	240	1/2	41
$\Sigma^{*\prime} \to N\bar{K}$	1790	519	1/6	18
$\Sigma^{*\prime} \to \Delta\bar{K}$	1790	214	5/6	50
$\Sigma^{*\prime} \to \Lambda\pi$	1790	535	1/4	34
$\Sigma^{*\prime} \to \Sigma\pi$	1790	484	1/6	22
$\Sigma^{*\prime} \to \Sigma^*\pi$	1790	338	5/6	89
$\Sigma^{*\prime} \to \Sigma\eta$	1790	196	1/4	13
$\Xi^{*\prime} \to \Lambda\bar{K}$	1955	525	1/4	31
$\Xi^{*\prime} \to \Sigma\bar{K}$	1955	461	1/4	29
$\Xi^{*\prime} \to \Sigma^*\bar{K}$	1955	239	5/4	86
$\Xi^{*\prime} \to \Xi\pi$	1955	520	1/4	36
$\Xi^{*\prime} \to \Xi^*\pi$	1955	358	5/16	36
$\Xi^{*\prime} \to \Xi\eta$	1955	269	1/4	19
$\Omega' \to \Xi\bar{K}$	2120	506	1	128
$\Omega' \to \Xi^*\bar{K}$	2120	274	5/4	101

TABLE II. Decays of the radially excited **56′** into ground state **56** plus meson. Seven of these decays are measured (indicated by a dagger), and the other 22 widths are predictions. If the mass of the initial state is known, the uncertainty in the width from our fitting procedure is ±15%. Uncertainties in the mass induce further and sometimes large uncertainties in the width. These of course are greatly reducible once the mass of the state is measured, as discussed in the text.

	Mass (MeV)	Decay	k (MeV)	Partial Width (MeV)
$N(1440)$	1450 ± 20			
		$N\pi$	397	227.5 ± 67.3
		$\Delta\pi$	143	87.5 ± 30.5
$\Delta(1600)$	1625 ± 75			
		$N\pi$	512	61.25 ± 31.5
		$\Delta\pi$	301	192.5 ± 76.0
$\Lambda(1600)$	1630 ± 70			
		$N\overline{K}$	343	33.75 ± 25.2
		$\Sigma\pi$	336	52.5 ± 51.3
$\Sigma(1660)$	1660 ± 30			
		$N\overline{K}$	405	24.0 ± 20.0

TABLE I. Input values used in the determination of $f(k)$.

MATRIX ELEMENTS

$$\langle \Sigma(B_f,\pi^a)\,|\,H_{eff}\,|\,\Sigma(B_i)\rangle = f(k)\,k^j\,\langle B_f\,|\,G^{ja}\,|\,B_i\rangle$$

↑ JUST SPIN-FLAVOR PART

$k = |\vec{k}|$

$f(k)$ PARAMETERIZES MOMENTUM DEPENDENCE OF AMPLITUDE (IN MODELS WOULD HAVE OVERLAP OF BARYON W. F.)

$$\Gamma = \frac{M_f}{6\pi M_i}\,k^3\,f(k)^2 \sum_{\substack{\text{final spins}\\ \& \text{ isospins}}} |\langle B_f\,|\,G^{ja}\,|\,B_i\rangle|^2$$

7 MEASURED DECAYS.

GOOD FIT USING SIMPLE FORM $f(k) = \frac{\text{CONST.}}{k} = \frac{2.8 \pm 0.2}{k}$

[$\chi^2/\text{d.o.f.} \simeq 1$. BEST LINEAR FIT HAS $\chi^2/\text{d.o.f.} \simeq 2$.]

PREDICT WIDTHS FOR 22 DECAYS

SOURCES OF UNCERTAINTIES

* $\pm 15\%$ FROM $f(k)$
* MASS UNCERTAINTY IN $56'$ FEEDS INTO PHASE SPACE E.g., $\Delta' \to \Xi^* K$. FIXABLE, OF COURSE.

SUMMARY

* LARGE N_c IDEAS LEAD TO GURSEY-RADICATI MASS FORMULA (L=0 BARYONS)

* KNOWN 56' DECAYS EASY TO FIT IN MODEL INDEPENDENT WAY. SUCESS OF PREDICTIONS WOULD BOLSTER VIEW OF ROPER AS $3q$ STATE. (MODEL DEPENDENT CALCULATIONS ....)

REFERENCES:

70-PLET RADIATIVE DECAYS

CARONE & me, PRD 58, 053005 (1998)
PL B 441, 363 (1998)

70-PLET MASS ANALYSIS

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PR D 59, 114008 (1999).

56' STRONG DECAYS

CARONE & me, ACCEPTED FOR PL B (2000).