Confinement
and
Flavor Symmetry Breaking
via
Monopole Condensation

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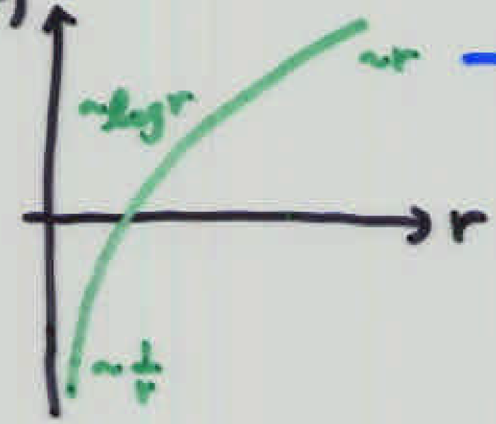
§ Introduction

two main dynamical issues in gauge theories

confinement

symmetry breaking

in QCD



→ confinement

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \simeq \langle \bar{s}s \rangle \neq 0$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

microscopic mechanism?

use $N=2$ SUSY gauge theories

to address this question

à la Seiberg-Witten

$\mathcal{N}=2$ SUSY $SU(n_c)$ QCD w/ n_f flavors $+ \mu \,\mathrm{tr}\, \Phi^2$

flavor symmetry breaking

$$U(n_f) \longrightarrow U(r) \times U(n_f - r)$$

caused by condensation of

magnetic degrees of freedom

<u>$r=1$</u>

magnetic monopoles themselves

<u>$r>1$</u>

magnetic monopoles "break up" into "dual quarks" before reaching the non-baryonic roots

In any case,

confinement

flavor symmetry breaking

caused by the same condensates

$N=1$ perturbation

$$\Delta W = \mu \, \mathrm{tr}\, \Phi^2$$

study the theory in two limits

① μ large

integrate Φ out $\Rightarrow$ $N=1$ theory

$$W = -\frac{1}{\mu}(\tilde{Q} T^a Q)(\tilde{Q} T^a Q)$$

use Seiberg's analysis

easy to identify vacua in terms of

$$M^{ij} = \tilde{Q}^i Q^j, \quad B^{i_1 \cdots i_{N_c}} = Q^{i_1} \cdots Q^{i_{N_c}}, \quad \tilde{B}$$

$\Rightarrow$ flavor symmetry breaking

② μ small

$\mu = 0 \Rightarrow N=2$ theory

$\Rightarrow$ study monopoles etc

$\Rightarrow$ $\mathcal{L}_{eff}$ at singularities

turn $\mu \neq 0$

$\Rightarrow$ microscopic dynamics

Very powerful checks

further perturb w/ $m_i \neq 0$

$\Rightarrow$ discrete vacua

count # vacua = Witten index

numbers should match among
various limiting cases

① A semi-classical
μ, m_i large $\rightarrow$ $Q, \tilde{Q}, \Phi$ large

① B large μ $N=1$ analysis
μ large, m_i small
$\rightarrow$ easy to identify flavor symm

② A Coulomb branch
$\mu=0$, m_i small, identify singularities
$\rightarrow$ fully quantum analysis

② B APS effective $\mathcal{L}$
μ, m_i small $\rightarrow$ microscopic picture

§ Large μ Analysis

$$W = \sqrt{2}\,\tilde{Q}_i \Phi Q_i + \mu \,\mathrm{tr}\,\Phi^2 + m_i \tilde{Q}_i Q_i$$

integrate out Φ

$$W = -\frac{1}{\mu}(\tilde{Q}_i T^a Q_i)(\tilde{Q}_j T^a Q_j) + m_i \tilde{Q}_i Q_i$$

Fierz tr. use $M_{ij} \equiv \tilde{Q}_i Q_j$

$$W = -\frac{1}{2\mu}\left[\mathrm{tr}\,M^2 - \frac{1}{n_c}(\mathrm{tr}\,M)^2\right] + \mathrm{tr}\,mM$$

when $n_f < n_c$ Affleck-Dine-Seiberg non-perturbative superpotential

$$\Delta W = (n_c - n_f) \frac{\Lambda_1^{(3n_c - n_f)/(n_c - n_f)}}{(\det M)^{1/(n_c - n_f)}}$$

matching

$$\Lambda_1^{3n_c - n_f} = \mu^{n_c} \Lambda^{2n_c - n_f}$$

Solve for M $\quad \partial W_{tot}/\partial M = 0$

$$M = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{n_f} \end{pmatrix}$$

$$\lambda_i = \frac{1}{2}\left(Y \pm \sqrt{Y^2 + 4\mu X} \right) + O(m)$$

2^{n_f} choices

choose r $+$'s, $(n_f - r)$ $-$'s

$$Y \propto \Lambda_\mu \left(e^{2\pi i/(2n_c - n_f)} \right)^k$$

$(2n_c - n_f)$ choices

$$X \propto \frac{1}{\mu} Y^2$$

double counting $\quad r \leftrightarrow n_f - r$

$(2n_c - n_f)\, 2^{n_f - 1}$ vacua

$m \to 0$ limit $\quad M = \left(\begin{array}{ccc|ccc} \lambda_1 & & & & & \\ & \ddots & & & & \\ & & \lambda_r & & & \\ \hline & & & \lambda_{n_f} & & \\ & & & & \ddots & \\ & & & & & \lambda_{n_f} \end{array}\right) \begin{array}{l} \}\, r \\ \\ \}\, n_f - r \end{array}$

$$U(n_f) \to U(r) \times U(n_f - r)$$

$n_f = n_c$ case the same result

§ Semi-classical monopoles

't Hooft, Polyakov

SU(2) gauge theory

Φ adjoint Higgs

$\langle \Phi \rangle = \begin{pmatrix} a & \\ & -a \end{pmatrix}$ $\quad SU(2) \to U(1)$

topologically non-trivial boundary cond

$r \to \infty$

$\Phi \to U^{+} \begin{pmatrix} a & \\ & -a \end{pmatrix} U = \left(\vec{\sigma} \cdot \frac{\vec{r}}{r} \right) a$

$\Rightarrow$ monopole

$\pi_2(G/H) = \pi_2(SU(2)/U(1)) = \mathbb{Z}$

$M \sim \frac{4\pi a}{g}$

weak coupling $\to$ heavy

strong coupling $\to$ light(?)

cf. $M_W \sim ga$ the opposite behavior

Jackiw - Rebbi $\mathcal{L} = \psi^i \psi^i \Phi$ $SO(2n_f)$ symm

couple to quarks ($\underline{3}$)

$$H_D \psi = [\vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta \Phi] \psi = 0$$

zero mode (real)

monopole configuration rotationally inv.

<u>if</u> both space + gauge rotated

because $\Phi \to (\vec{\sigma} \cdot \frac{\vec{r}}{r}) a$

$\vec{J} + \vec{I}$ conserved

2π rotation of fermion zero mode

$$R(2\pi) = e^{2\pi i \vec{\theta} \cdot \vec{J}} e^{2\pi i \vec{\theta} \cdot \vec{I}}$$

$$= (-1)^{2S} (-1)^{2I} = 1$$

$\Rightarrow$ boson!

monopole multiplet

$|M\rangle$, $\psi |M\rangle$

both bosons, degenerate

with many flavors

ψ^i : real oscillators

$\{\psi^i, \psi^j\} = \delta^{ij}$

monopole multiplet is a representation of the Clifford algebra

$\psi^i = \gamma^i \frac{1}{\sqrt{2}}$ of $SO(2n_f)$

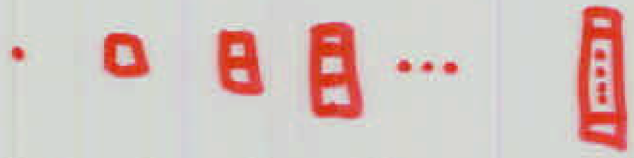
monopoles = $SO(2n_f)$ spinor 2^{n_f}

in general $Sp(n_c)$ theories

monopoles = $SO(2n_f)$ spinor

in general $SU(n_c)$ theories

$\mathcal{L} = \bar{\xi}\, \Phi\, \xi$ only $U(n_f) \subset SO(2n_f)$ sym

monopoles = 

${}_{n_f}C_0 + {}_{n_f}C_1 + {}_{n_f}C_2 + {}_{n_f}C_3 + \cdots + {}_{n_f}C_{n_f} = 2^{n_f}$

Possible conjecture:

condensation of monopoles

in  multiplet

$$\Rightarrow \begin{cases} \text{confinement} \\ U(n_f) \to U(r) \times U(n_f - r) \end{cases}$$

§ Moduli Space of $N=2$ Theories

$$W = \sqrt{2}\, \tilde{Q}_i \Phi Q_i$$

$$\Rightarrow \Phi Q_i = 0,$$

$$\tilde{Q}_i \Phi = 0,$$

$$\sum_i \left\{ Q_i \tilde{Q}_i - \frac{1}{n_c} \mathrm{tr}\, Q_i \tilde{Q}_i \right\} = 0$$

$$[\Phi, \Phi^\dagger] = 0$$

$$Q^\dagger T^a Q - \tilde{Q}^\dagger T^{a*} \tilde{Q} = 0$$

moduli space = space of vacua

3 distinct branches

① Coulomb branch

② Non-baryonic branch

③ Baryonic branch

Argyres, Plesser, Seiberg

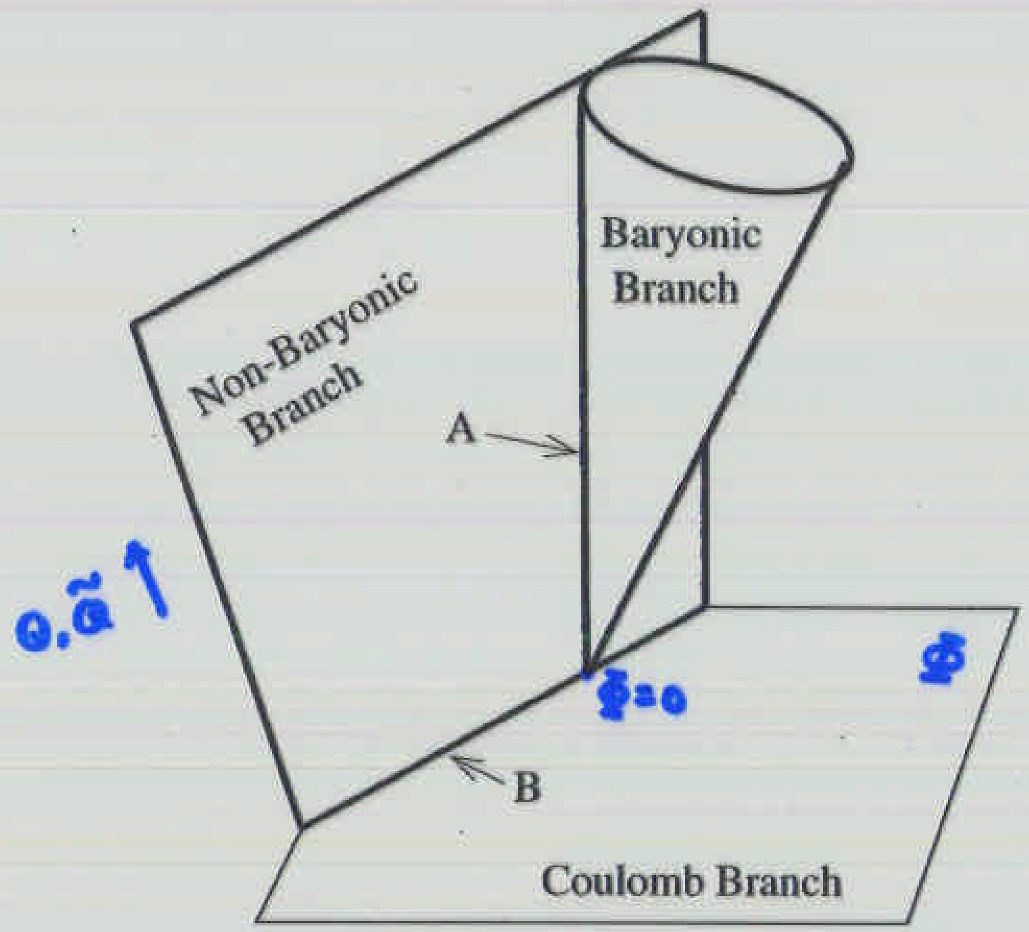


Fig. 1: Map of the classical moduli space of $N=2$ $SU(n_c)$ QCD with n_f fundamental flavors. The baryonic and non-baryonic Higgs branches intersect along a submanifold A, while the non-baryonic branch intersects the Coulomb branch along submanifold B where there is an unbroken $SU(r)\times U(1)^{n_c-r}$ gauge symmetry with n_f massless fundamental hypermultiplets. A and B intersect at a point where the full $SU(n_c)$ with n_f hypermultiplets is unbroken. There are separate non-baryonic branches for $1 \leq r \leq [n_f/2]$.

① Coulomb branch

$$Q, \tilde{Q} = 0$$

$$\Phi = \begin{pmatrix} \phi_1 & & & \\ & \phi_2 & & \\ & & \ddots & \\ & & & \phi_{n_c} \end{pmatrix}$$

$$\mathrm{tr}\,\Phi = \sum_a \phi_a = 0$$

$SU(n_c) \longrightarrow U(1)^{n_c - 1}$ gauge group

complex $n_c - 1$ dim moduli space

generic points:

$U(1)^{n_c-1}$ gauge multiplets

massive quarks, monopoles

singular submanifolds

some massless hypermultiplets

enhanced gauge groups

② Non-baryonic branch ($n_f \geq 2$)

$$Q = \left(\begin{array}{c|c|c} \begin{matrix}\kappa_1 & & \\ & \ddots & \\ & & \kappa_r\end{matrix} & 0 & \\ \hline 0 & 0 & 0 \end{array}\right)$$

$$\tilde{Q} = \left(\begin{array}{c|c|c} 0 & \begin{matrix}\kappa_1 & & \\ & \ddots & \\ & & \kappa_r\end{matrix} & 0 \\ \hline 0 & 0 & \end{array}\right)$$

$$\Phi = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{matrix}\phi_{r+1} & & \\ & \ddots & \\ & & \phi_{n_c}\end{matrix} \end{array}\right)$$

separate r-branches

$$1 \leq r \leq \min\left\{ \left[\frac{n_f}{2}\right], n_c - 2 \right\}$$

$\kappa \to 0$ touches Coulomb branch w/ enhanced $SU(r) \times U(1)^{n_c - r}$ gauge group

IR free theory

③ Baryonic branch $(n_f \geq n_c)$

$$Q = \left(\begin{array}{ccc|c|c} \kappa_1 & & & 0 & 0 \\ & \ddots & & & \\ & & \kappa_{n_f-n_c} & & \\ \hline & 0 & & \begin{matrix}\kappa_0 & & \\ & \ddots & \\ & & \kappa_0\end{matrix} & 0 \end{array}\right)$$

$$\tilde{Q} = \left(\begin{array}{ccc|c|c} \tilde{\kappa}_1 & & & 0 & \begin{matrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_{n_f-n_c}\end{matrix} \\ & \ddots & & & \\ & & \tilde{\kappa}_{n_f-n_c} & & \\ \hline & 0 & & \begin{matrix}\tilde{\kappa}_0 & & \\ & \ddots & \\ & & \tilde{\kappa}_0\end{matrix} & 0 \end{array}\right)$$

$$\Phi = 0$$

$$\kappa_a \tilde{\kappa}_a = \kappa_0 \tilde{\kappa}_0 = \rho$$

$$|\tilde{\kappa}_a|^2 + |\lambda_a|^2 - |\kappa_a|^2 = |\tilde{\kappa}_0|^2 - |\kappa_0|^2 = \nu$$

$\kappa, \tilde{\kappa}, \lambda \to 0$ touches Coulomb branch

Argyres, Plesser, Seiberg

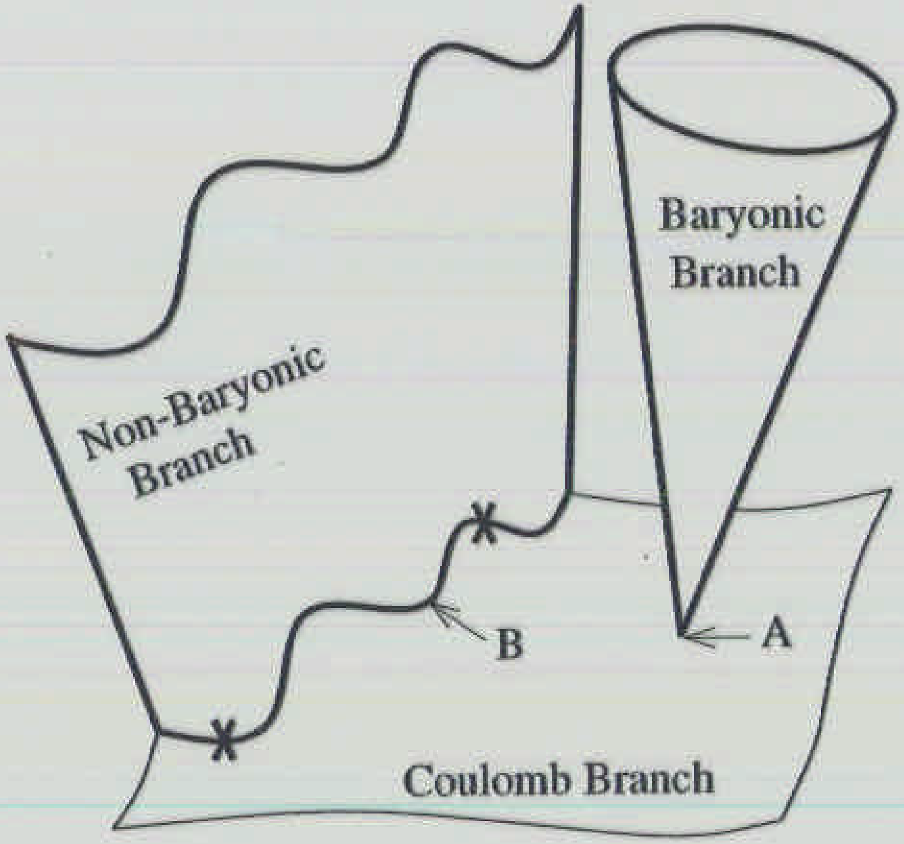


Fig. 2: Map of the quantum moduli space of $N=2$ $SU(n_c)$ QCD with n_f fundamental flavors. Point A has unbroken gauge group $SU(n_f-n_c)$ with n_f massless fundamental hypermultiplets as well as various extra monopole singlets. Submanifold B has unbroken gauge group $SU(r)\times U(1)^{n_c-r}$ with n_f fundamental hypermultiplets. The X's mark points (submanifolds) on B where there are extra massless singlets.

auxiliary curve known

Argyres - Plesser - Shapere

Hannany - Oz

$$y^2 = \prod_{a=1}^{n_c} (x - \phi_a)^2 + 4\,\Lambda^{2n_c - n_f} \prod_{i=1}^{n_f} (x + m_i)$$

$(n_f \leq 2n_c - 2)$

study singularities

$\Rightarrow$ locate "roots" of

non-baryonic r-branches

baryonic branch

<u>remember</u> r-branch root $\Rightarrow$ $SU(r) \times U(1)^{n_c - r}$

IR free gauge theory

$$\Phi = (\underbrace{0, \cdots, 0}_{r}, \phi_{r+1}, \cdots, \phi_{n_c - r})$$

choose remaining ϕ's

$\Rightarrow$ all a_D's vanish

$n_c - r - 1$ massless monopoles

§ Coulomb branch description

identify r-branch roots

baryonic root

mass perturbation $m_i \neq 0$

$\Rightarrow$ count # vacua around each root

technically quite complicated

but doable

r-branch root $\Rightarrow$ ${}_{n_f}C_r$ vacua $U(n_f) \to U(r) \times U(n_f - r)$

$2n_c - n_f$ of them for each r

baryonic root

$\parallel$ $\Rightarrow$ ${}_{n_f}C_{n_f-n_c} \times (2n_c - n_f) + N_2$

$(n_f - n_c)$-branch root

(APS thought they were separate)

total $2^{n_f-1}(2n_c - n_f) + N_2$

consistent with large μ analysis

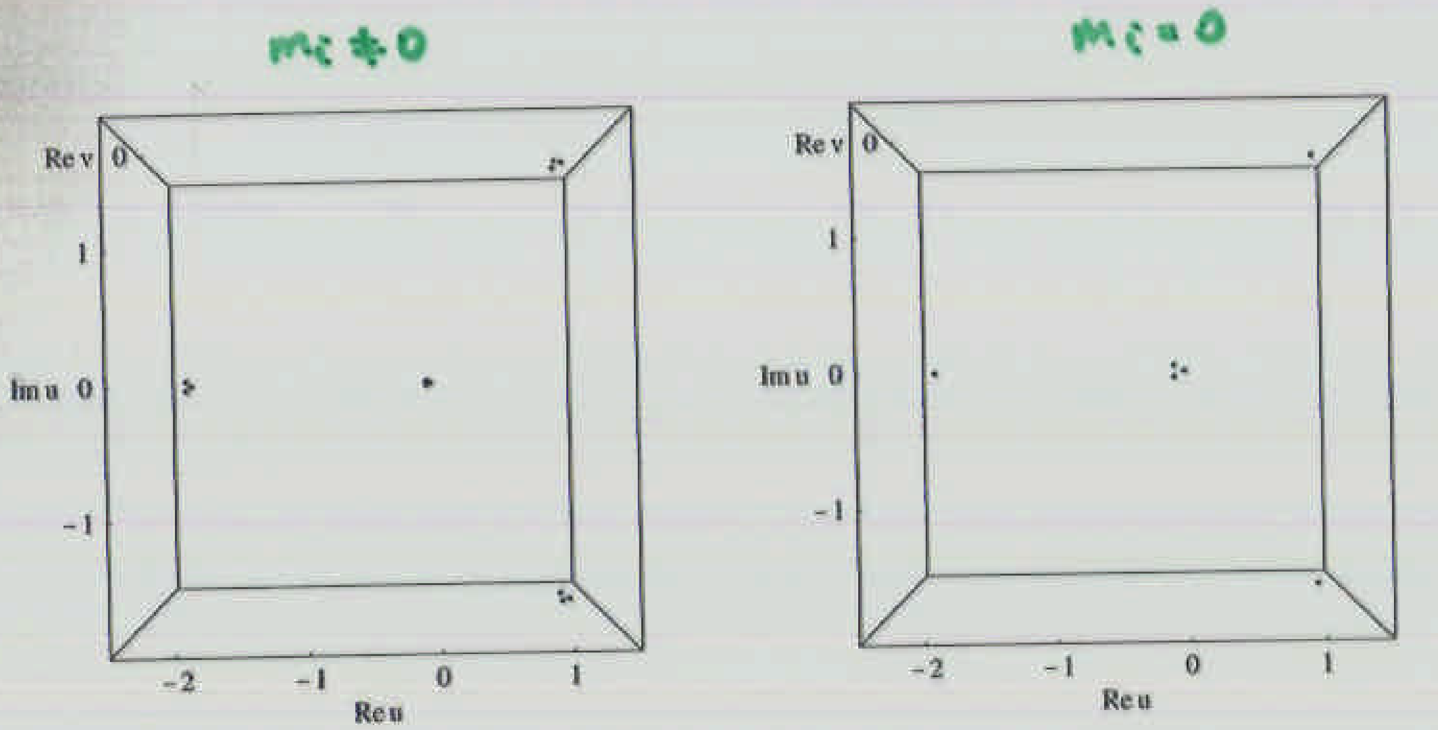


Figure 3: Twelve vacua of the $SU(3)$ theory with $n_f = 3$ in the projection $(\mathrm{Re}\,u, \mathrm{Im}\,u, \mathrm{Re}\,v)$. $\Lambda_3 = 2$, $m_1 = 1/64$, $m_2 = i/64$, $m_3 = -i/64$. The same projection in the right with equal masses: $m_i = 1/64$.

$$W = a_D \tilde{M}_{i_1 \dots i_r} M_{i_1 \dots i_r} + \mu u$$

$$\Rightarrow \quad M, \tilde{M} \neq 0$$

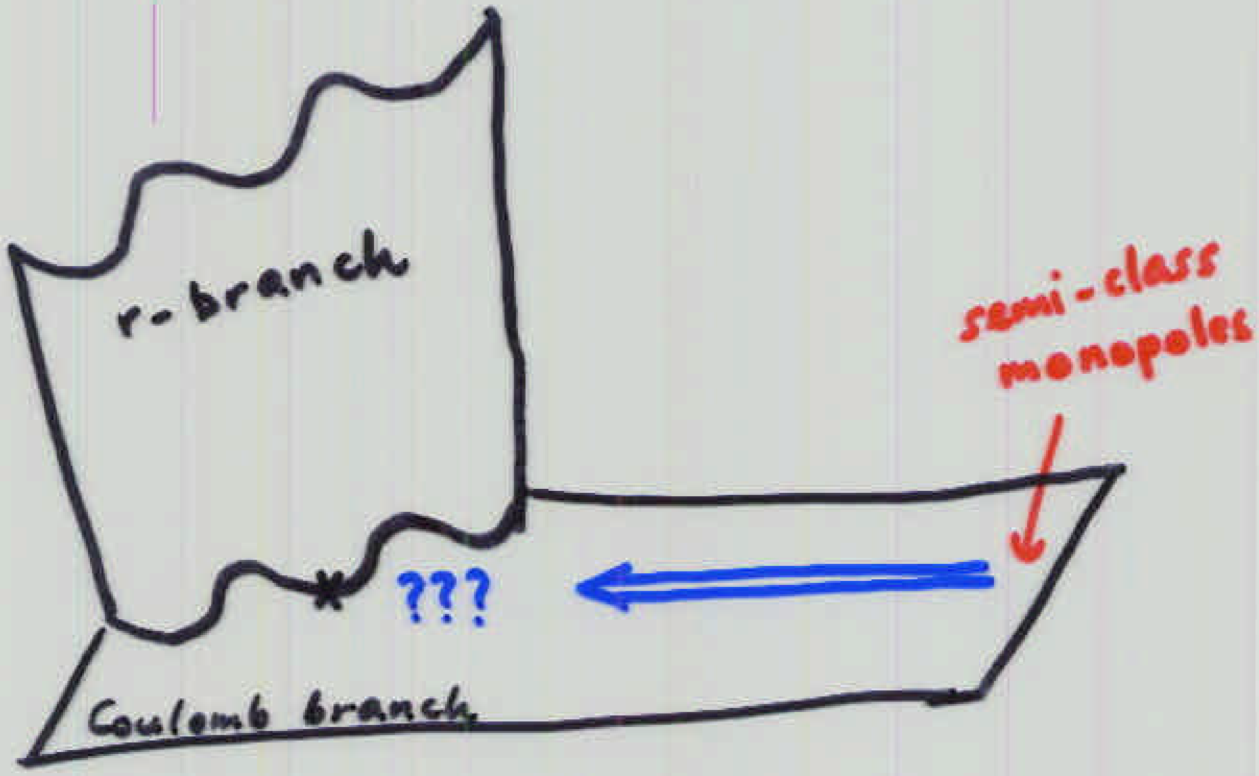
$$U(n_f) \to U(r) \times U(n_f - r)$$

paradox

too large accidental low-energy symm

$U({}_{n_f}C_r)$

→ too many Nambu-Goldstone



§ Low-energy Effective Lagrangians

r-branch root approached from the non-baryonic branch

⇒ low-energy degrees of freedom

$SU(r)$ gauge multiplet

n_f hypermultiplets

$U(1)^{n_c-r}$ gauge multiplets

n_c-r-1 monopole hypermultiplets

	$SU(r)$	$U(1)_0$	$U(1)_1$	....	$U(1)_{n_c-r-1}$
n_f q's	$\underline{r}$	1	0	.....	0
e_1	$\underline{1}$	0	1	....	0
⋮	⋮	⋮	⋮		⋮
e_{n_c-r-1}	$\underline{1}$	0	0	....	1

$$W = \sqrt{2}\,\tilde{q}_i \phi q_i + \sqrt{2}\,\psi_0 \tilde{q}_i q_i + \sqrt{2} \sum_{k=1}^{n_c-r-1} \psi_k \tilde{e}_k e_k$$

$N=1$ perturbation $\mu \,\mathrm{tr}\,\Phi^2$

$$\Delta W = \mu \Lambda \sum_{j=0}^{n_c-r-1} x_j \psi_j + \mu \,\mathrm{tr}\,\phi^2$$

vacua

$$q = \tilde{q} = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \end{pmatrix} \sqrt{-\frac{\mu\Lambda}{\sqrt{2}r}}$$

choose r flavors

${}_{n_f}C_r$ choices

correct!

$$e_k = \tilde{e}_k = \sqrt{-\mu\Lambda}$$

$$\psi_0 = \psi_k = 0$$

$U(n_f) \to U(r) \times U(n_f - r)$

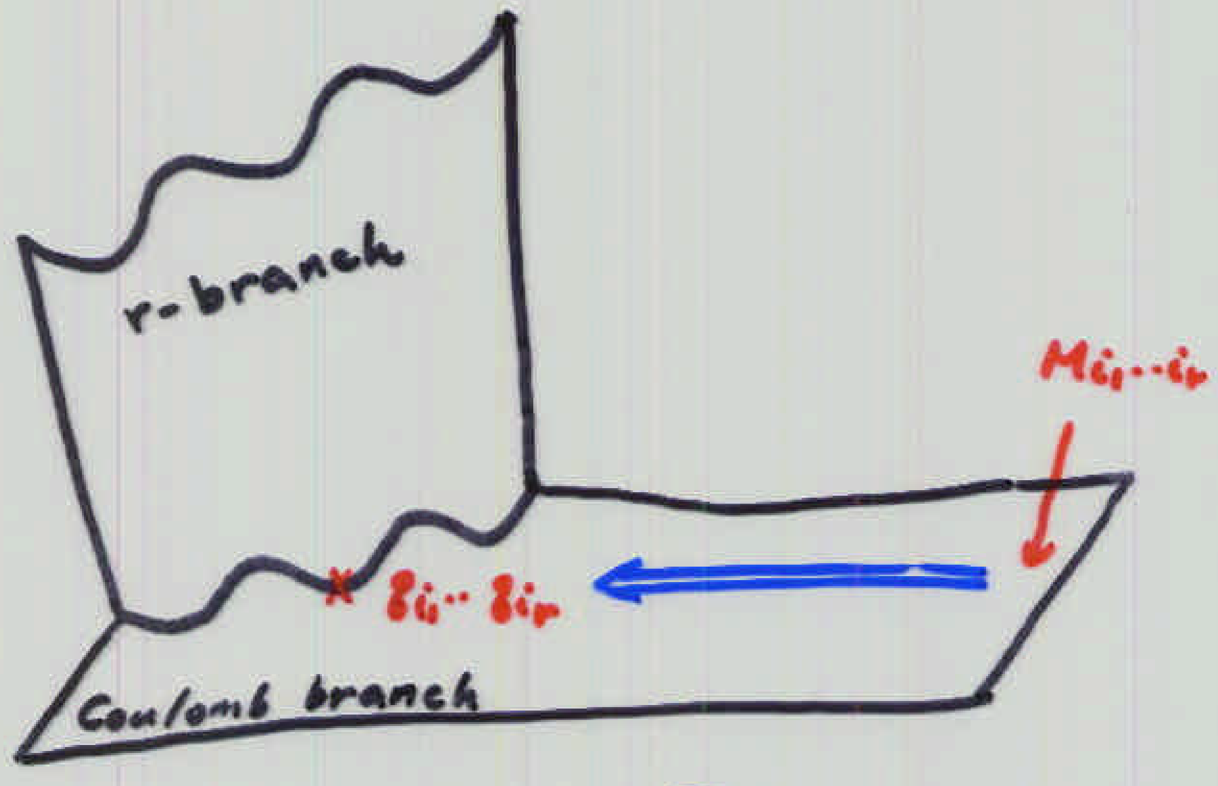
due to $q \cdot \tilde{q} \neq 0$

APS quoted more sol'ns where $\psi_0 \sim \Lambda$

they are beyond the validity of $\mathcal{L}_{\text{eff}}$

and should be discarded

relationship to semi-classical monopole



monopole in $\left.\begin{array}{|c|}\hline \\ \hline \vdots \\ \hline \\ \hline\end{array}\right\} r$ rep $M_{i_1 \cdots i_r}$

breaks up into "dual quarks"

$$M_{i_1 \cdots i_r} = q_{i_1} \cdots q_{i_r}$$

su(r) baryon

before it condenses

when $r = 1$

$M_i = q_i$ monopoles themselves

evidence for this identification

mass perturbation

$$W = \sqrt{2}\,\tilde{q}_i \phi q_i + \sqrt{2}\,\psi_0 \tilde{q}_i q_i + m_i \tilde{q}_i q_i + e\text{'s}$$

can choose

$$\phi + \psi_0 = \begin{pmatrix} -m_1 & & \\ & \ddots & \\ & & -m_r \end{pmatrix}$$

to make r out of n_f flavors massless

$$SU(r) \times U(1) \longrightarrow U(1)^r$$

turn $\mu \neq 0$

$$q_i \tilde{q}_i = \mu \left(m_i - \frac{1}{r} \sum_j^r m_j \right) - \frac{\mu \Lambda}{\sqrt{2}\, r}$$

simple monopole condensation
for each U(1) factors

verified further by explicit
monodromy calculations

§ Conclusion

$N=2$ SUSY QCD $SU(n_c)$, n_f

$\begin{cases} \text{confinement} \\ \text{flavor symmetry breaking} \end{cases}$

both caused by the same mechanism

condensation of magnetic objects

$Sp(n_c)$ theories

monopoles: $SO(2n_f)$ spinor

can't break up into q's

$SO(2n_f)$ vector

monopoles + quarks coexist at the singularity

⇒ superconformal theory

still, $SO(2n_f) \to U(n_f)$ breaking

consistent w/ monopole condensation