Spontaneous susy breaking in N=2 super Yang-Mills theories

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Topics:

1) basic fields in N=2 super Yang-Mills theories
   - central anomalies ⟷ $U2_R$ symmetry

2) trace anomaly and potential susy breaking for all supersymmetries

3) effective potential ⟷ minimal source extension. axial current anomaly ⟷ thermodynamic limit - Θ, Θ'

4) nontrivial relaxation of external sources the case of complete susy breaking

5) conclusions & outlook.

work done in collaboration with L. Bergamin

basic field content of N=2 s. Yang-Mills theories

integral susy charges
(exist only if no spontaneous susy breaking prevails)

$$\{ Q^i{}_\alpha , \bar{Q}_{k \dot\beta} \} = \delta^i{}_k P_{\alpha\dot\beta} \quad ; i,k = 1,2.(N)$$

'once' local form

$$\{ J^i_{\mu\alpha}(x) , \bar{Q}_{k\dot\beta} \} = \delta^i_k \, \vartheta_{\mu\alpha\dot\beta}(x)$$

$$\downarrow$$

$$\vartheta_{\mu\nu} \sigma^\nu{}_{\alpha\dot\beta}$$

$$\int_t J^i_{0\alpha}(x) d^3x = Q^i{}_\alpha \quad / \quad \int_t d^3x \, \vartheta_{0\alpha\dot\beta} = P_{\alpha\dot\beta}$$

supercurrents — energy momentum tensor.

exact conservation

$$\partial^\mu J_\mu{}^i{}_\alpha (x) = 0 \qquad \partial^\nu \vartheta_{\nu\mu}(x) = 0.$$

$$\{ Q^i{}_\alpha , Q^j{}_\beta \} = 0$$

no central charges

$$Q \to \bar{Q}$$

universal (potential) obstruction
for integral susy charges →

$$\{ J_\mu{}^i{}_\alpha (x) \,,\, \bar{Q}_{\kappa\dot\beta} \} = \delta^i{}_\kappa \, \vartheta_{\mu\,\alpha\dot\beta} (x)$$

$[\dim]_{mass}$ $\quad 7/2 \quad 1/2 \quad 4.$

trace anomaly $\quad \vartheta^\mu{}_\mu = \frac{2\beta}{g}(-\mathcal{L})$ with

$$\langle \Omega | \vartheta_{\mu\nu} | \Omega \rangle = -\frac{1}{4} g_{\mu\nu} \, B^2 \neq 0$$

$|\Omega\rangle$ true (as opposed to trial-) ground state

$$\to \langle \Omega | \{ J_\mu{}^i{}_\alpha (x) \,,\, \bar{Q}_{\kappa\dot\beta} \} \Omega \rangle$$

$$= \left(-\frac{1}{4} B^2 \right) (\sigma_\mu)_{\alpha\dot\beta} \, \delta^i{}_\kappa$$

$\to \quad \bar{Q}_{\kappa\dot\beta} |\Omega\rangle \neq 0. \; \forall \kappa$ → universally

⟷ spontaneous breaking of all N supersymmetries.
→ N goldstino modes

will argue in the following that this situation prevails adjoint representation:

$$\left(\tilde{t}^A\right)_{BC} = \frac{1}{i} f_{ABC}$$, f: structure constants of local (semi) simple gauge group G.

$$\sum_A \left(\tilde{t}^A\right)^2 = C_2(G) \cdot \mathbb{1}_{(\text{adjoint})}.$$

$C_2(SU_n) = n$ conventional normalization.

constrained (Lie algebra valued) chiral N=2 superfield

$$W(x^+, \vartheta^\alpha_i) = \left\{ \begin{array}{l} \sqrt{2}\, \Phi(x^+) \\ + \sqrt{2}\, \vartheta^\alpha_i \, \lambda^i_\alpha(x^+) \\ \vartheta^{(2)\{\alpha\beta\}} F_{\{\alpha\beta\}}(x^+) \\ + \vartheta^{(2)}_{\{ik\}} H^{\{ik\}}(x^+) \\ + \vartheta^{(3)\,\alpha}_i \, \chi^i_\alpha(x^+) \\ + \vartheta^{(4)} D(x^+). \end{array} \right\}_{BC.}$$

$$\| \quad \left(W^A \tilde{t}^A\right)_{BC}$$

$$\varepsilon_{\alpha\beta} = \varepsilon_{ik} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\substack{ik \\ \alpha\beta}} = -\varepsilon^{\alpha\beta} = -\varepsilon^{ik}.$$

$$\vartheta^{\alpha}{}_{i}\,\vartheta^{\beta}{}_{k} = -\varepsilon_{ik}\;\vartheta^{(2)\{\alpha\beta\}} - \varepsilon^{\alpha\beta}\,\vartheta^{(2)}{}_{\{ik\}}.$$

$$\vartheta^{(2)\{\alpha\beta\}} = \frac{1}{2}\,\vartheta^{\alpha}{}_{i}\,\vartheta^{\beta i}$$

$$\vartheta^{(2)}{}_{\{ik\}} = \frac{1}{2}\,\vartheta^{\alpha}{}_{i}\,\vartheta_{\alpha k}.$$

$$\vartheta^{(4)} = \frac{1}{12}\,\vartheta^{\alpha}{}_{i}\,\vartheta^{\beta i}\,\vartheta_{\alpha k}\,\vartheta_{\beta}{}^{k}.$$

$$\vartheta^{(3)}{}^{i}{}_{\alpha} = \partial_{\vartheta^{\alpha}{}_{i}}\,\vartheta^{(4)} = \partial^{i}{}_{\alpha}\,\vartheta^{(4)}.$$

$$D^{i}{}_{\alpha} = \partial^{i}{}_{\alpha} - \frac{i}{2}\,\sigma^{\mu}{}_{\alpha\dot\beta}\,\bar\vartheta^{\dot\beta i}\,\partial_{\mu}$$

$$\bar D_{\kappa\dot\beta} = -\partial_{\kappa\dot\beta} + \frac{i}{2}\,\sigma^{\mu}{}_{\alpha\dot\beta}\,\vartheta^{\alpha}{}_{\kappa}\,\partial_{\mu}$$

$$\{D^{i}{}_{\alpha}, \bar D_{\kappa\dot\beta}\} = \delta^{i}{}_{\kappa}\,\sigma^{\mu}{}_{\alpha\dot\beta}\,(i\,\partial_{\mu})$$

$$x^{+\mu} = x^{\mu} + \frac{i}{2}\,\bar\vartheta^{\kappa\dot\beta}(\sigma^{\mu})_{\alpha\dot\beta}\,\vartheta^{\alpha}{}_{\kappa}, \quad \bar D_{\kappa\dot\beta}(x^{+\mu}) = 0.$$

constraint on $\left(W = W^A \tilde{T}^A\right)_{BC}$

not chosen $\theta^{\dot\alpha}$: chiral

$$W^+ = e^{-iX}\,\overline{W}\,e^{+iX}.$$

gauge transformation chiral $\Lambda(x,\theta)$

$$W \to e^{i\Lambda}\,W\,e^{-i\Lambda}$$

$$W^+ \to e^{i\overline{\Lambda}}\,W^+\,e^{-i\overline{\Lambda}}$$

$$e^{iX} \to e^{i\Lambda}\,e^{iX}\,e^{-i\overline{\Lambda}}$$

$$\nabla^{ik}\cdot W = \overline{\nabla}^{ik}\cdot\overline{W}\;;\; \nabla^{ik} \sim D^{\alpha i} D_\alpha{}^k$$

$$\overline{\nabla}^{ik} \sim \overline{\nabla}^{\alpha i}(X, W, \overline{W}) \cdot \overline{\nabla}_\alpha{}^k$$

$\longrightarrow$

in N=2 generalized Wess-Zumino gauge

$$\nabla^{ik} W = \bar{\nabla}^{ik} \cdot \bar{W} \rightarrow$$

$$W = \left\{ \begin{array}{l} \sqrt{2}\ \Phi \\ + \sqrt{2}\ \vartheta^{\alpha}{}_{i}\ \lambda^{i}{}_{\alpha} \\ + \vartheta^{(2)\{\alpha\beta\}}\ F_{\{\alpha\beta\}} \\ + \vartheta^{(2)}{}_{\{ik\}}\ H^{\{ik\}} \end{array} \right.$$

$$+ \vartheta^{(3)\alpha}{}_{i}\ \chi^{i}{}_{\alpha}$$

$$- \vartheta^{(4)}\ D.$$

$(E + iB)_{\{\alpha\beta\}}$.

base auxiliary fields $(H^{\{ik\}})$ B.c.

potentials

$$\rightarrow.\ \chi^{\alpha}{}_{i} = \sqrt{2}\, i\, \sigma^{\mu\, \dot{\beta}\alpha} \left[D_{\mu}(V) \cdot \bar{\lambda}_{i \dot{\beta}} \right]$$

$$D = \sqrt{2}\ D^{\mu} \cdot \left(D_{\mu} \cdot \Phi \right)$$

$$- \frac{1}{\sqrt{2}} \left[\bar{\Phi}, \left[\Phi, \bar{\Phi} \right] \right].$$

$$- i \left\{ \bar{\lambda}^{i}{}_{\dot{\beta}}, \bar{\lambda}^{\dot{\beta}}{}_{i} \right\}.$$

$$H^{i}{}_{k} = \frac{1}{2} (\tau^{a})^{i}{}_{k}\ H_{a}{}^{A}\, \tilde{T}^{A},\ H_{a}^{A} = H_{a}^{*A};\ a = 1,2,3$$

$\mathcal{U}(\underset{1}{2})$ $(R, SU2)$ classical symmetries

$$\underline{\mathcal{U}}^{-1} Q^i{}_\alpha \underline{\mathcal{U}} = \mathcal{U}^i{}_k Q^k{}_\alpha.$$

$$\underline{\mathcal{U}}^{-1} \lambda^\alpha{}_i \underline{\mathcal{U}} = \bar{\mathcal{U}}^i{}_k \lambda^\alpha{}_k.$$

$(U1 \times SU2)$ matrices.

generators $(R \otimes I_k)$.

$$[Q^i{}_\alpha , R] = Q^i{}_\alpha$$

$$[Q^i{}_\alpha \; I_r] = \tfrac{1}{2}(\tau_r)^i{}_k \; Q^k{}_\alpha.$$

associated $\lambda^\alpha{}_i$ currents

$$R \to Tr(\lambda^{*\dot\beta}{}_i \; \sigma_{\mu\alpha\dot\beta} \; \lambda^\alpha{}_i) = J^{(5)}_\mu(\lambda) \cdot C_2(G)$$

$\to$ chiral anomaly $\leftarrow$ total chiral fermion

broken R-symmetry 'like' Q^5_1 in QCD.

(not changed in susy framework)

SU2: nonanomalous $\to$ conserved

not (obviously) broken

$$I_r \to \lambda^{*\dot\beta}{}_i \; \tfrac{1}{2}(\tau_r)^{*}{}_i{}^k \; \lambda^\alpha_k \; (\sigma_\mu)_{\alpha\dot\beta}$$

central anomalies

$$J_\mu^5 = \frac{1}{C_2(G)} \underset{(BC)}{Tr} \left(\lambda^{*\dot\rho}_{\ i} \sigma_{\mu\alpha\dot\rho} \lambda^{\alpha}_{\ i} \right)$$

$$= \sum_A \lambda^{*\dot\rho A}_{\ i} \sigma_{\mu\alpha\dot\rho} \lambda^{\alpha A}_{\ i})$$

$$\left(\lambda^{\alpha}_{\ i} = \lambda^{\alpha A}_{\ i} \tilde{T}^A \right)_{BC}$$

$$Tr\left(\tilde{T}^A \tilde{T}^B\right) = C_2(G)\, \delta^{AB}$$

$A, B = 1, \ldots \dim(G).$

axial central anomaly

$$\partial^\mu J_\mu^5 = 2\, C_2(G) \underbrace{\frac{1}{16\pi^2} \sum_A F^A_{\mu\nu} \tilde{F}^{\mu\nu A}}_{\frac{1}{16\pi^2} \frac{1}{C_2(G)} Tr\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)} + \ldots$$

$$\underbrace{Tr\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)}_{4\vec{E}\cdot\vec{B}}$$

in parallel with trace anomaly

$$\vartheta^\mu{}_\mu = \underbrace{\frac{2\beta}{g^3}}\, \frac{1}{4}\, \frac{1}{C_2(G)} Tr\left(F_{\mu\nu} F^{\mu\nu}\right)$$

$$= -\frac{1}{16\pi^2} \frac{1}{C_2(G)} Tr\left(F_{\mu\nu} F^{\mu\nu}\right)$$

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perfect chiral structure of both
central anomalies.

$$2\cdot ch_2(F)$$

$$P = \partial^\mu J^5_\mu = \frac{1}{16\pi^2}\,\frac{1}{C_2(G)}\,Tr\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\underline{S} = \vartheta^\mu_\mu = -\frac{1}{16\pi^2}\,\frac{1}{C_2(G)}\,Tr\, F_{\mu\nu}F^{\mu\nu}$$

action functional density

$$S_\chi = -\frac{1}{4}\left(Tr\left(\frac{1}{g^2}+\frac{i\Theta}{8\pi^2}\right)\cdot W^2\right)\frac{1}{C_2(G)}$$

$$S = Re\ S_\chi = (S_\chi + S_\chi^*)/2.\ (x)$$

$\Theta - \Theta'(\Omega) \to 0$
relaxation.
like in QCD
$\cong$ no Θ, no Θ'

chiral dilaton field

$$\underline{S} - iP = \frac{1}{4\pi^2}\ S_\chi\ [\Theta = \Theta' = 0]\ (x)$$

ren. group invariant relations.

$$\sim Tr\left[F_{\mu\nu}F^{\mu\nu} + i F_{\mu\nu}\tilde{F}^{\mu\nu}\right]\frac{1}{g^2}$$

Θ, Θ' relaxation (in short)

Θ at in $\underline{S_\Theta}$ fixed arbitrarily but

<u>trial</u> ground states involving

Θ', Θ'' representing of winding numbers

$$Ch_2 = \int d^4x \, ch_2(F) \Rightarrow \nu$$

$\nu = 0, \pm 1, \ldots$

$$\Omega(\Theta') = \sum_{\nu=-\infty}^{+\infty} e^{-i\nu\Theta'} |\nu\rangle$$

$$Ch_2 |\nu\rangle = \nu |\nu\rangle$$

$$\langle \Omega(\Theta'') | \Omega(\Theta') \rangle = \delta(\Theta' - \Theta'')$$

fixed <u>angle</u> coherent state on the circle

$c = e^{i\Theta'}$

$$Z(\Theta, \Theta') = \sum e^{i\nu(\Theta - \Theta')} z_\nu$$

$\downarrow$

$\Omega(\Theta')$

$z_\nu = (z_{-\nu})^*$

minimize effective action w.r.t. Θ' given Θ $\longrightarrow$

f (functional) $i\bar{W}_5 = \log Z\ (\Theta, \Theta')$

$$\left.\frac{\delta \bar{W}}{\delta \Theta'(x)}\right|_{\Theta'(x) \to \Theta'} = 0$$

boundary condition
for finite $V^{(4)} = V \to \infty$

$\to \quad \Theta'$ trial $\to \Theta'^{(*)}$ true groundstate

$\to \langle \Omega(\Theta'^{(*)}) | P | \Omega(\Theta'^{*}) \rangle$

$= 0$. restoration of CP invariance

yields $\Theta'^{(*)} = \Theta$
this nontrivial relaxation is (in the end) equivalent to restoration of full $U(2)_{R,I_r}$ symmetry despite the anomaly but only in the thermodynamic limit $\simeq$ $\left.\begin{matrix}\Theta \to 0\\ \Theta'^{*} \to 0\end{matrix}\right\}$ simultaneously

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$\Omega(\Theta'^{*}) \to \Omega$ $\quad / (\Theta = \Theta'^{4} = 0)$

after performing the dynamically nontrivial th.dyn. limit

$$\langle \Omega | P | \Omega \rangle = 0$$

(CP=-)

and potential

(universal / all N) susy breaking

$\Updownarrow$

$$\langle \Omega | s | \Omega \rangle = - B^2.$$

$$= - \frac{1}{16\pi^2} \times$$

$$\langle \Omega | \frac{1}{c_2(G)} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} | \Omega \rangle$$

$$+ \dots$$

$$\propto \mathcal{L}(\Theta = 0)$$

$s = s(x)$

$$\langle \Omega | s | \Omega \rangle = -B^2$$

and $\langle \Omega | p | \Omega \rangle = 0$

override in essentially
non semiclassical way

$$s \sim -\mathcal{L}$$

the semiclassical form of $\mathcal{L}(\phi)$

scalar field potential $V_\phi = -\mathcal{L} \sim \frac{1}{g^2} f_{ABC} \phi^B \phi^{*C} f_{AB'C'} \phi^{C'} \phi^{*B'}$

$$\left(\phi = \phi^A F^A\right)_{BC}$$

no Θ, Θ' dependence here

suggesting persistence of
semiclassical flat directions
suggesting
V_ϕ minimal for any 'modulus'
$\phi = \phi_{cl}$ mit $[\phi_{cl}, \phi_{cl}^*] = 0$
Seiberg and Witten assumption.

a simple 'hint' to be complemented ↓.

from 6-d $\mathcal{N}=1$ super Yang Mills system.

$$V_\phi \propto \mathrm{Tr}\left(F_{56}\right)^2 \quad (\geq 0)$$

at long range i.e. in th. dyn. limit
also restoration of 6-d symmetry
(not at finite momenta).

g large

$$\langle \Omega | \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} | \Omega \rangle \neq 0$$

$\mu, \nu = 0, \dots, 3$

↕

$$\langle \Omega | V_\phi | \Omega \rangle = \langle \Omega | c.\, \mathrm{Tr}\left(F_{56}\right)^2 | \Omega \rangle$$

in $\mathcal{L}$ $\qquad \neq 0$ also.

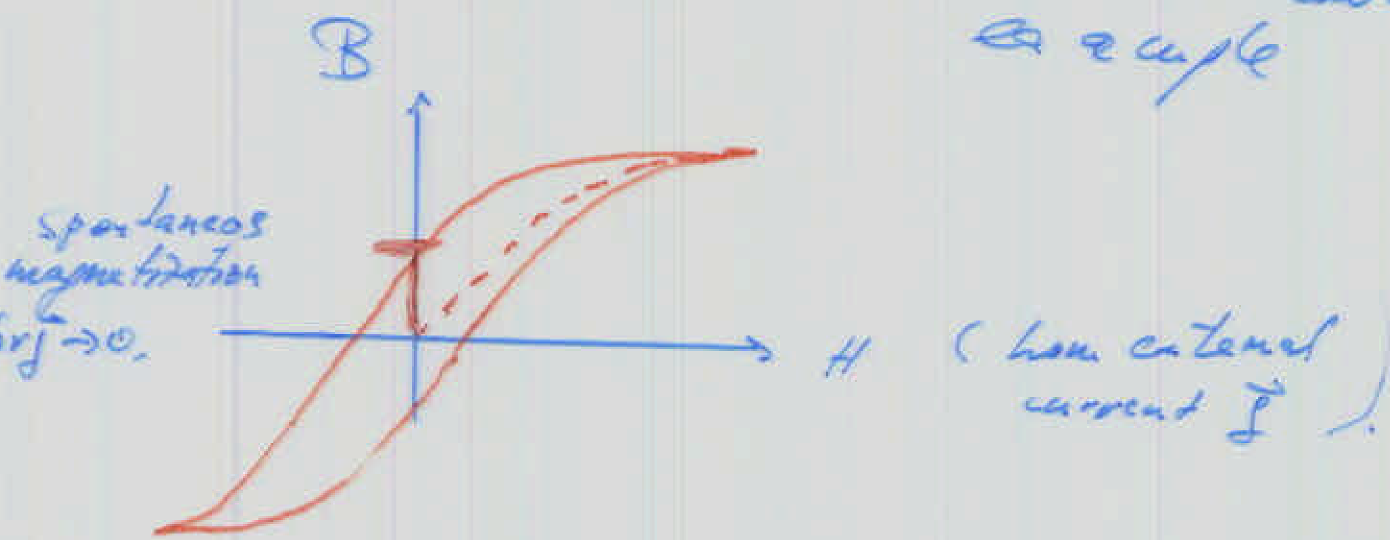
since $s \propto \mathcal{L}$ all susy ($N=1,2$)
are spontaneously broken
for $\langle \Omega | s | \Omega \rangle \neq 0$ at the same time

sketch of nontrivial external source extension

effective action codetermined from { } Green-functions and th. dyn. limit:

Idea: use explicite minimal and eventually susy breaking external sources which do not spoil renormalizability

'hysteresis' like a well known example



here external sources chiral coupling constant multiplet set freely

lowest ϑ^i_α... component

$$\frac{1}{g^2} \to \frac{1}{g^2(x)} - i\,\frac{\Theta(x)}{8\pi^2}$$ with bound. cond.

$$\frac{1}{g^2} \to \underbrace{\frac{1}{g^2(x)} - i\,\frac{\Theta(x)}{8\pi^2}}_{\tau(x)} \Bigg\} \; J(x) \to$$

$$\lim_{x\to\infty} \frac{1}{g^2(x)} = \frac{1}{g^2} \Bigg\} \; J_0$$

$$\lim_{x\to\infty} \Theta(x) = \Theta \to 0 \,.\; \begin{pmatrix}\Theta, \Theta' \\ \text{relaxation}\end{pmatrix}$$

and extension to full chiral superfield of external sources with controlled arbitrary but dynamically controlled boundary values

$$J(x,\vartheta) = \left\{ \begin{array}{l} \tau(x) \\ + \vartheta^{\alpha}_{i}\, \xi^{i}_{\alpha}(x) \\ - 4\, \vartheta^{(2)}_{\{ij\}}\, m^{\{ij\}}(x) \\ + \vartheta^{(2)\{\alpha\beta\}}\, w_{\{\alpha\beta\}}(x) \\ + \vartheta^{(3)\alpha}_{i}\, \kappa^{i}_{\alpha}(x) \\ + 4\, \vartheta^{(4)}\, M^2(x) \end{array} \right\}$$

$x \to x^+$ if full $\vartheta, \bar\vartheta$ exploited.

no gauge group dependence.

$$\mathcal{L} \to \mathcal{L}_J = \operatorname{Re}\int d^4\vartheta \, J(x,\vartheta)\left(-\frac{1}{4}\right) \times$$

$$\frac{1}{C_2(G)} \operatorname{Tr} W^2(x,\vartheta)$$

Lconstrained

if $m^{\{J\}}(x) \underset{x\to\infty}{\to} m^{\{J\}} \neq 0$.

$M^2(x) \to M^2$

in $J(x) \to J_\infty$

explicit mass breakings of susy multiplet are induced.

eliminate all auxiliary primary fields H_a^A in W

$$Z(J\bar{J}) = \int \mathcal{D}X \exp i(S_0 + S_J)$$

$$= \exp i\, W_{(f)}(J,\bar{J})$$

Legendre transform $\Gamma(\ell, \bar{\ell}) = \int d^4x \left[J(x) \frac{\delta W_f}{\delta J(x)} + h.c. \right]$

$$- W_f(J,\bar{J})$$

$\ell(x) \leftrightarrow J(x)$

component by component

$$\frac{\delta W_f}{\delta J(x)} = e(x)\,,$$

$$\frac{\delta \Gamma(e,\bar e)}{\delta e(x)} = J(x)$$

relaxation of external sources

$$\to \frac{\Gamma(e,\bar e)}{\delta e(x)} = 0 \qquad \text{maximum condition}$$

also determine in th. dyn. limit $(T=0)$
non trivial dependence
of Γ, for $J \to J_\infty \leftrightarrow e_\infty$

susy covariance (not on shell invariance)
restricts $\Gamma(e,\bar e)$ in an interesting way
(for details see L. Rupanik P.M.)

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$$e(x) \longleftrightarrow \mathrm{tr}\, W^2 \quad \frac{1}{g^2}.$$

$$(\dot{=}) \propto \underbrace{(s - ip(x))}_{\delta(x)_\alpha}{}_{\alpha.}$$ discussed before

in the dyn. limit.

general Kähler (not hyper Kähler) form but with restored chiral symmetry

$x \to \infty$ $$T(\delta_\infty, \bar{\delta}_\infty) = T(\cdot \delta\bar{\delta}).$$

$\delta_\infty \downarrow \delta$, $\bar{\delta}_\infty \downarrow \bar{\delta}$

again $\ominus$ relaxation

$T = T_{max}$ $\nearrow$ $\delta = \langle \Omega | s - ip | \Omega \rangle \neq 0$; $\searrow$ $\delta = 0.$

on a circle in complex δ plane

$\left[\delta\bar{\delta} = \text{const.} \; \substack{> 0 \\ = 0.} \right]$ bad.

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susy covariance implies

for $\delta = 0$ all condensates vanish.

thus infrared confinement

problem is not resolved in s-Yang Mills system

→ if gauge invariance does not for G break

⇒ $\langle \Omega | s | \Omega \rangle = 0$

$\langle \Omega | \bar{p} | \Omega \rangle = 0$

only logical choice in analogy with QCD

no contradiction to N=2 susy invariance of local operators

Conclusions.

1) even though the step from N=1 s-Yang Mills theories to N=2 is nontrivial the basic condensate structure

$$\langle \Omega | \mathcal{L} | \Omega \rangle \neq 0 \Longleftrightarrow \partial^{\mu}{}_{\mu}$$

remains the same.

2) contrary to N=1 due to restored $\mathcal{U}(2)$ invariance and R-chiral structure $\langle \cdot | \Omega \rangle$ there are no binary gaugino condensates in N=2.

$$Tr\{\lambda^{i\alpha}, \lambda^{j}{}_{\xi\alpha}\}$$ is

a triplet representation of SU2(...)

at most triple condensates of the form

$$Tr\,\phi^{*}\{\lambda^{i\alpha}\,\lambda_{i\alpha}\}$$

can develop. singlet but adjoint gauge multiplet

3) separation into different condensates appearing in $\mathcal{L}$.
necessitate a non minimal source extension

on Proof: if N=2 susy is broken spontaneously (s.4.11) two goldstino modes definitely appear in flat space time realized theory. (exactly massless fermion modes)

$\Rightarrow$ if we consider such modes as absent there implies a necessary extension to include N=2 supergravity whereby the goldstino modes are incorporated into 2 massive gravitinos.

$\rightarrow$ N=2 $\rightarrow$ N=4 .....