EXTRA DIMENSIONS

AND

BENDING OF LIGHT

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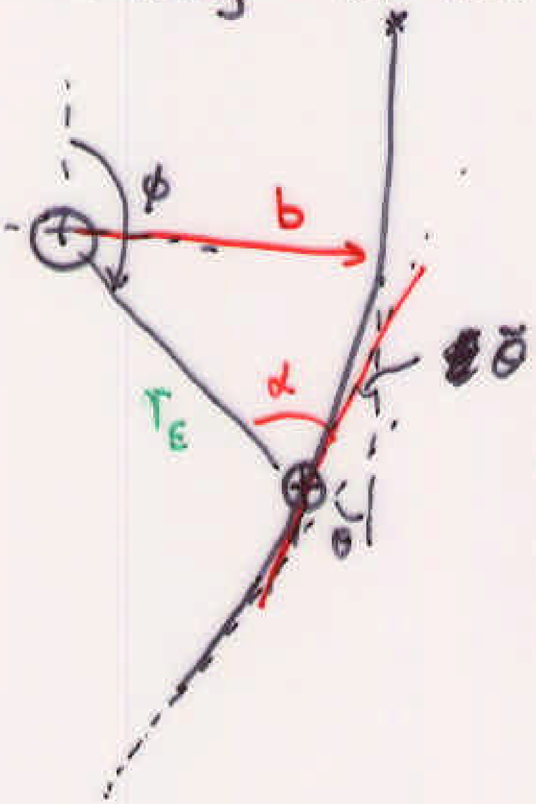
THE CLASSIC CALCULATION

Light bending in GR.



$\alpha = \pi - \phi_E + \delta\alpha$

observed angle between star & sun.

The geodesic is

$$\frac{b}{r} = \sin\phi + G\frac{(1+\gamma)M_\odot}{b}(1-\cos\phi)$$

$$\phi(r=\infty) = \pi + \theta$$

$$\theta = 2\frac{(1+\gamma)}{b}M_\odot G = \left(\frac{1+\gamma}{2}\right)\times 1.''75$$

at $b = r_\odot$

The deflection angle is

$$\tilde{\Theta} = (1+\gamma)\frac{M_\odot G}{r_E}\left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)^{1/2}$$

at $\alpha = \frac{\pi}{2}$, $\tilde{\Theta} = (1+\gamma)\frac{M_\odot G}{r_E} \sim \frac{1+\gamma}{2}\, 0''.0041$

THE MOST ACCURATE CURRENT DATA is

FOR $40° \leq \alpha \leq 140°$

[HIPPARCOS SATELLITE]

RESULTS

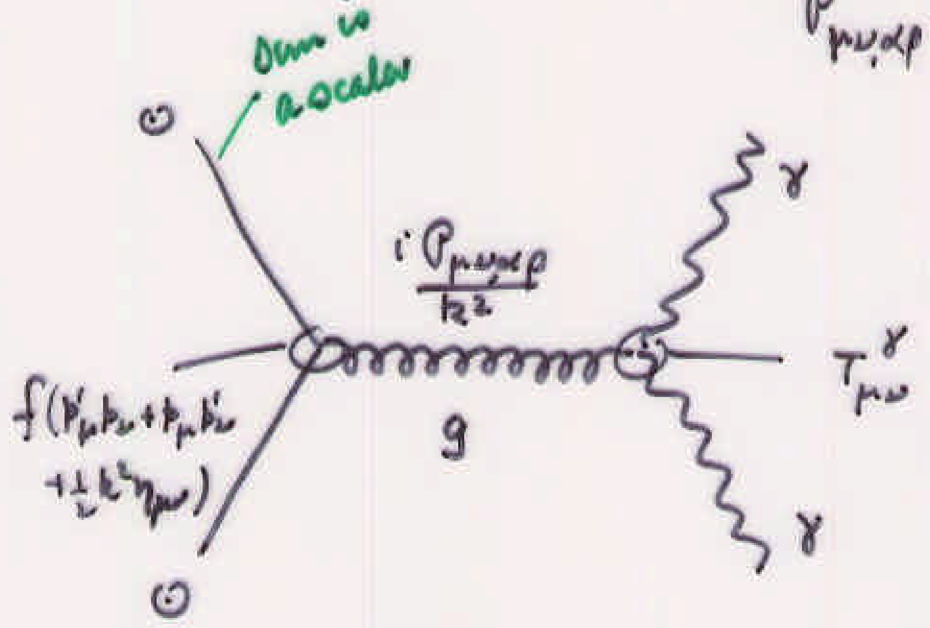
$$\gamma = 0.997 \pm 0.003$$

GR $\gamma = 1$ ✓

ALTERNATIVE CALCULATION!

GRAVITON EXCHANGE ~ LIGHT BENDING.

$$P_{\mu\nu;\alpha\beta} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}$$



$$T^\gamma_{\mu\nu} = F_\mu{}^\alpha F_{\alpha\nu} + \frac{1}{4}\eta_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$
$$= \epsilon^{*\beta} T^\gamma_{\mu\nu\beta\alpha}\epsilon^\alpha$$

$$T^\gamma_{\mu\nu,\alpha\beta} = \frac{1}{2}\Big[q'_\alpha(q_\mu\eta_{\beta\nu} + q_\nu\eta_{\alpha\mu}) + q_\beta(q'_\mu\eta_{\nu\alpha} + q'_\nu\eta_{\mu\alpha})$$

$$- \eta_{\alpha\beta}(q'_\mu q_\nu + q_\mu q'_\nu) + \eta_{\mu\nu}(q'\cdot q\,\eta_{\alpha\beta} - q'_\alpha q_\beta)$$

$$- q'\cdot q(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha})\Big]$$

$$S_{fi} = (-i)^2\, 4 f^2\, p'_\delta\, p_\sigma\; i\,\frac{P^{\delta\sigma\mu\nu}}{k^2}$$

$$\left[q'_\alpha q_\mu \eta_{\beta\nu} + q_\beta q'_\mu \eta_{\alpha\nu} - \eta_{\alpha\beta} q'_\mu q_\nu - \frac{1}{4}\eta_{\alpha\beta} q_\beta q'_\alpha\right] \varepsilon^{\beta\alpha}(q')\,\varepsilon^{\alpha}(q)$$

$$\to \frac{1}{2}\left|\frac{f^2 M_\odot^2}{\sin^2\frac{1}{2}\theta}\right|^2 \cdot \sum_{\text{pol}} |\varepsilon'\cdot\epsilon|^2 = \left|\frac{f^2 M_\odot^2}{\sin^2\frac{1}{2}\theta}\right|^2$$

$$\frac{d\sigma}{d\Omega} = \left|\frac{T_{fi}}{8\pi M_\odot}\right|^2 = \frac{16\, G^2 M_\odot^2}{\theta^4}$$

$$b\,db = -\frac{d\sigma}{d\Omega}\cdot \sin\theta\, d\theta$$

$$\text{so} \int_{b_{min}}^{\cancel{b}\,\theta_{max}} b\,db = -\int_{\theta_{min}}^{\tilde{\theta}} \frac{d\sigma}{d\Omega}\,\theta\, d\theta = -16\, G^2 M_\odot^2 \int_{\theta_{min}}^{\tilde{\theta}} \frac{d\theta}{\theta^3}$$

$$2 R_\odot^2 = 16\, G^2 M_\odot^2\, \frac{1}{\cancel{2}\,\tilde{\theta}^{\cancel{2}}}$$

$$\tilde{\theta} = \frac{4\, G M_\odot}{R_\odot} \qquad \text{At } b = R_\odot$$

as <u>usual</u>

WHICH TECHNIQUE DO WE USE FOR
GENERALISING TO EXTRA DIMENSIONS?

GRAVITON EXCHANE HAS BEEN USED
TO INTRODUCE 1 LOOP CORRECTIONS
TO EINSTEIN'S RESULT

FOLLOW THIS WAY TO EXTRA
DIMENSIONS — EXCHANGE
KALUZA KLEIN TOWERS AS
WELL AS GRAVITONS.

$\boxed{\text{GENERALISATION}}$

With Extra Dimensions

Now allow the

to represent
- graviton
- KK towers of spin 0, 1, 2

But only spin 2 couples to both scalar matter field and photon.

Additional KK propagator term

$$\sum_{\ell} \frac{B^{KK}_{\mu\nu,\alpha\rho}(k)}{k^2 - m_\ell^2}$$

$$B^{KK}_{\mu\nu,\alpha\rho}(q) = \left(\eta_{\mu\alpha} - \frac{q_\mu q_\alpha}{m_\ell^2}\right)\left(\eta_{\nu\rho} - \frac{q_\nu q_\rho}{m_\ell^2}\right) + (\mu \leftrightarrow \nu) - \frac{2}{3}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_\ell^2}\right)\left(\eta_{\alpha\rho} - \frac{q_\alpha q_\rho}{m_\ell^2}\right)$$

Only $\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}$ terms survive

[Gauge Invariance]

Just replace $\frac{1}{k^2} \longrightarrow \frac{1}{k^2} + \sum_{\ell} \frac{1}{k^2 - m_\ell^2}$

The density of states are very large

$$\Delta = \sum_{\ell} \frac{1}{q^2 - m_\ell^2} = -\frac{2}{M_S^2 G_N} \left(\frac{|q|^2}{M_S^2}\right)^{\frac{n}{2}-1}$$

$$I_n\left(M_S; \tfrac{n}{2}\sqrt{|q^2|}\right)$$

$$I_n = \int_{M_{min}/\sqrt{q^2}}^{M_S/\sqrt{q^2}} \frac{y^{n-1}}{\sqrt{1+y^2}}\, dy$$

M_{min} - min KK mass $\frac{2\pi}{R} \sim 10^{-3}\,eV$

$R \sim 10^{-3}\,m.$

$$\Delta = \frac{1}{M_S^4 G_N} \ln \frac{M_S^2}{|q|^2 + M_{\min}^2} \qquad n = 2$$

$$\Delta = \frac{1}{M_S^4 G_N} \frac{2}{n-2}$$

used $G_N = (4\pi)^{-n/2}\, \Gamma(n/2)\, R^{-n} M_S^{-(n+2)}$

REPEAT CROSS SECTION CALCULATION:

$$\frac{d\sigma}{d\Omega} = 16\, G_N^2 m^2 \left(\frac{1}{\theta^2} + \omega^2 \Delta \right)^2$$

AND BENDING ANGLE CALCULATION:

$$\theta = \frac{4 G_N M_\odot}{b} \left(1 - 2\omega^2 \Delta \cdot \left(\frac{4 G_N M_\odot}{b} \right)^2 \ln \frac{4 G_N M_\odot}{b} \right)$$

$$\boxed{\text{GR + correction due to Extra Dimensions}}$$

Correction

- Strongly ω dependent — also true of 1 loop corrections
- $\propto \frac{1}{b^2}$

Only significant for $b \sim R_\odot$, where $\gamma = 1 + \Delta\gamma$

$$\Delta\gamma = -0.50 \left(\frac{\omega^2}{eV^2}\right)\left(\frac{1\,TeV}{M_s}\right)^4 \delta$$

$$\delta = \frac{2}{n-2} \qquad n > 2$$

$$\doteq \ln\left(\frac{M_S^2}{\omega^2 b^2 + M_{min}^2}\right) \qquad n = 2$$

General result away from grazing incidence

$$\delta\alpha = (1+\text{"}\gamma\text{"})\,\frac{G_N M_\odot}{r_E}\left(\frac{1+\cos\alpha}{\sin\alpha}\right)$$

$$\text{"}\gamma\text{"} = 1 - 4\omega^2\Delta\left(\frac{4G_N M_\odot}{r_E\sin\alpha}\right)^2 \ln\left(\frac{4G_N M_\odot}{r_E\sin\alpha}\right)$$

The Hipparcos results are $\sin\alpha \sim 1$

$\delta\gamma \ll 1$ — no constraint

Go back to eclipse data $r_E \sin\alpha = r_\odot$

$$\gamma = 0.95 \pm 0.11$$

$$M_S \geq 1.4\left(\frac{2}{n-2}\right)^{1/4}\,\text{TeV} \qquad n>2$$

$$\sim 4.2\ \text{TeV} \qquad n \sim 2.$$

OPEN QUESTIONS

1. EXPERIMENTAL

CAN WE GET 10% ACCURACY AT $\omega \sim 1\,MeV$

THIS WOULD PERMIT $10^3\,TeV$ LIMITS ON M_s

2. THEORETICAL

CALCULATING THE KK CONTRIBUTION TO THE POTENTIAL, AND INSERTING THAT INTO THE LINEARISED EINSTEIN EQUATIONS GIVES A NEGLIGIBLE CORRECTION — SUPRESSED BY $e^{-M_s R_\odot}$

A GOOD THEORETICAL ARGUMENT PREFERRING ONE CALCULATION OVER THE OTHER IS NEEDED.