Multi-instanton and monopole results and applications to gauge/string dynamics

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Progress in $\mathcal{N}=4 \Rightarrow$ AdS/CFT

$\mathcal{N}=2 \Rightarrow$ Seiberg-Witten

$\mathcal{N}=1 \Rightarrow$ SQCD

$\langle\lambda\lambda\rangle$ monopoles on $R^3 \times S^1$

$\oplus$ D-instantons in string theory

UKSQCD collaboration : 1996-2000

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- 1996 - 1997 : DKM : $\mathcal{N}=2$ Seiberg-Witten 2 instanton calc-s
- 1997 : DKM + DHKM : $\mathcal{N}=4$, $\mathcal{N}=2$, $\mathcal{N}=1$ $\forall$ multi-instanton measure
- 1998 - 1999 : DHKMV : $\mathcal{N}=4$ AdS/CFT $\forall$ multi instantons
- 1999 - 2000 : HKM + DaHKM + HK + DaK + DHK : $\mathcal{N}=4$, $\mathcal{N}=2$, $\mathcal{N}=1$ instantons + monopoles + branes

$\forall$ Exact results for supersymmetric F-terms [superpotentials, condensates, prepotentials, effective actions] can be calculated semiclassically (sometimes in slightly unusual settings).

1.) Non-Abelian gauge group is broken by VEVs to an Abelian subgroup (or completely broken)
$\Rightarrow$ use constrained instantons
$\mathcal{N}=2$ Seiberg-Witten prepotential;
$\mathcal{N}=1$ SQCD with $N_f = N-1$ Affleck-Dine-Seiberg superpotential

2.) Gauge group is Non-Abelian and $\beta = 0$
$\Rightarrow$ use standard ADHM instantons
$\mathcal{N}=4$ superconformal YM $\Leftrightarrow$ AdS/CFT correspondence
$\oplus$ D-instantons in string theory

3.) Gauge group is Non-Abelian and $\beta < 0$
$\Rightarrow$ standard ADHM instantons give incomplete answers
use monopoles on $\mathbb{R}^3 \times S^1$.
$\mathcal{N}=1$ $\langle \lambda\lambda \rangle$; ADS superpotential for $N_f < N-1$

$\mathcal{N}=2$ SQCD $d=4$ Coulomb phase :

$$\mathcal{F}^{k\text{-inst}}_{SW}(\text{vevs}) = 8\pi i \int \frac{d\mu_{k\text{-inst}}}{\int d^4x_0 \int d^4\xi_{SS}} e^{-S_{k\text{inst}}}$$

Complete agreement w SW et al for :

1) $N=2$ $N_f = 0, 1, 2$ $k = 1, 2$

[2) $N=2$ $N_f = 3, \underline{\underline{4}}$ $k = 1, 2$

No disagreement

after modification of SW.

$N_f = 4 \Rightarrow \tau \rightsquigarrow \tau_{eff} = \tau + \sum_{\substack{n>0 \\ \text{even}}}^{\infty} c_n q^n$]

3) $\forall N$ $N_f < 2N$

agreement at $k=1$ inst level

[4) $\forall N$; $N_f = 2N$ $k=1$ ...

τ_{eff} $[N-1]\times[N-1]$ matrix (ratio of vevs)

KMS + Argyres-Pelland results ; need more study !]

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$\mathcal{N}=2$ D=4 Coulomb phase SU(N) $N_f \leq 2N$

K ≡ instanton number.

- ADHM multiinstanton supermultiplet is known in components [ADHM, CWS, CGFT, DKM '96]
- multi-instanton action [DKM '96] and collective-coordinates integration measure are known [K=2 SU(2) ⇐ DKM '96
  ∀K SU(2) ⇐ DKM '97, ∀K SU(N) KMS '98
  Sp(N), O(N) HKM '99]

⊛ The prepotential $\mathcal{F}(a)$ is expressed as an unambiguous finite-dimensional integral for ∀K [DKM '97, KMS '98]

⊛ $\mathcal{F}(a)$ calculated at 2-instanton level for SU(2) [DKM '96] [AHSW '96]
at 1-instanton level for SU(N) [IS, KMS '98]

In complete agreement with exact results of SW, AF, KLTY, HO, APS, MN, W for $N_f < 2N$

[more work is needed !]

... and for $N_f = 2N$ After a general reparametrization of parameters of the curves [Argyres-Pelland '99]

[At what price ! ⇒ need explicit instanton calculations to match!]

Ways to get multi-instanton measure:

① Quantum Field Theory :

change variables in the path integral to instanton collective coordinates
Use supersymmetry ! $\forall K$ DKM '97

② String Theory :

K-instantons in $N=4$ SYM $SU(N)$ are described by eff. theory of

K D(-1) branes $\oplus$ N D3 branes

DHKMV '99

K D5 $\oplus$ N D9 in type IIB

easy to include α' corrs

NS B-field

③ Topological Field Theory :

Bellisai - Fucito - Tanzini - Travaglini

$\mathcal{N}=4$ D=4 superconformal YM SU(N)

A theory with $16 + 16 = 32$ supercharges $\stackrel{\text{BPS}}{\Rightarrow}$ need to calculate correlators G_n of composite operators which saturate 16 fermion zero modes in the instanton background.

$n = 16, 8, 4.$ i.e. $G_{16} = \langle \operatorname{tr} \sigma^{\mu\nu} F_{\mu\nu} \lambda(x_1) \ldots \operatorname{tr} \sigma^{\mu\nu} F_{\mu\nu} \lambda(x_{16}) \rangle$

in IIB SUGRA corr. to 16 dilatinos. etc

★ Multi-instanton calculation

is performed at

[DHKMV '99]

$$\begin{cases} \forall K & \text{all orders in instantons} \\ g^2 \to 0 & \text{weak coupling } g^2 N \ll 1 \\ N \to \infty & \text{leading order in } g^2 \text{ and in } \frac{1}{\sqrt{N}} \end{cases}$$

Revealing a compelling evidence for the AdS/CFT correspondence:

1) Large-N K-instanton collective coordinate space has a geometry of a <u>single copy</u> of <u>$AdS_5 \times S^5$</u>

2) and it contains the partion function of 10 dimensional $\mathcal{N}=1$ SU(K) theory reduced to Ø dimensions matching the description of K D-instantons in IIB string theory

K - instanton level

for arbitrary K

$$\underbrace{\int d\mathcal{M}_{\text{k-inst}}\, e^{-S_{\text{k-inst}}}}\ \langle \ldots \rangle \xrightarrow[N\to\infty]{}$$

$$\left\{ \begin{array}{l} \int d^4x_0 \frac{d\rho}{\rho^5} \int dS^5 \int d^{16}\xi_{sc+ss} \cdot e^{-\frac{k 8\pi^2}{g^2} + \frac{ik\theta}{2\pi}} \cdot Z_K \\ \text{and:} \\ Z_K = \sqrt{N}\ K^{-7/2} \left(\sum \frac{1}{(d|k)^2} \right) g^8 \end{array} \right.$$

Our result DHKMV hep-th/9810243
hep-th/9901128

$$\sum_{k=1}^{\infty} K^{-7/2 + 16}\, e^{-2\pi k \tau} \left(\sum_{d|k} \frac{1}{(d|k)^2} \right) (1 + o)$$

$\uparrow$ from 16-point function

$$\in f_{16}(\tau, \bar{\tau})$$

3) In exact agreement with IIB string calculation at k-instanton level

$$G_n = \underbrace{\sqrt{N}}_{\frac{1}{\alpha'}} \; \underbrace{g^8 \, k^{n-7/2} \, e^{2\pi i k\tau} \sum_{d|k} \frac{1}{d^2}}_{\subset\, f_{n-4,\,\bar{n}-4}(\tau)} \cdot \int_{d(AdS_5\times S^5)\times d^{16}\xi} \underbrace{F(x_1 \ldots x_n)}_{n \text{ bulk-to-boundary propagators}}$$

expected non-holomorphic modular form of $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$

Similar results were derived for

$\mathcal{N}=4$ $Sp(N)$, $O(N)$ $\beta = 0$

$\mathcal{N}=2, 1, 0$ $SU(N)\times SU(N)\times \ldots SU(N)$ $\beta = 0$

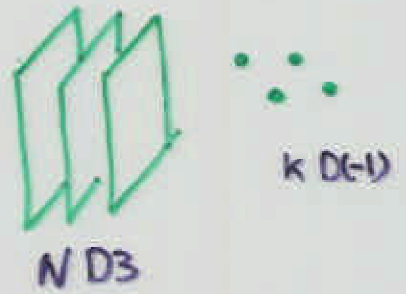
[HK '99 , HKM '99]

Matching leading weak coupling $g^2N \ll 1$ result to a leading strong coupling $g^2N \gg 1$ result !?

New D-instanton effects in string theory

Dorey - Hollowood - Khoze - in preparation
Green - Gutperle hep-th/0002011

$\star$ k D(-1) branes $\oplus$ N D3 branes in type IIB string theory



$\odot$ partition function

$$Z_{k,N} = \int d\mu_k \, e^{-S_{k,N}}$$

is known

DHKMV '99 + DHK - in prep

Here the D3 branes are static (not integrated over)
Integrate out D(-1)'s collective coordinates

$$Z_{k,N} \, [\text{ D3 fields} \; ; \; \alpha' ; \; B ; \text{ Coulomb phase VEVs }]$$

After integrating out D-instantons
get the $\underbrace{\text{instanton-induced}}$ effective action of the N D3 branes

$\boxed{\text{Take } N=1}$

$$S_{eff}^{N=1\,3\text{ branes}} = S_{\text{Dirac-Born-Infeld}} + S_{\text{Wess-Zumino}}$$
$$+ \frac{\pi^2 \alpha'^4}{4} \int d^4x \sqrt{g^{(E)}} \, h(\tau,\bar\tau) \, \Big[(\partial^2 \varphi)^4 +$$
$$+ \tau_2 (\partial^2\varphi)^2 \partial F^+ \partial F^- + \tau_2^2 (\partial F^+)^2 (\partial F^-)^2$$
$$+ \text{fermions} \Big] + \ldots$$

$$h(\tau,\bar\tau) = \log |\tau_2 \, \eta(\tau)|^4$$ Green-Gutperle Conjecture

$N=1$:

$$h(\tau,\bar{\tau}) = \Big(-\tfrac{\pi}{3}\tau_2 + \log\tau_2 - 2\sum_{k=1}^{\infty}\sum_{d|k}\tfrac{1}{d}\big(e^{2\pi i k\tau} + e^{-2\pi i k\bar{\tau}}\big)\Big)$$

$\uparrow$ string Born

$\uparrow$ string 1 loop

$\uparrow$ k-instanton + k-antiinstanton

Green-Gutperle conjecture

based on string Born term : $SL(2,\mathbb{Z})$ duality and holomorphy.

1-instanton test is successful.

Note : no non-perturbative corrs apart from (anti) instantons

no perturbative corrections in the instanton background.

k-instanton result

$$\sum_{d|k} \frac{1}{d}\cdot e^{-S_{k\,inst}}$$

is exact

Need to check for $\forall k > 1$ ✓

Need to generalize to $\forall N > 1$ ✓

1. set D3 brane fields to zero

But keep N D3 branes separated $\Leftrightarrow$ Coulomb phase VEVs $\neq 0$

2. keep NS B-fields and α' corrections in the D-instanton measure

$$\ast \quad \boxed{\frac{Z_{K,N}\,[\alpha', B, VEV]}{\mathbb{R}^4 \int d^8\xi_{ss}} = N \sum_{d|K} \frac{1}{d}\, e^{-2\pi i \tau K}}$$

mod out translational coll. coord. of the K Dinstanton as a whole and 8 supersym. fermion zero modes

Exact result.
$\forall N$; $\forall K$

Dorey-Hollowood-Khoze
in preparation.

Does not depend on B, α' or VEV.

= Bulk contribution to a regularized Witten index

Note $\sum \frac{1}{d^1}$ not $\sum \frac{1}{d^2}$ as in AdS/CFT !

$$\int \frac{d\mu_{K,N}\, e^{-S'_{K,N}[\alpha', B, VEV]}}{d^4x\, d^8\xi_{ss}} = e^{2\pi i\tau K} N \sum_{d|K} \frac{1}{d^2}$$

Strong conjecture. (DHK.)

Tests :

① $N=1$, $\forall K$ $\Rightarrow \sum_{d|K} \frac{1}{d^2}$

This is proven.

Use $U(1) \Rightarrow$ no vevs ; Witten index $= 1$

deficit contr. is known $= -\sum_{d|K \neq 1} \frac{1}{d}$

$\Rightarrow$ bulk $= \sum_{d|K} \frac{1}{d}$.

(1.2) $N=1$ $K=2$ explicit calculation $\Rightarrow \frac{3}{2}$

② $\forall N$; $K=1$ $\Rightarrow N$.

for $K=1$ there are no classically flat directions (instanton separations)

Instanton size is lifted by VEV, α', B.

$\Downarrow$

Nakajima index$_{K=1}$ $= N$ no deficit.

bulk $= N$

Knowing this bulk contr. to the index

we can get the K-instanton contribution to the effective action on N D3 branes:

$$S^{ND3}_{KD(-1)} \propto \alpha'^4 \boxed{e^{2\pi i \tau K} \sum_{d|K} \frac{1}{d} \operatorname{tr}_N} \int d^4x \left((\partial^2 \varphi)^4 + \ldots \right)$$

⊙ Proof of Green - Gutperle conjecture for $N=1$ $\forall K$

⊙ Generalization to $\forall N$

⊙ as $\Rightarrow h(\tau,\bar\tau)$ no perturbative corrections in the instanton background

$\Downarrow$

K-instanton measure is perturbatively exact.

⊙ VEVs are necessary!

even though the action does not depend explicitly on VEVs.

if VEV $\equiv 0$ — different 'strange' result.

$\mathcal{N}=1$ D=4 SU(N) SYM

The story of the gluino condensate:

(I) $\left\langle \frac{\mathrm{tr}\,\lambda^2(x)}{16\pi^2} \right\rangle = \Lambda_{PV}^3$

weak-coupling calculation [NSVZ]
and also an exact result ⇐ [SW]

(II) $\left\langle \frac{\mathrm{tr}\,\lambda^2(x_1)}{16\pi^2} \cdots \frac{\mathrm{tr}\,\lambda^2(x_N)}{16\pi^2} \right\rangle = \frac{2^N}{(N-1)!\,(3N-1)!} \Lambda_{PV}^{3N}$

↑ direct strong-coupling SU(N) 1-instanton calculation [NSVZ, AKMRV + ...]

- the right hand side does not depend on $x_1 \ldots x_N$ ⇐ Supersymmetric Ward identity
- No perturbative corrections.

If there are no additional to instantons non-perturbative contributions, then clustering in quantum field theory will give

$$\mathrm{II} \Rightarrow \left\langle \frac{\mathrm{tr}\,\lambda^2}{16\pi^2} \right\rangle = \frac{2}{[(N-1)!\,(3N-1)!]^{1/N}} \Lambda_{PV}^3 < \Lambda_{PV}^3$$

Something is missing!

a) another vacuum (Kovner - Shifman)

[HKLM, DHKM] → b) other non-perturbative effects

c) both

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In fact, the situation with the strong-coupling instanton calculation gets even worse at the multi-instanton level.

K-instanton has $2KN$ adjoint fermion zero modes

$$\Rightarrow \quad \langle \mathrm{tr}\,\lambda^2(x_1) \ldots \mathrm{tr}\,\lambda^2(x_{KN}) \rangle \Big|_{K\,\mathrm{inst}} \neq 0$$

superselection rule.

[HKLM '99] ⇒

For $N \to \infty$

$$\Big\langle \prod_{i=1}^{KN} \mathrm{tr}\,\lambda^2(x_i) \Big\rangle \Big|_{K\,\mathrm{inst}} \ll \Big\langle \prod_{i=1}^{pN} \mathrm{tr}\,\lambda^2(x_i) \Big\rangle \Big|_{p\,\mathrm{inst}} \times$$

$$\times \Big\langle \prod_{j=pN+1}^{KN} \mathrm{tr}\,\lambda^2(x_j) \Big\rangle \Big|_{(K-p)\,\mathrm{inst}}$$

Multi-instanton contributions at strong-coupling do not cluster!

But for weakly coupled theories (no-non-Abelian group) they do.

With or without an additional vacuum.

Monopoles on $\mathbb{R}^3 \times S^1$ as instanton partons:

[DHKM '99, DK '99] [all semisimple gauge groups: Davies-Hollowood-Khoze 2000]

It has been suspected for a long time that at strong coupling instantons should be thought as composite states of 'instanton partons'
[CDG '78, BFST '79, FFS '79, BL '79 ...]

These partons (and not instantons) presumably would give dominant contributions to strong coupling dynamics.

1. $\mathbb{R}^4 \rightsquigarrow \mathbb{R}^3 \times S^1$

two reasons:

a) $x_4 \in [0, \beta]$ monopoles have finite action.

b) $\langle A_4 \rangle \sim \frac{1}{\beta} \begin{pmatrix} \cdot & & \\ & \ddots & \\ & & \cdot \end{pmatrix}_{N\times N}$

holomorphy of F-terms allows to analytically continue results in β

[Weakly coupled theory] $\Leftarrow$ β small (semiclassical regime) $\longrightarrow$ $\beta_{large} \longrightarrow \infty$ [Strongly coupled theory]

$\boxed{\mathbb{R}^4}$

2. $\mathbb{R}^3 \times S^1 \xrightarrow[\beta\to\infty]{} \mathbb{R}^4$ and SU(N): $\left.\frac{\langle \mathrm{tr}\,\lambda^2 \rangle}{16\pi^2}\right|_{monopoles} = 1 \cdot \Lambda^3_{PV}$!