MASSIVE GHOSTS IN SOFTLY BROKEN SUSY GAUGE THEORIES

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The Main Statement:

Softly Broken SUSY Theory	$\approx$	Rigid SUSY Theory in External Spurion Superfield

Avdeev, Kazakov & Kondrashuk NP '98
Jack & Jones, PL '97
Giudice & Rattazzi, NP '98
Yamada, PR '94

The Couplings $g \Rightarrow$ External Superfields $S(\theta,\bar\theta)$

Singular Part of Effective Action:

$$S^{eff}_{Sing}(g) \Rightarrow S^{eff}_{Sing}(S, \cancel{D^2 S}, \cancel{\bar{D}^2 S}, \cancel{D^2 \bar{D}^2 S})$$

Consequence:

Renormalizations of the Soft Terms follow from those of a Rigid Theory

Renormalization Constants: $\tilde{Z}_i(g) = Z_i(S)$

Grassmannian Taylor Expansion !

Kazakov PL '99

The Rigid N=1 SUSY Gauge Theory

- The Lagrangian

$$\mathcal{L}_{rigid} = \int d^2\theta \, \frac{1}{4g^2} \mathrm{Tr} W^\alpha W_\alpha + \int d^2\bar\theta \, \frac{1}{4g^2} \mathrm{Tr} \bar W_{\dot\alpha} \bar W^{\dot\alpha} + \int d^2\theta d^2\bar\theta \, \bar\Phi^i (e^V)_i^j \Phi_j + \int d^2\theta \, \mathcal{W} + \int d^2\bar\theta \, \bar{\mathcal{W}}$$

$$W_\alpha = -\frac{1}{4}\bar D^2 e^{-V} D_\alpha e^V, \quad \bar W_{\dot\alpha} = -\frac{1}{4} D^2 e^{-V} \bar D_{\dot\alpha} e^V$$

$$\mathcal{W} = \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M^{ij} \Phi_i \Phi_j$$

- Gauge fixing

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{16} \int d^2\theta d^2\bar\theta \, \mathrm{Tr}\left(\bar f f + f \bar f\right)$$

$$f = \bar D^2 \frac{V}{\sqrt{\xi g^2}}, \quad \bar f = D^2 \frac{V}{\sqrt{\xi g^2}}$$

- Ghost Term

$$\mathcal{L}_{ghost} = i \int d^2\theta \, \frac{1}{4} \mathrm{Tr} \, b \, \delta_c f - i \int d^2\bar\theta \, \frac{1}{4} \mathrm{Tr} \, \bar b \, \delta_{\bar c} \bar f$$

$$\delta_\Lambda f = \bar D^2 \delta_\Lambda \frac{V}{\sqrt{\xi g^2}} = i \bar D^2 \frac{1}{\sqrt{\xi g^2}} \mathcal{L}_{V/2} [\Lambda + \bar\Lambda + \coth(\mathcal{L}_{V/2})(\Lambda - \bar\Lambda)]$$

$$
\begin{aligned}
\mathcal{L}_{ghost} \;=\; & -\int d^2\theta\, \frac{1}{4}\mathrm{Tr}\; b\bar{D}^2 \frac{1}{\sqrt{\xi g^2}} \mathcal{L}_{V/2}\Big[c+\bar{c} \\
& \qquad + \coth(\mathcal{L}_{V/2})(c-\bar{c})\Big] + h.c. \\
= & \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right) \mathcal{L}_{V/2}\Big[c+\bar{c} \\
& \qquad + \coth(\mathcal{L}_{V/2})(c-\bar{c})\Big] \\
= & \int d^2\theta d^2\bar{\theta}\; \mathrm{Tr}\left(\frac{b+\bar{b}}{\sqrt{\xi g^2}}\right) \Big((c-\bar{c}) + \frac{1}{2}[V,(c+\bar{c})] \\
& \qquad + \frac{1}{12}[V,[V,(c-\bar{c})]] + \ldots\Big)
\end{aligned}
$$

where $\mathcal{L}_X Y \equiv [X, Y]$

⋆ BRST transformations

$$
\begin{aligned}
&\delta V = \epsilon \mathcal{L}_{V/2}[c+\bar{c}+\coth(\mathcal{L}_{V/2})(c-\bar{c})] \\
&\delta\, c^a = -\frac{i}{2}\epsilon f^{abc} c^b c^c\,, \quad \delta\, \bar{c}^a = -\frac{i}{2}\epsilon f^{abc}\bar{c}^b\bar{c}^c \\
&\delta\, b^a = \frac{1}{8}\epsilon \bar{D}^2 \bar{f}^a\,, \quad \delta\, \bar{b}^a = \frac{1}{8}\epsilon D^2 f^a
\end{aligned}
$$

Soft SUSY Breaking

⋆ Soft Terms

$$-\mathcal{L}_{\text{soft-breaking}} = \left[\frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + h.c.\right] + (m^2)^i_j\,\phi^*_i\phi^j$$

⋆ Spurion Fields $\eta = \theta^2$ and $\bar{\eta} = \bar{\theta}^2$

Girardello & Grisaru
NP '82

$$\begin{aligned}
\mathcal{L}_{\text{soft}} \;=\; & \int d^2\theta\, \frac{1}{4g^2}(1-2M\eta)\mathrm{Tr}W^\alpha W_\alpha \\
& + \int d^2\bar{\theta}\, \frac{1}{4g^2}(1-2\bar{M}\bar{\eta})\mathrm{Tr}\bar{W}^{\dot\alpha}\bar{W}_{\dot\alpha} \\
& + \int d^2\theta d^2\bar{\theta}\bar{\Phi}^i(\delta^k_i - (m^2)^k_i\eta\bar{\eta})(e^V)^j_k\Phi_j \\
& + \int d^2\theta\left[\frac{1}{6}(y^{ijk} - A^{ijk}\eta)\Phi_i\Phi_j\Phi_k + \frac{1}{2}(M^{ij} - B^{ij}\eta)\Phi_i\Phi_j\right] + h.c.
\end{aligned}$$

⋆ Replacement

$$\frac{1}{g^2} \;\rightarrow\; \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2}{g^2}$$

$$y^{ijk} \;\rightarrow\; \tilde{y}^{ijk} = y^{ijk} - A^{ijk}\eta$$

$$M^{ij} \;\rightarrow\; \tilde{M}^{ij} = M^{ij} - B^{ij}\eta$$

⋆ **New!**

$$\frac{1}{g^2} \to \frac{1}{\tilde{g}^2} = \frac{1 - M\theta^2 - \bar{M}\bar{\theta}^2 - \Delta\theta^2\bar{\theta}^2}{g^2}$$

$$\tilde{g}^2 = g^2\left(1 + M\theta^2 + \bar{M}\bar{\theta}^2 + 2M\bar{M}\theta^2\bar{\theta}^2 + \Delta\theta^2\bar{\theta}^2\right)$$

⋆ Gauge Parameter

$$\tilde{\xi} = \xi\left(1 + x\theta^2 + \bar{x}\bar{\theta}^2 + (x\bar{x} + z)\theta^2\bar{\theta}^2\right)$$

⋆ Modified Gauge Fixing

$$f \to \bar{D}^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}\ , \quad \bar{f} \to D^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}$$

$$\mathcal{L}_{gauge-fixing} = -\frac{1}{8}\int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\left(\bar{D}^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}D^2\frac{V}{\sqrt{\tilde{\xi}\tilde{g}^2}}\right)$$

⋆ Modified Ghost Term

$$\mathcal{L}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\tilde{\xi}\tilde{g}^2}}\left(b + \bar{b}\right)\mathcal{L}_{V/2}[c + \bar{c} + \coth(\mathcal{L}_{V/2})(c - \bar{c})]$$

$$\mathcal{L}^{(2)}_{ghost} = \int d^2\theta d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}}\left(1 - \frac{1}{2}\Delta\xi\theta^2\bar{\theta}^2\right)(b + \bar{b})(c - \bar{c}) - \frac{1}{2}\int d^2\theta\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}}M\xi bc + \frac{1}{2}\int d^2\bar{\theta}\ \mathrm{Tr}\frac{1}{\sqrt{\xi g^2}}\bar{M}\xi\bar{b}\bar{c}$$

$$V(x,\theta,\bar\theta) = \mathbb{C}(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \frac{i}{2}\theta\theta N(x) - \frac{i}{2}\bar\theta\bar\theta\bar N(x)$$
$$-\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\left[\bar\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right]$$
$$-i\bar\theta\bar\theta\theta\left[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right] + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left[D(x) - \frac{1}{2}\Box\mathbb{C}(x)\right]$$

$$\mathcal{L}_{g-f} = \frac{1}{2\xi g^2}\left[-\left(D - \Box\mathbb{C} - \Delta\xi\mathbb{C} + \frac{i}{2}M\xi\bar N - \frac{i}{2}\bar M\xi N\right)^2\right.$$
$$-(\partial^\mu v_\mu)^2 + (\bar N - i\bar M\xi\mathbb{C})\,\Box\,(N + iM\xi\mathbb{C})$$
$$-2i\left(\lambda + \frac{1}{2}\bar M\xi\chi\right)\sigma^\mu\partial_\mu\left(\bar\lambda + \frac{1}{2}M\xi\bar\chi\right)$$
$$-2\left(\lambda + \frac{1}{2}\bar M\xi\chi\right)\Box\chi - 2\left(\bar\lambda + \frac{1}{2}M\xi\bar\chi\right)\Box\bar\chi$$
$$\left.-2i\Box\chi\sigma^\mu\partial_\mu\bar\chi\right]$$

⋆ The Meaning of Masses

M – gaugino field λ mass
– auxiliary field χ mass
– spinor ghost mass

Δ – soft scalar ghost mass
– auxiliary fields $\mathbb{C}$ and N mass

RGE's For The Soft Terms

⋆ Complete Set of Substitutions

$$\tilde{g}_i^2 = g_i^2 \left(1 + M_i \eta + \bar{M}_i \bar{\eta} + (2 M_i \bar{M}_i + \Delta_i)\, \eta\bar{\eta}\right)$$

$$\tilde{y}^{ijk} = y^{ijk} - A^{ijk}\eta + \frac{1}{2}\left(y^{njk}(m^2)^i_n + y^{ink}(m^2)^j_n + y^{ijn}(m^2)^k_n\right)\eta\bar{\eta}$$

$$\tilde{\bar{y}}_{ijk} = \bar{y}_{ijk} - \bar{A}_{ijk}\bar{\eta} + \frac{1}{2}\left(y_{njk}(m^2)^n_i + y_{ink}(m^2)^n_j + y_{ijn}(m^2)^n_k\right)\eta\bar{\eta}$$

Notation: $\Sigma_{\alpha_i} = M_i \bar{M}_i + \Delta_i$

⋆ RGE For The Soft Scalar Masses

$$[\beta_{m^2}]^i_j = D_2 \gamma^i_j\,, \quad \beta_{\Sigma_{\alpha_i}} = D_2 \gamma_{\alpha_i}$$

$$\begin{aligned} D_2 \;=\;& \bar{D}_1 D_1 + \Sigma_{\alpha_i} \alpha_i \frac{\partial}{\partial \alpha_i} \\ +\;& \frac{1}{2}(m^2)^a_n \left(y^{nbc}\frac{\partial}{\partial y^{abc}} + y^{bnc}\frac{\partial}{\partial y^{bac}} + y^{bcn}\frac{\partial}{\partial y^{bca}} \right. \\ & \left. + y_{abc}\frac{\partial}{\partial y_{nbc}} + y_{bac}\frac{\partial}{\partial y_{bnc}} + y_{bca}\frac{\partial}{\partial y_{bcn}} \right) \end{aligned}$$

$$D_1 = M_i \alpha_i \frac{\partial}{\partial \alpha_i} - A^{ijk}\frac{\partial}{\partial y^{ijk}} \quad \bar{D}_1 = \bar{M}_i \alpha_i \frac{\partial}{\partial \alpha_i} - A_{ijk}\frac{\partial}{\partial y_{ijk}}$$

Jack, Jones & Pickering
PL '98 $(\Delta \equiv X)$

⋆ Illustration (3 loops) Jack, Jones & North PL '96

$$\gamma_\alpha = \alpha Q + 2\alpha^2 Q C(G) - \frac{2}{r}\alpha\gamma^{i\,(1)}_j C(R)^j_i$$
$$- \alpha^3 Q^2 C(G) + 4\alpha^3 Q C^2(G) - \frac{6}{r}\alpha^3 Q C(R)^i_j C(R)^j_i$$
$$- \frac{4}{r}\alpha^2 C(G)\gamma^{i\,(1)}_j C(R)^j_i + \frac{3}{r}\alpha y^{ikm} y_{jkn}\gamma^{n\,(1)}_m C(R)^j_i$$
$$+ \frac{1}{r}\alpha\gamma^{i\,(1)}_j \gamma^{j\,(1)}_p C(R)^p_i + \frac{6}{r}\alpha^2\gamma^{i\,(1)}_j C(R)^j_p C(R)^p_i$$
$$\gamma^i_j = \frac{1}{2} y^{ikl} y_{jkl} - 2\alpha C(R)^i_j + 2\alpha^2 Q C(R)^i_j$$
$$- \left(y^{imp} y_{jmn} + 2\alpha C(R)^p_j \delta^i_n\right)\left(\frac{1}{2} y^{nkl} y_{pkl} - 2\alpha C(R)^n_p\right)$$

⋆ Solution for Σ_α: Jack, Jones & Pickering PL '98 $(\Delta \equiv X)$

$$\Sigma_\alpha{}^{(1)} = M^2$$
$$\Sigma_\alpha{}^{(2)} = \Delta^{(2)}_\alpha = -2\alpha[\frac{1}{r}(m^2)^i_j C(R)^j_i - M^2 C(G)]$$
$$\Sigma_\alpha{}^{(3)} = \Delta^{(3)}_\alpha = \frac{\alpha}{2r}\Big[\frac{1}{2}(m^2)^i_n y^{nkl} y_{jkl} + \frac{1}{2}(m^2)^n_j y^{ikl} y_{nkl}$$
$$+ 2(m^2)^m_n y^{ikn} y_{jkm} + A^{ikl} A_{jkl} - 8\alpha M^2 C(R)^i_j\Big] C(R)^j_i$$
$$- 2\alpha^2 Q C(G) M^2 - 4\alpha^2 C(G)\Big[\frac{1}{r}(m^2)^i_j C(R)^j_i - M^2 C(G)\Big]$$

⋆ All loop Solution in NSVZ Scheme

Novikov, Shifman, Vainshtein & Zakharov
NP '83

$$\gamma_\alpha^{\mathsf{NSVZ}} = \alpha \frac{Q - 2r^{-1}\mathrm{Tr}[\gamma C(R)]}{1 - 2C(G)\alpha}$$

Solution:

$$\Delta_\alpha^{\mathsf{NSVZ}} = -2\alpha \frac{r^{-1}\mathrm{Tr}[m^2 C(R)] - M^2 C(G)}{1 - 2C(G)\alpha}$$

Jack, Jones & Pickering
PL '98
$(\Delta \equiv X)$

Illustration: The MSSM

⋆ Notation:

$$\alpha_i \equiv \frac{g_i^2}{16\pi^2}, \quad i = 1,2,3; \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad k = t, b, \tau$$

⋆ Substitution:

$$\begin{aligned} \tilde{\alpha}_i &= \alpha_i (1 + M_i\eta + \bar{M}_i\bar{\eta} + (M_i\bar{M}_i + \Sigma_{\alpha_i})\eta\bar{\eta}) \\ \tilde{Y}_k &= Y_k (1 - A_k\eta - \bar{A}_k\bar{\eta} + (A_k\bar{A}_k + \Sigma_k)\eta\bar{\eta}) \end{aligned}$$

$$\Sigma_t = \tilde{m}_Q^2 + \tilde{m}_U^2 + m_{H_2}^2, \ \Sigma_b = \tilde{m}_Q^2 + \tilde{m}_D^2 + m_{H_1}^2, \ \Sigma_\tau = \tilde{m}_L^2 + \tilde{m}_E^2 + m_{H_1}^2$$

⋆ RGE: $\dot{a}_i = a_i \gamma_i(a), \qquad a_i = \{\alpha_i, Y_k\}$

⋆ Soft RGE:

Kazakov
PL '99

$$\dot{\tilde{a}}_i = \tilde{a}_i \gamma_i(\tilde{a}).$$

$$\tilde{a}_i = a_i(1 + m_i\eta + \bar{m}_i\bar{\eta} + S_i\eta\bar{\eta}),$$

$$m_i = \{M_i, -A_k\}, \quad S_i = \{M_i\bar{M}_i + \Sigma_{\alpha_i}, A_k\bar{A}_k + \Sigma_k\}$$

$$\dot{m}_i = \gamma_i(\tilde{a})|_F = \sum_j a_j \frac{\partial \gamma_i}{\partial a_j} m_j$$

$$\dot{\Sigma}_i = \gamma_i(\tilde{a})|_D = \sum_j a_j \frac{\partial \gamma_i}{\partial a_j}(m_j m_j + \Sigma_j) + \sum_{j,k} a_j a_k \frac{\partial^2 \gamma_i}{\partial a_j \partial a_k} m_j m_k$$

⋆ Solutions for Σ_α:

$$\Sigma_{\alpha_1} = M_1^2 - \alpha_1\sigma_1 - \frac{199}{25}\alpha_1^2 M_1^2 - \frac{27}{5}\alpha_1\alpha_2 M_2^2 - \frac{88}{5}\alpha_1\alpha_3 M_3^2$$
$$+\frac{13}{5}\alpha_1 Y_t(\Sigma_t + A_t^2) + \frac{7}{5}\alpha_1 Y_b(\Sigma_b + A_b^2) + \frac{9}{5}\alpha_1 Y_\tau(\Sigma_\tau + A_\tau^2)$$

$$\Sigma_{\alpha_2} = M_2^2 - \alpha_2(\sigma_2 - 4M_2^2) - \alpha_2^2(4\sigma_2 + 9M_2^2) - \frac{9}{5}\alpha_2\alpha_1 M_1^2$$
$$-24\alpha_2\alpha_3 M_3^2 + 3\alpha_2 Y_t(\Sigma_t + A_t^2) + 3\alpha_2 Y_b(\Sigma_b + A_b^2)$$
$$+\alpha_2 Y_\tau(\Sigma_\tau + A_\tau^2)$$

$$\Sigma_{\alpha_3} = M_3^2 - \alpha_3(\sigma_3 - 6M_3^2) - \alpha_3^2(6\sigma_3 - 22M_3^2) - \frac{11}{5}\alpha_3\alpha_1 M_1^2$$
$$-9\alpha_3\alpha_2 M_2^2 + 2\alpha_3 Y_t(\Sigma_t + A_t^2) + 2\alpha_3 Y_b(\Sigma_b + A_b^2)$$

$$\sigma_1 = \frac{1}{5}\left[3(m_{H_1}^2 + m_{H_2}^2) + 3(\tilde{m}_Q^2 + 3\tilde{m}_L^2 + 8\tilde{m}_U^2 + 2\tilde{m}_D^2 + 6\tilde{m}_E^2)\right]$$
$$\sigma_2 = m_{H_1}^2 + m_{H_2}^2 + 3(3\tilde{m}_Q^2 + \tilde{m}_L^2)$$
$$\sigma_3 = 3(2\tilde{m}_Q^2 + \tilde{m}_U^2 + \tilde{m}_D^2)$$

Martin & Vaughn
PR '94

Resume

- The component approach of J&J and our superfield one finally match
- The extra terms are related to the mass of unphysical degrees of freedom
- The paradox: Unphysical degree of freedom via its boundary condition enters the solution for a physical mass

Solution to the paradox: Absorb the unphysical parameter into the scheme redefinition.

Jack, Jones, Martin, Vaughn & Yamada, PR '94

$DRED \to DRED'$

$$(m^2)^j_i\Big|_{\overline{DR}'} = (m^2)^j_i\Big|_{\overline{DR}} - \frac{2g_A^2 C_A(i)}{(4\pi)^2}\delta^j_i \tilde{m}_\epsilon^2$$

In our approach: One can add to a particular solution an arbitrary solution of a homogeneous equation

$$\Sigma_i = \mathcal{C}\gamma_i, \quad i = \alpha_1, \alpha_2, \alpha_3, t, b, \tau$$

where $\mathcal{C}$ is an overall constant. This is equivalent to the change of a scale.

The pole mass, which is observable, is scheme independent!