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An Exact Solution of the Randall-Sundrum Model & the Mass Hierachy Problem

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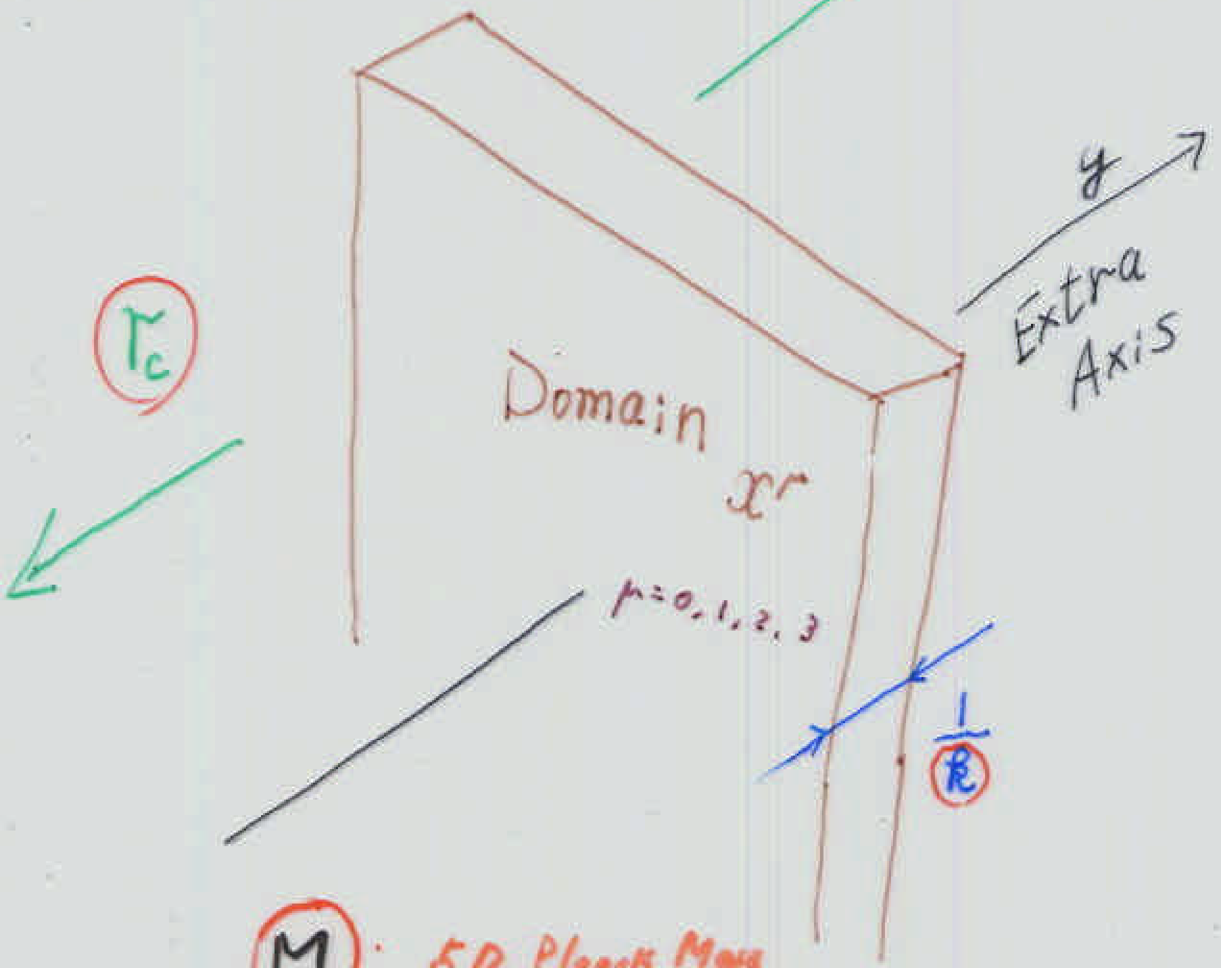
Mass hierarchy problem

$\Lambda_{4d} = M_{Pl}^2 \lambda_{4d}$

	Planck	Weak	Neutrino	Cosmo. Const
	M_{Pl}	M_W	m_ν	$\sqrt{\lambda_{4d}}$
GeV	10^{19}	10^{2}	10^{-11}	10^{-41}

Explain these scales naturally.

$\boxed{\text{simplified Randall \& Sundrum Model}}$



M : 5D Planck Mass (fundamental scale)

$$\frac{1}{r_c} \ll R \ll M$$

§1 Model Set-Up

Let us start with 5D Gravity

$$S = \int d^5 X \sqrt{-G} \left(M^3 \hat{R} - \frac{1}{2} \partial_A \Phi \, \partial_B \Phi \cdot G^{AB} + V(\Phi) \right)$$

$M > 0$ 5D Planck Mass

$\lambda > 0$

$$V(\Phi) = \frac{\lambda}{4} \left(\Phi^2 - v_0^2 \right)^2 + \Lambda$$ Higgs Potential

$v_0 > 0$

5D real scalar 5D Cosm. Const

$\Lambda < 0$
(Anti de Sitter later)

$$(X^M) = (\underbrace{x^0, x^1, x^2, x^3}_{x^\mu}, y)$$

x^μ: 1+3 Minkowski

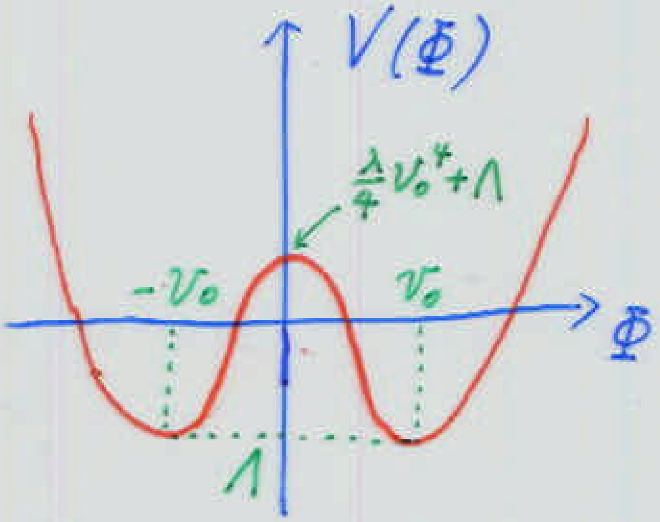
y: Extra dim. (a space coord.)

Physical dim.

$[v_0] = [\Phi] = (\text{mass})^{3/2}$,

$[\Lambda] = (\text{mass})^5$,

$[\lambda] = (\text{mass})^{-1}$



Assume 4D Poincare inv $\rightarrow (ds^2)_{4D} \propto \eta_{\mu\nu} dx^\mu dx^\nu$

Assume $ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + (dy)^2$

warp factor

non-Factorizable (x^μ vs y)

Einstein Eq.

$$\left\{\begin{array}{l} M^3\left(\hat{R}_{MN}-\frac{1}{2}G_{MN}\hat{R}\right)=-\partial_M\Phi\,\partial_N\Phi \\ \qquad\qquad +G_{MN}\left(\frac{1}{2}G^{KL}\partial_K\Phi\,\partial_L\Phi+V(\Phi)\right), \\ \nabla^2\Phi=\frac{\delta V}{\delta\Phi}\ , \end{array}\right.$$

Assume $\Phi = \Phi(y)$

$$\left\{\begin{array}{ll} -6M^3(\sigma')^2=-\frac{1}{2}(\Phi')^2+V\ , & ① \\ 3M^3\sigma''=\Phi'^2\ , & ② \end{array}\right.$$

$$' = \frac{d}{dy}$$

Let us find an exact solution

$\sigma(y)\ ,\ \Phi(y)$

$$\boxed{\begin{array}{l}\text{Condition Ⓐ} \\ \kappa \ll M \quad \text{No Quantum Effect of 5D Gravity}\end{array}}$$

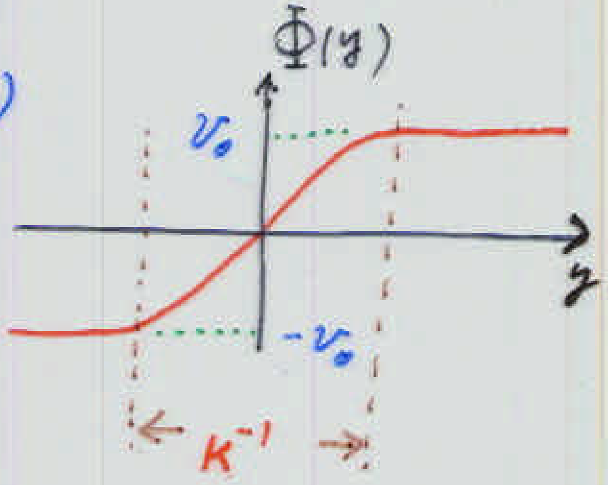
§2 Exact Solution

We take the extra space to be $\mathbb{R} = (-\infty, \infty)$
(y-axis)

cf Randall & Sundrum 9905 $\quad S^1/Z_2$

Boundary Cond. for Φ

Kink $\begin{cases} \Phi(y = +\infty) = v_0 \ (>0) \\ \qquad -\infty \qquad\quad -v_0 \end{cases}$



Asymptotic Behaviour

$y \to \pm\infty \quad , \ \Phi' \to 0$

From ② $\quad , \ \sigma'' \to 0$

$\therefore \ \sigma' \to c_1 \ (\text{const.}) = \pm\sqrt{\frac{-\Lambda}{6}} \quad \because$ ①

$\sigma \to \underbrace{c_1 y}_{= \sqrt{\ } |y|} + c_2 \qquad \Lambda < 0$

We can expect 'gross' form of solution.

$v_0 \tanh(\kappa y) \leftarrow \Phi(y) \sim v_0 \,\theta(\kappa y) \qquad \kappa^{-1}$: 'thickness' of the Wall

$\kappa > 0 \qquad \Phi'(y) \sim \kappa v_0 \,\delta(\kappa y)$

$\sigma(y) \sim \sqrt{\ } \,|y|$

$\sqrt{\ } \tanh(\kappa y) \leftarrow \sigma'(y) \sim \sqrt{\ }\, \theta(\kappa y)$

$\sigma''(y) \sim \kappa \sqrt{\ }\, \delta(\kappa y)$

Let us take the following form

as an exact solution

to be determined

odd

$$\begin{cases} \sigma'(y) = k \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!} \left\{ \tanh(ky + \underset{\xi}{\ell}) \right\}^{2n+1} & \text{③} \\ \Phi(y) = v_0 \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} \left\{ \tanh(ky + \underset{\xi}{\ell}) \right\}^{2n+1} & \text{④} \end{cases}$$

translation inv.

$\ell = 0$

Constraints for c's & d's

$$\begin{cases} \sqrt{-\frac{\Lambda}{6M^3}} = k \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!} & \text{⑤} \\ 1 = \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} & \text{⑥} \end{cases}$$

③, ④ → ① ② $(\tanh y)' = 1 - (\tanh y)^2$

$\Downarrow$

Recursion relations between c's & d's first few terms

$\Phi \leftrightarrow -\Phi$ symmetry, we take +

$$d_1 = \pm \frac{\sqrt{2}}{v_0 k} \sqrt{\underbrace{\Lambda + \frac{\lambda v_0^4}{4}}_{\geq 0}}, \quad c_1 = \frac{2}{3k^2}\left(\Lambda + \frac{\lambda v_0^4}{4}\right),$$

$$\frac{d_3}{d_1} = 2 - \frac{\lambda v_0^2}{k^2} + \frac{8}{3k^2}\left(\Lambda + \frac{\lambda}{4} v_0^4\right),$$

$$\frac{c_3}{c_1} = 2 - 2\frac{\lambda v_0^2}{k^2} + \frac{16}{3k^2}\left(\Lambda + \frac{\lambda}{4} v_0^4\right),$$

$$\vdots$$

$$-\frac{\lambda v_0^4}{4} \leq \Lambda \leq 0$$

So far, constraints ⑤ ⑥

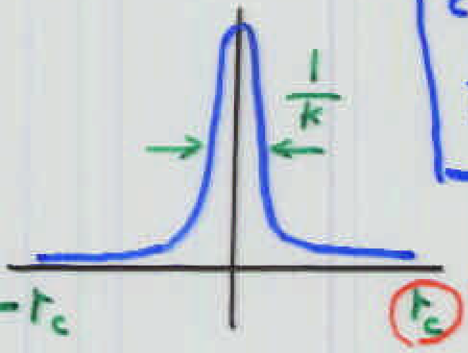
are not taken into account.

§3. Vacuum Parameters & Final Result

$$(\Lambda, \lambda, v_0) \longrightarrow \left(\Omega \equiv \Lambda + \frac{\lambda}{4} v_0^4 ,\ \tau = \lambda v_0^2 ,\ v_0 \right)$$

to be fixed

$$\begin{cases} \Omega = M^3 k^2 \left\{ \alpha_0 + \dfrac{\alpha_1}{(k r_c)^2} + \cdots \right\} & (7) \\ \tau = k^2 \left\{ \gamma_0 + \dfrac{\gamma_1}{(k r_c)^2} + \cdots \right\} & (8) \\ v_0 = M^{3/2} \left\{ \beta_0 + \dfrac{\beta_1}{(k r_c)^2} + \cdots \right\} & (9) \end{cases}$$



Condition (B)
$\frac{1}{k} \ll r_c$ Thin Wall expansion

Infrared Reg.

one freedom for each n-th set $(\alpha_n, \beta_n, \gamma_n)$

(7), (8), (9) $\longrightarrow$ (5), (6)

Sample Sol. $n = 0, 1$ Order

$$\begin{cases} \alpha_0 = 1 , \quad \alpha_1 = 1 \quad \text{(Input)} \\ \beta_0 = 1.48 , \quad \beta_1 = 0.749 \\ \gamma_0 = 4.38 , \quad \gamma_1 = 2.63 \end{cases}$$

for general (k, M, r_c) with $k r_c \gg 1$

$k r_c = 10$

$$\frac{V_0}{M^{3/2}} = 1.49$$

$$\frac{\Lambda}{k^2 M^3} = -1.43$$

Condition on k Ⓐ, Ⓑ

$$\boxed{\frac{1}{r_c} \ll k \ll M}$$

- Choice 2
- kr_c = 10. , 20

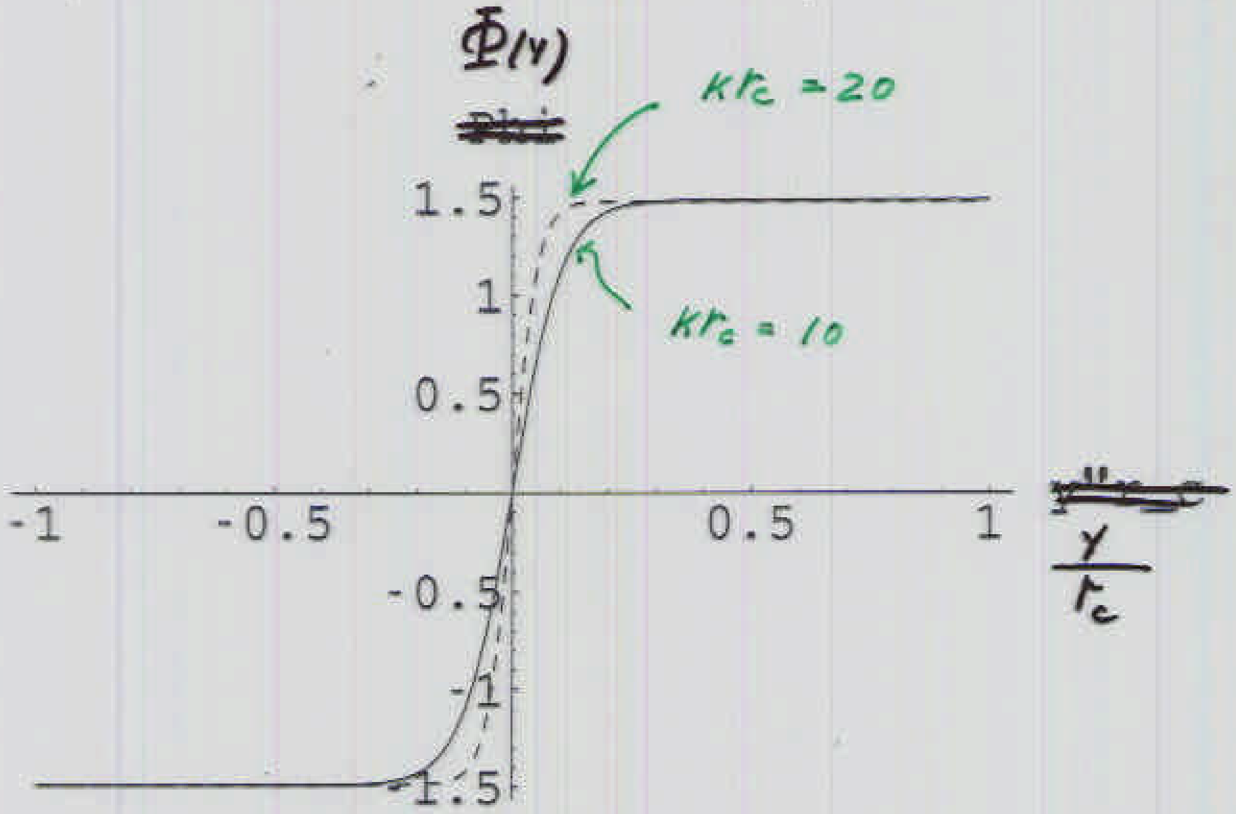


Fig.1

Ph.2k.10a20.eps

solid

dotted

- Choice 2
- $kr_c = 10, 20$

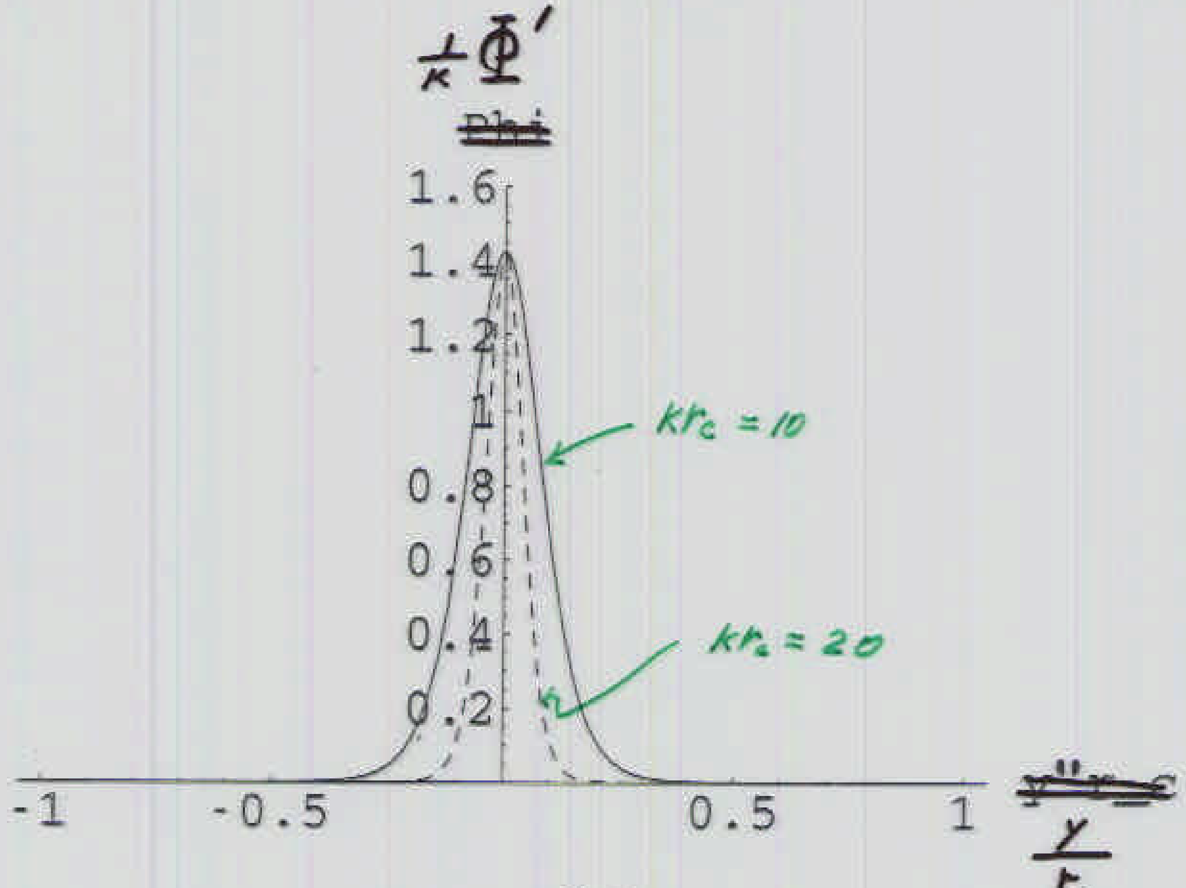


Fig.2

DPhi2R $\underset{\text{solid}}{\underline{10}}$ a $\underset{\text{dotted}}{\underline{20}}$ .eps

A parameter choice

$$K = 10^{4}\ \mathrm{GeV}\quad (\text{input})$$

$$\sqrt{\frac{M^3}{K}} \sim M_{P\ell} = 10^{19}\ \mathrm{GeV}\quad (\text{we require})$$

$$K \sim \sqrt{-\lambda_{4d}} = 10^{4}\ \mathrm{GeV}\qquad \text{cf. } \sqrt{\lambda_{obs}} \sim 10^{-41}\ \mathrm{GeV}$$

$$M = 10^{14}\ \mathrm{GeV}$$

$$K e^{-K r_c} \sim m_\nu = 10^{-11}\ \mathrm{GeV}\quad (\text{we require})$$

$$r_c = 3. \times 10^{-3}\ \mathrm{GeV}^{-1}$$

§4. Discussion & Conculusion

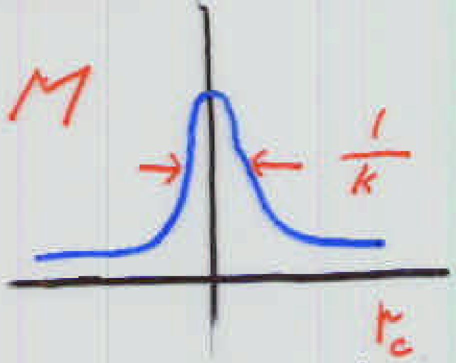
⊙ An exact solution for

a family of vacua (Higgs)

in the one-wall case

$\frac{1}{k^2}$ expansion

thin wall



★ Similarity in other 'regularization'

Lattice $\underset{m_{quark}}{\overset{}{0}} < m_{5D\,fermion} < \frac{1}{a}$ Vranas '97

IIB Matrix Model $\frac{1}{\sqrt{n}}\frac{1}{\ell_{NC}} < \rho < \sqrt{n}\frac{1}{\ell_{NC}}$ KEK group '99

★ $\mathbb{R} \rightarrow S^1/Z_2$

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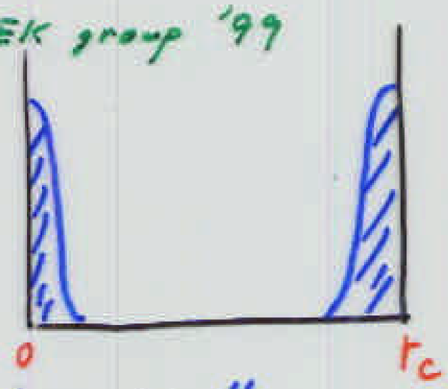

two walls

★ vortex in 6D