B and D Mesons in Lattice QCD

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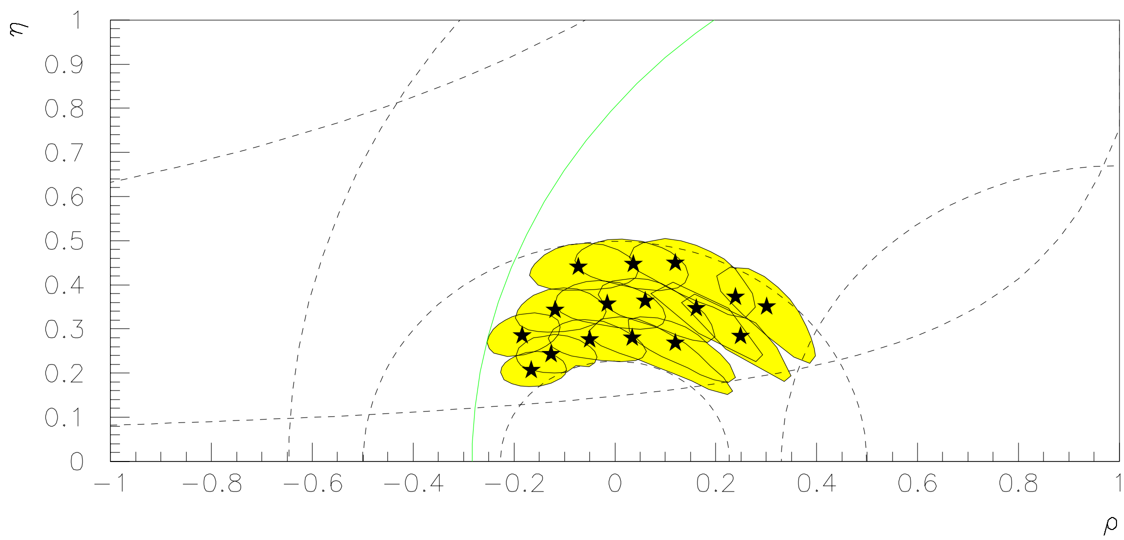
Status of CKM Matrix



Plaszczynski and Schune

$V_{ub} = A\lambda^3(\rho - i\eta)$: KM CPV $\Leftrightarrow \eta \neq 0$.

Only ε_K (CPV in mixing) pulls η away from 0, but $\varepsilon' \neq 0 \Rightarrow \eta \neq 0$.

Each blob shows experimental uncertainties for fixed theoretical inputs. Range of blobs shows theoretical uncertainties.

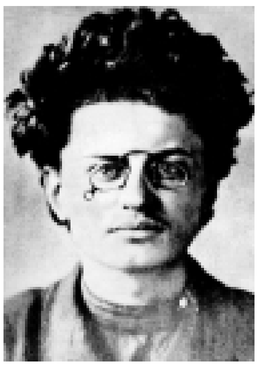
So, *reduce* theoretical uncertainties (while making measurements less sensitive to them).

Especially since . . .

Three Wise Men

High-energy physics is exciting and will remain exciting, precisely because it exists in a state of permanent revolution.

Joe Lykken



Revolution, whose every successive stage is rooted in the preceding one, and which can end only in complete liquidation.

Leon Trotsky

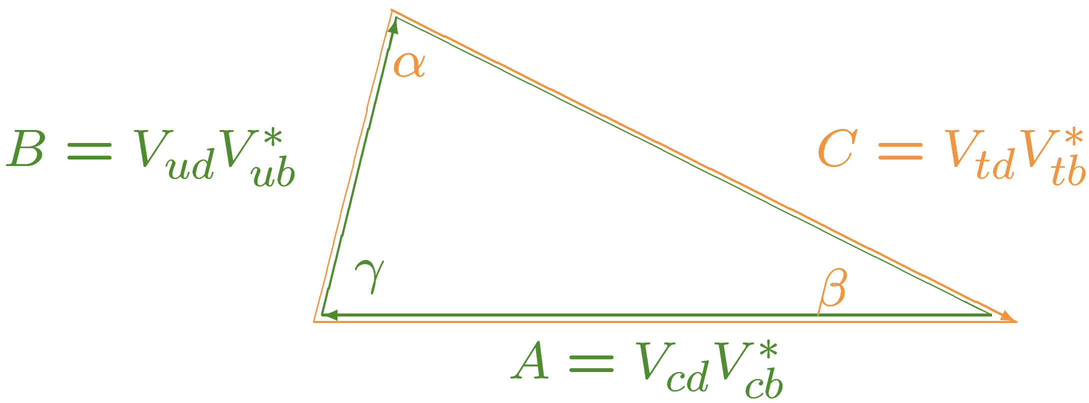
So, let us assume the CKM picture of CPV is established, and see how lattice QCD can aid the discovery of new sources of CPV.

It is possible, likely, unavoidable, that the SM picture of CPV is incomplete.

Yossi Nir

Unitarity Triangles

Want to test CKM picture: are other sources of CPV lurking in B (and D) decays?



Tree processes:

$|V_{ud}|$ from $n \to pe^-\bar{\nu}$
$|V_{ub}|$ from $B^- \to \rho^0 l^- \bar{\nu}$ or $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$
γ from $B^\pm \to D^0_{CP} K^\pm$ (or $B_s \to D_s^\pm K^\mp$)
$|V_{cd}|$ from $D^0 \to \pi^- l^+ \nu$
$|V_{cb}|$ from $B^- \to D^{0(*)} l^- \bar{\nu}$

$SAS \Rightarrow$ the "tree triangle".

Mixing processes (including interference of decays with and without mixing):

α from $B \to \rho\pi$
$|V_{td}|$ from $B_d^0 \leftrightarrow \bar{B}_d^0$
$|V_{tb}|$ from $t \to W^+ b$
β from $B^0 \to J/\psi K_S$

$ASA \Rightarrow$ the "mixing triangle".

Checking whether the mixing triangle agrees with the tree triangle tests whether there is new physics in, say, amplitude of B_d^0-$\bar{B}_d^0$ mixing.

New physics in magnitude modifies side C and new physics in phase modifies angles α and β.

Similarly, taking $B_s^0 \leftrightarrow \bar{B}_s^0$, $B_s \to D_s^{\pm} K^{\mp}$, and $B_s \to J/\psi\, \eta^{(\prime)}$ (or $J/\psi\, \phi$) sorts out new physics in B_s^0-$\bar{B}_s^0$ mixing.

Finally, mixing and CPV in D mesons from CKM is small.

Obtaining Sides

To obtain the sides, hadronic matrix elements are needed.

In isolated cases a symmetry provides it. E.g., $n \to pe^{-}\bar{\nu}$ & isospin cleanly determine $|V_{ud}|$.

For the others, we must "solve" nonperturbative QCD. You need lattice calculations.

Caveman striving to comprehend charm and beauty.

How Dark is the Cave?

Lattice QCD calculates matrix elements by integrating the functional integral, using a Monte Carlo with importance sampling. Hence, (correlated) statistical error bars.

This part of the method is well understood and, these days, rarely leads to controversy. When conflicting results arise, they originate in different treatments of the systematics.

The consumer does not need to know how the Monte Carlo works, but should develop an intuition of how the systematics work.

Of course, cavemen tend to grunt incomprehensibly, so it's not an easy task. I'll try to speak plainly.

The main tool for understanding the systematics is effective field theory:

1. lattice spacing a: Symanzik's LE$\mathcal{L}$

2. finite box size L: Lüscher's massive QFT

3. light-quark mass m_q: chiral Lagrangian

4. heavy-quark mass m_Q: HQET or NRQCD

The utility of these techniques is that they allow us to control the extrapolation of artificial, numerical data to the real world, provided the data start "near" enough.

The beauty is that most of you are familiar with the logic of effective field theories, and are accustomed to judging their range of validity.

Then there's the quenched approximation. Your guess could be as good as mine.

Symanzik's LE$\mathcal{L}$

Match the lattice theory to continuum QCD:

$$\mathcal{L}_{\text{lat}} \doteq \mathcal{L}_{\text{QCD}} + \sum_i a^{s_i} C_i(g^2, m_q a; \mu) \ \mathcal{O}_i(\mu)$$

$\overline{\text{MS}}(\mu)$ short long

If $\Lambda_{\text{QCD}} a$ is small enough the higher terms can be treated as perturbations:

$$m_p(a) = m_p + a C_{\sigma F}(c_{\text{SW}}) \langle p | \bar{\psi} \sigma \cdot F \psi | p \rangle$$

using the leading term for Wilson fermions as an example. c_{SW} is so-called clover coupling.

To eliminate or reduce lattice spacing effects

1. reduce a greatly, but CPU time $\sim a^{-(5 \text{ or } 6)}$
2. combine several data sets and extrapolate
3. adjust c_{SW} so $C_{\sigma F}$ is $O(\alpha^\ell)$ or $O(a)$

For light hadrons, combining of 2 & 3 is best in practice.

Large $m_Q a$

With the bottom and charm quarks, the mass is large in lattice units: $m_b a \sim$1–2 and $m_{ch} a$ about a third of that.

It will not be possible to reduce a enough to make $m_b a \ll 1$ for many, many years.

This leads to some silly statements.

It is correct to say that all lattice calculations of B and D mesons appeal to HQET in some way. (For Υ- and ψ-systems to NRQCD.)

The methods:

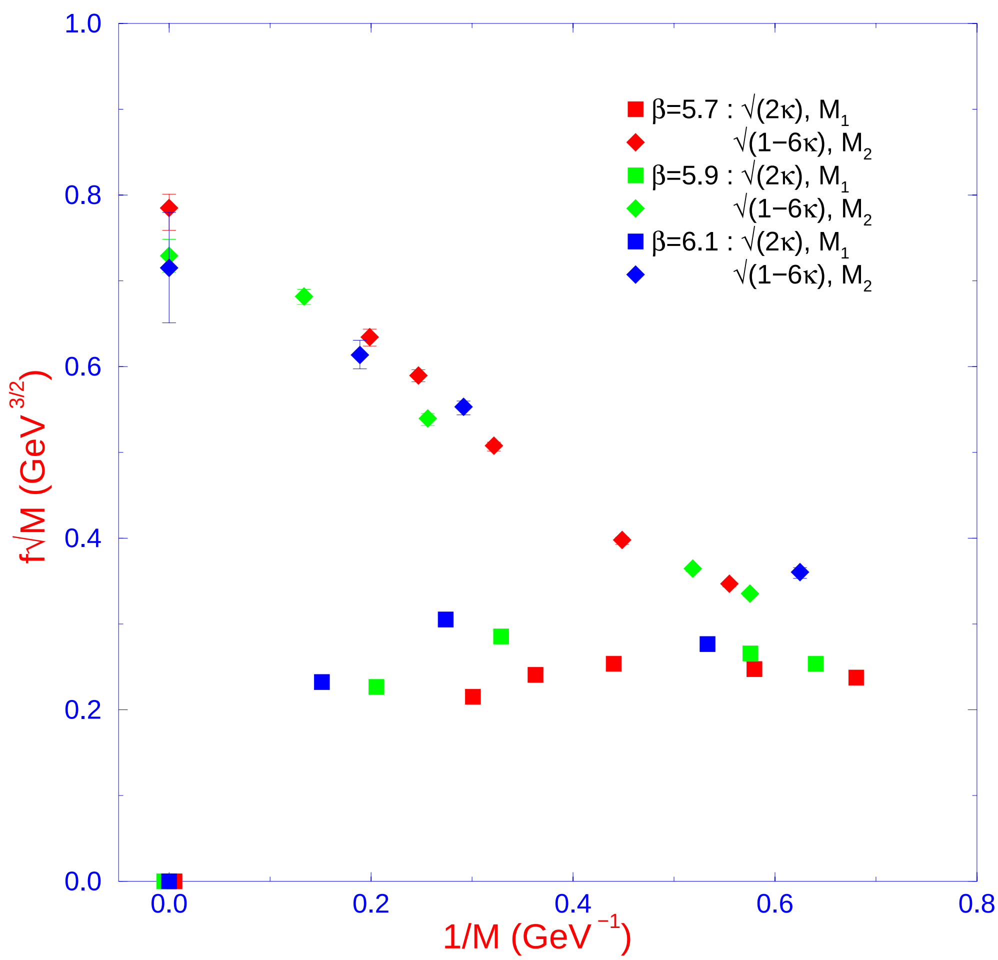
1. static approximation
2. lattice NRQCD
3. extrapolate Wilson from $m_Q < m_b$

$3'$. $3 + 1$

4. normalize Wilson to HQET

Let's compare their strengths and weaknesses.

#	method	strength	weakness
1.	static	$a \to 0$ fast	$1/m_Q^n$
2.	latNRQCD	$1/m_Q^n$ fast	$a \to 0$ PT C_i
3.	extrap	$a \to 0$ nonPT C_i	$m_Q < 2$ GeV $m_Q a > 2 - \sqrt{2}$
3′.	3+1	anchors $m_Q \to \infty$	same as 3
4.	"Fermilab"	$a \to 0$ $1/m_Q^n$	not just a^n PT C_i



HQET and Lattice QCD

The strengths and weaknesses can be organized by matching lattice theories to the HQET:

$$\mathcal{L}_{\text{lat}} \doteq \mathcal{L}_{\text{HQET}}$$

where the effective Lagrangian

$$\mathcal{L}_{\text{HQET}} = \sum_n \mathcal{L}^{(n)}$$

contains the same operators as the usual HQET, with *modified* short-distance coefficients.

The coefficients are different, because the lattice changes the dyanamics at short distances. As a short-distance effect, these effects may, however, be lumped into coefficients.

As long as $m_Q \gg \Lambda_{\text{QCD}}$ this matching procedure makes sense, just as in continuum QCD.

The static approximation is a discretization of the first term. There is no m_Q in this formulation, so higher-dimension terms are suppressed by powers of a.

Lattice NRQCD is a discretization of the first few terms. Some of the coefficients have power-law divergences, so must keep $a \sim m_Q^{-1}$.

Wilson fermions satisfy the Isgur-Wise heavy quark symmetries. Therefore, the HQET description holds for them, even when $m_Q a \sim 1$. Lattice artifacts of heavy quarks then appear in the deviation of the coefficients from the continuum value.

The HQET provides the normalization conditions of the "Fermilab" approach. As $m_b a \to 0$ (which is impractical) these conditions merge into Symanzik-style matching.

If $m_Q a$ is reduced by reducing m_Q, one must be careful not to leave the domain of HQET. Otherwise, extrapolations back up to m_b are not controllable.

$B^- \to \pi^0 l^- \bar{\nu}$ and V_{ub}

The decay rate for $B \to \pi l \nu$ is

$$\frac{d\Gamma}{dp} = \frac{G_F^2 m_B}{12\pi^3} |V_{ub}|^2 \frac{p^4}{E} |f_+(E)|^2,$$

where $E = v \cdot p_\pi$, $p^2 = E^2 - m_\pi^2$. The form factor $f_+(E)$ is obtained from the matrix element

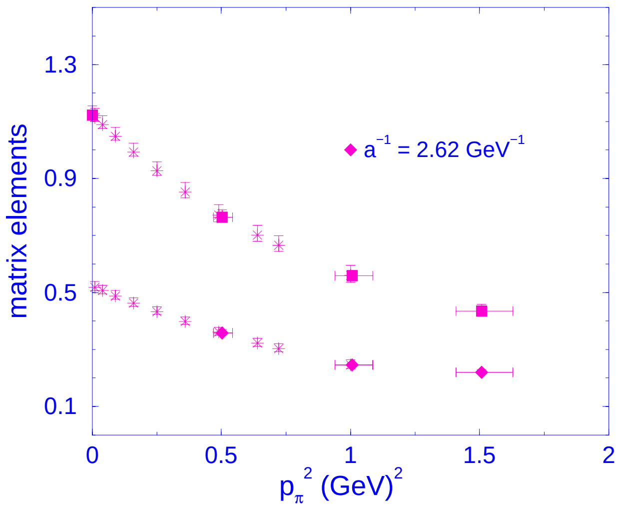
$$\langle \pi | V^\mu | B \rangle = \sqrt{2 m_B} \left[v^\mu f_\parallel(E) + p_\perp^\mu f_\perp(E) \right]$$

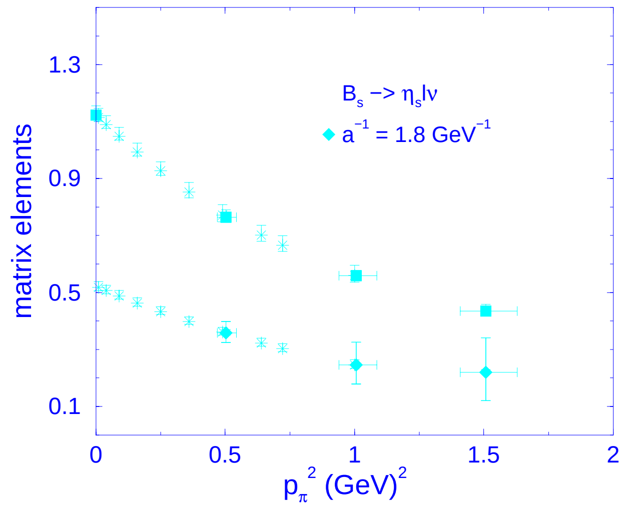
The pion energy E is related to the more familiar variable $q^2 = m_B^2 + m_\pi^2 - 2 m_B E$.

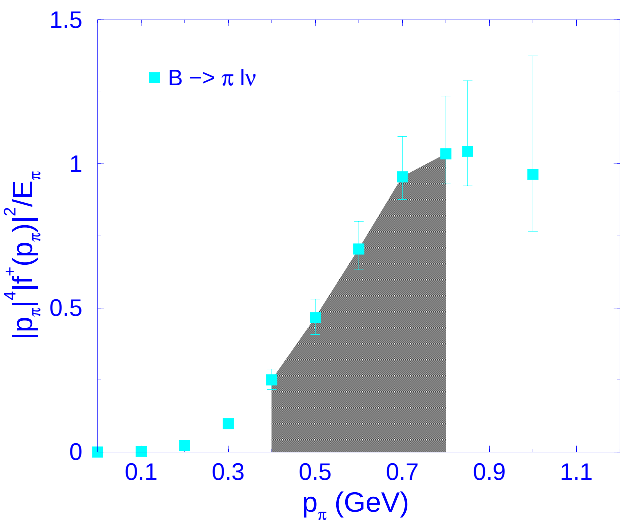
This is a good example to discuss, because it is timely, and because there are three calculations to compare

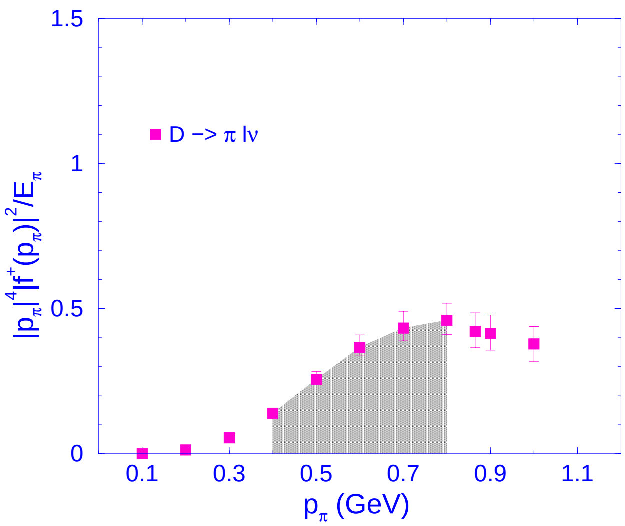
who	how	a^{-1} (GeV)	quark mass
UKQCD	3	2.54	$0.168 < m_Q a < 0.485$
JLQCD	2	1.64	$1.3 < m_Q a < 3.0$
D-F-I	4	2.62	from Υ (ψ)
		1.80	
		1.16	

D-F-I = Dublin-Fermilab-Illinois

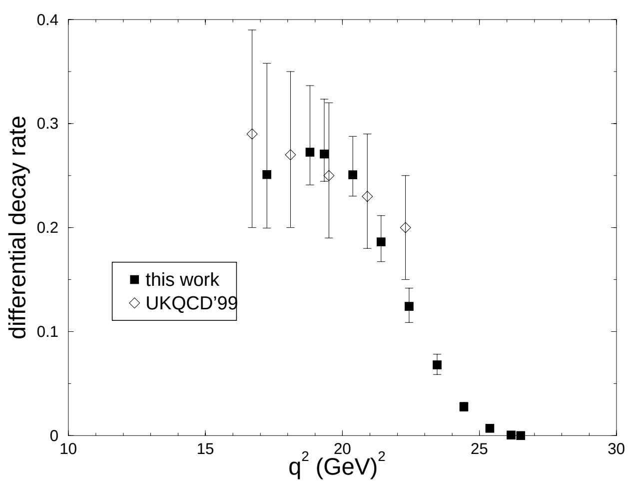


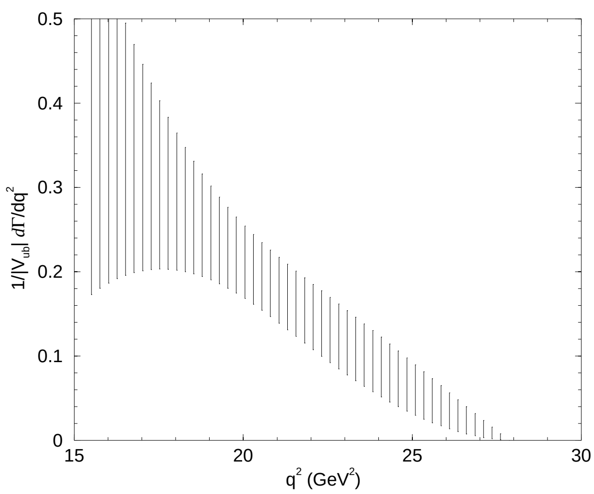






Differential decay rate





JLQCD

$\bar{\Lambda}$ and λ_1

The matching of lattice gauge theory to HQET can be used to determine "parameters" (actually, matrix elements) of the continuum heavy-quark expansion.

The spin-averaged B^*-B mass is given by

$$\bar{M} = m + \bar{\Lambda} - \frac{\lambda_1}{2m},$$

where m is the heavy quark mass, and $\bar{M} = \frac{1}{4}(3M_{B^*} + M_B)$.

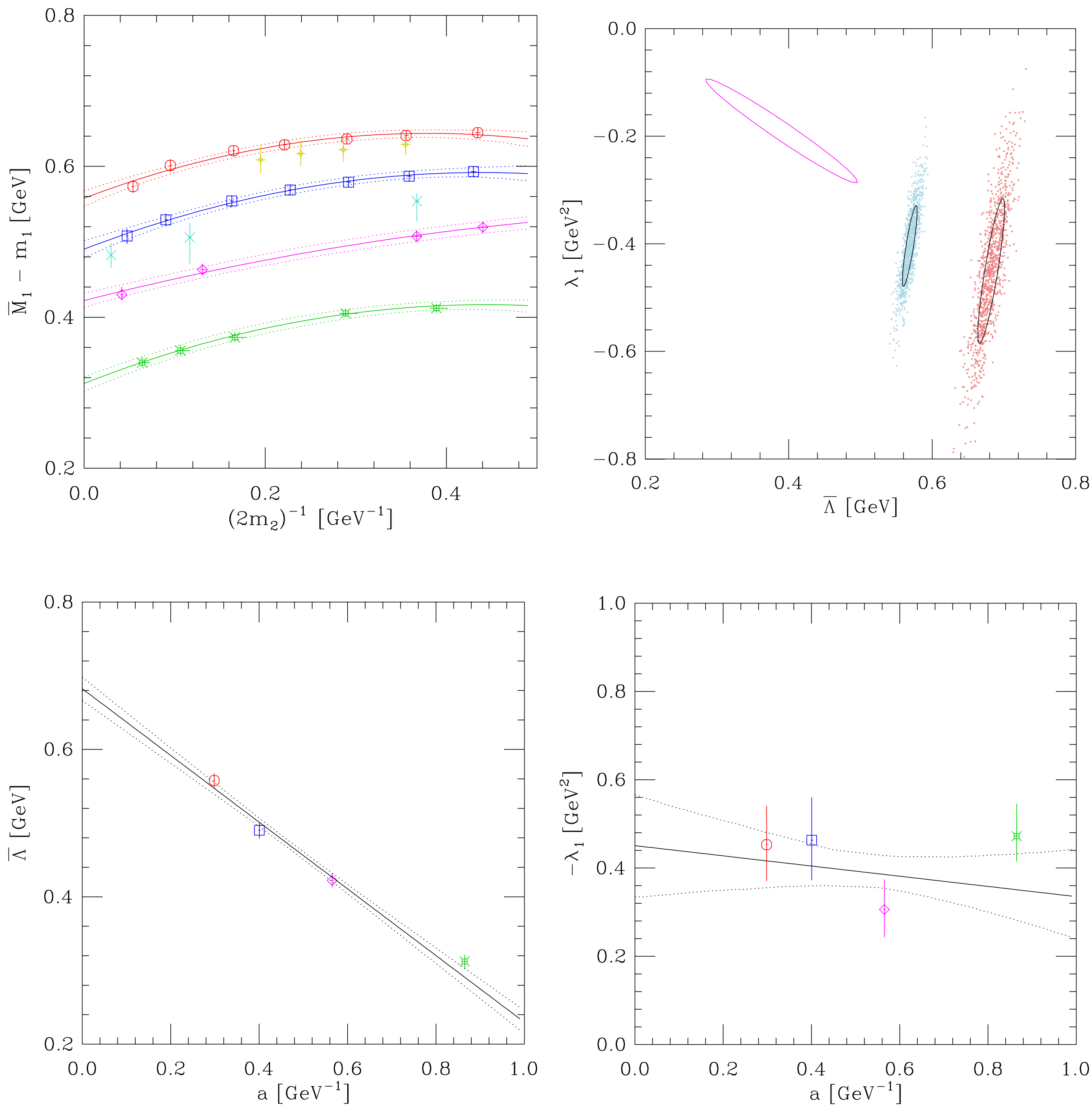
The lattice changes only the short-distance definition of the quark mass:

$$\bar{M}_1 - m_1 = \bar{\Lambda}_{\text{lat}} - \frac{\lambda_{1\,\text{lat}}}{2m_2}.$$

Because the lattice breaks Lorentz symmetry, $m_1 \neq m_2$, but still calculable in perturbation theory.

HQ expansions of inclusive decay distributions contain $\bar{\Lambda}$ and λ_1, so they're relevant to $|V_{ub}|$.

Simone and ASK, hep-ph/0006345:



Compared to Gremm et al. analysis of charged lepton moments.

$B^- \to D^{0(*)} l^- \bar{\nu}$ and V_{cb}

The decay rate for $B \to Dl\nu$ is

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_{BD}^5}{48\pi^3}(w^2 - 1)^{3/2}|V_{cb}|^2|\mathcal{F}(w)|^2,$$

where $w = v \cdot v'$ and $M_{BD}^5 = (m_B + m_D)^2 m_D^3$

$$\mathcal{F}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w).$$

The lattice yields $h_\pm(w)$.

As $w \to 1$ HQS normalizes h_+ and suppresses h_-. Thus, one actually seeks the deviations from the symmetry limit.

Hashimoto *et al.* devised a method based on double ratios, in which all the uncertainties cancel in the HQS limit. For example,

$$\frac{\langle D|\bar{c}\gamma_0 b|\bar{B}\rangle\langle \bar{B}|\bar{b}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 c|D\rangle\langle \bar{B}|\bar{b}\gamma_0 b|\bar{B}\rangle} = |h_+(1)|^2,$$

The masses of the "c" and "b" quarks can be varied to obtain $h_+(1) - 1$.

In the end (published)

$$\mathcal{F}_{B\to D}(1) = 1.058 \pm 0.016 \pm 0.003^{+0.014}_{-0.005},$$

where error bars are from statistics, adjusting the quark masses, and higher-order radiative corrections.

A similar technique leads to the form factor for $B \to D^* l\nu$. Our preliminary result

$$\mathcal{F}_{B\to D^*}(1) = 0.935 \pm 0.022^{+0.008}_{-0.011} \pm 0.008 \pm 0.020,$$

where the last uncertainty is from $1/m_Q^3$.

In both cases, lattice spacing dependence is not yet studied, nor is the effect of quenching. These, like the other uncertainties are a fraction of $\mathcal{F} - 1$.

These results will be updated soon, with calculations at a second lattice spacing and refinements in the radiative corrections.

$B_q^0 \leftrightarrow \bar{B}_q^0$ and V_{tq}

The mass difference of CP eigenstates is

$$\Delta m_{B_q^0} = \left(\frac{G_F^2 m_W^2 S_0}{16\pi^2} |V_{tq}^* V_{tb}|^2 + ? \right) \eta_B \langle Q_q^{\Delta B=2} \rangle$$

where $q \in \{d, s\}$, S_0 is an Inami-Lim function, and

$$\langle Q_q^{\Delta B=2} \rangle = \langle \bar{B}_q^0 | Q_q^{\Delta B=2} | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}.$$

μ dependence in η_B and $Q_q^{\Delta B=2}$ cancels.

Note that new physics could compete with the Standard Model (W-t) box diagrams.

Since lattice QCD gives matrix elements, the basic results are $\langle Q_q^{\Delta B=2} \rangle$ and, separately, f_{B_q}.

It is conventional wisdom that uncertainties in B_{B_q} and $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_s}^2 B_{B_s}$ should be small and easy to control, because they are ratios.

This is true, but that doesn't mean that the quoted uncertainties should be taken at face value.

Recent results for $B(\mu = 4.8\text{ GeV})$ agree:

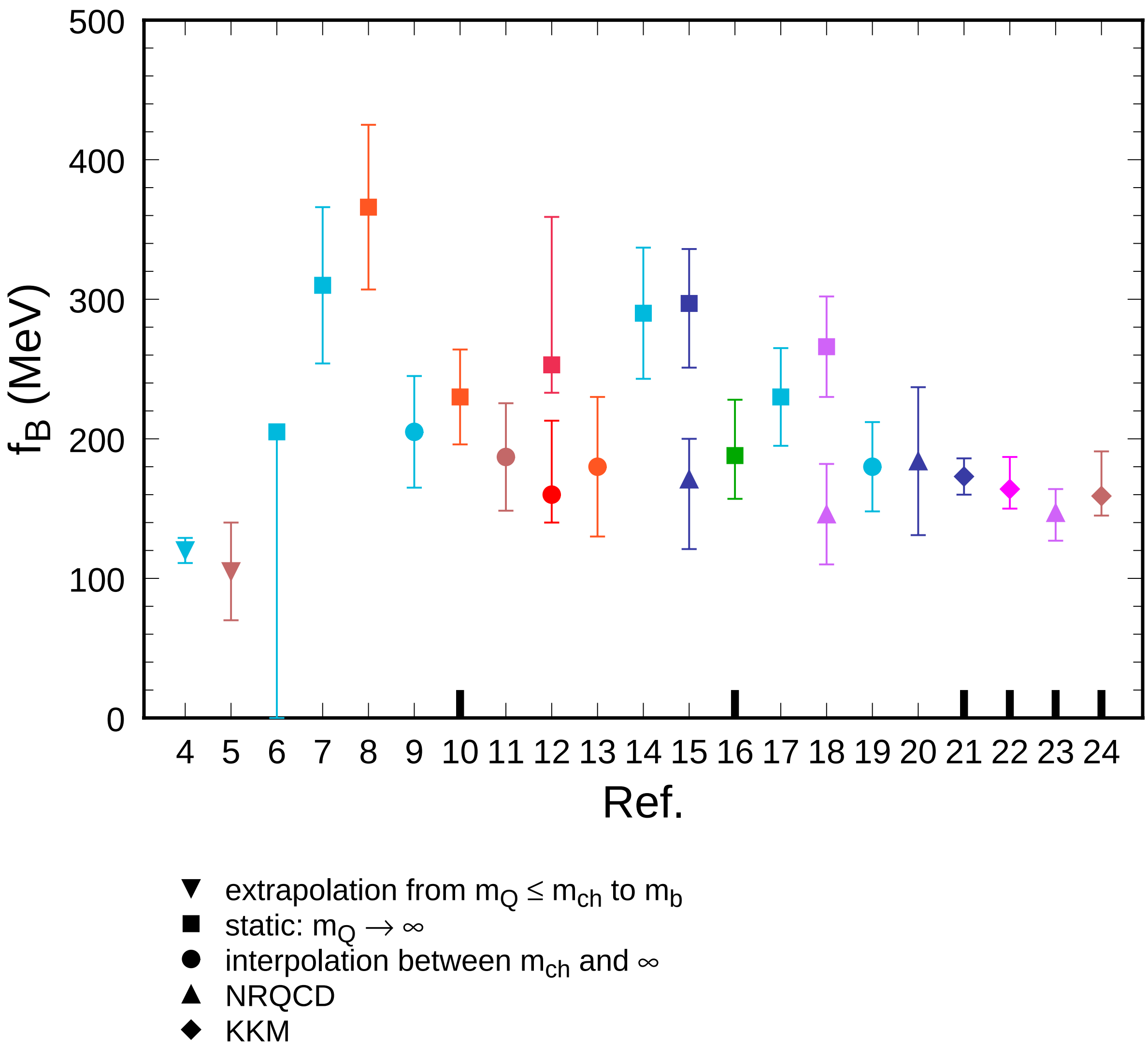
who	how	$B_{B_d}(4.8\text{ GeV})$
JLQCD	latNRQCD	$0.85 \pm 0.03 \pm 0.11$
UKQCD	extrap	$0.92 \pm 0.04^{+03}_{-00}$
APE	extrap	$0.93 \pm 0.08^{+00}_{-06}$

Note that JLQCD now includes the short-distance part of the $1/m_Q$ contribution.

UKQCD (preliminary) and APE (final) both extrapolate linearly in $1/m_Q$ from charm (e.g., 1.75 GeV$< m_P <$2.26 GeV for APE), without an anchor from the static limit. One should expect that the true systematic uncertainty, from the treatment of the heavy quark, is larger than quoted.

In older work, the extrapolation did not point at the static limit. This is a symptom of incorrectly normalized $1/m_Q$ terms, and the HQET-motivated normalization works better.

Timeline of f_B



Infancy 4–9; childhood 10–15; youth 16–19; adulthood 20–23. (Calculations before October 1998.)

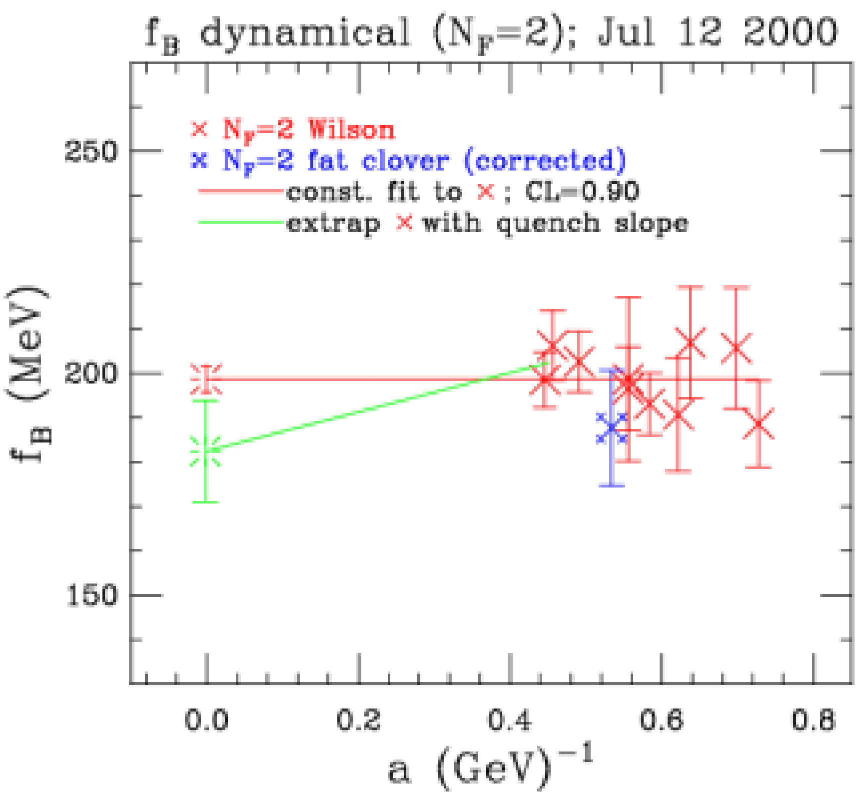
C. Quigg: When does senility set in?

Unquenched f_B

There are new unquenched calculations of f_B, with several lattice spacings.

	$n_f = 0$ (MeV)	$n_f = 2$ (MeV)
f_B	$171 \pm 6 \pm 17^{+21}_{-\ 4}$	$190 \pm 6^{+20+9}_{-15-0}$
f_{B_s}	$197 \pm 5 \pm 23^{+25}_{-\ 6}$	$218 \pm 5^{+26+11}_{-23-0}$
f_D	$199 \pm 6 \pm 12^{+14}_{-\ 0}$	$213 \pm 4^{+14+7}_{-13-0}$
f_{D_s}	$222 \pm 5 \pm^{+19}_{-17}{}^{+15}_{-\ 0}$	$240 \pm 4^{+25+9}_{-23-0}$

MILC preliminary; CP-PACS similar, but 1σ higher



Clusters of PCs

For B physics it is important to remove the quenched approximation. To do so, we need more computing. It sounds expensive, but isn't. Still, we need your help.

Fermilab Theory & CD, MILC, and Cornell are building a cluster of PCs.

8 nodes at left,
with Myrinet switch

Scale to 48–64, then to thousands.

Similar ideas at JLab/MIT, Wuppertal,. . . .

Replace 1/3 yearly.

http://www-theory.fnal.gov/pcqcd/

Thus, our collaboration wants to be in a state of permanent *evolution*.

Conclusions

The last few years have seen great progress in understanding heavy quarks in lattice QCD.

The progress has been both computational and theoretical. One guides the other.

Calculations shown here, for

$$\begin{array}{l} \bar{B}^0 \to \pi^+ l^- \bar{\nu} \\ D^0 \to \pi^- l^+ \nu \\ B^- \to D^{0(*)} l^- \bar{\nu} \\ B_d^0 \leftrightarrow \bar{B}_d^0 \end{array}$$

are a subset, but they're as basic as A, B, C.

The numerical results have, for the most part, been in the quenched approximation, except f_B from MILC.

If the aim is to determine the sides of the triangles, several upgrades—or a permanently evolving pile of PCs—will be needed.