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APPLICATIONS OF NON-PERTURBATIVE RENORMALIZATION

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Topics

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  - Quantities studied on the lattice
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- Non-perturbative renormalization group via a recursive finite-size scaling procedure
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  - Determination of the renormalization group invariant mass of the strange quark
  - Preliminary results for the renormalization of the static-light axial current
- Summary

Introduction

Lattice QCD:

non-perturbative framework to calculate relations between parameters in the Standard Model and experimental quantities from first principles

renormalization problem:

- application of renormalized perturbation theory is limited to processes at high E, where the QCD coupling constant α_s is sufficiently small

$\Rightarrow$ inadequate for bound state properties at low E, i.e. non-perturbative solution of the theory required

$\Rightarrow$ numerical (MC = Monte Carlo) simulations of the Euclidean path integral of QCD on a space-time lattice

renormalization = ultraviolet phenomenon

$\leadsto$ relevant momentum scales $\mu \sim a^{-1}$, $\alpha_s(\mu) \overset{\mu \text{ large}}{\sim} \frac{1}{\ln(\mu)}$

$\leadsto$ question: renormalize perturbatively as $a \to 0$?

$\leadsto$ in general the answer is no:
want tractable simulation effort $\leftrightarrow$ $a \not\ll L_{\text{phys}}^{(\text{obs})}$,
thus truncation of perturbative series not justified

$\Downarrow$

far more safe to
perform renormalizations non-perturbatively

ideally regard Lattice QCD as phenomenological tool:

- renormalization employing the hadron spectrum

$$am_{\rm hadron} = am_{\rm hadron}(g_0, \{am_0^f\}) \text{ via MC}$$

$$\left.\begin{array}{r} \text{lattice spacing } a = (am_{\rm proton})/m_{\rm proton}^{\rm (exp)} \\ \text{fix } (am_{\rm hadron})/(am_{\rm proton}) = m_{\rm hadron}^{\rm (exp)}/m_{\rm proton}^{\rm (exp)} \end{array}\right\} g_0 \hookrightarrow a$$

$$\rightsquigarrow \text{ predictions like } m_\Delta = a^{-1}\,[am_\Delta]\,[1 + \mathrm{O}(a)]$$

- address fundamental parameters that escape a *direct* determination, for instance

$$\left\{ F_\pi\,,\, m_{\rm K} \right\} \text{ from experiment } \hookrightarrow \left\{ \alpha_{\rm s}(M_{\rm Z})\,,\, m_{\rm s}(2\,{\rm GeV}) \right\}$$

prominent observables of interest for the lattice:

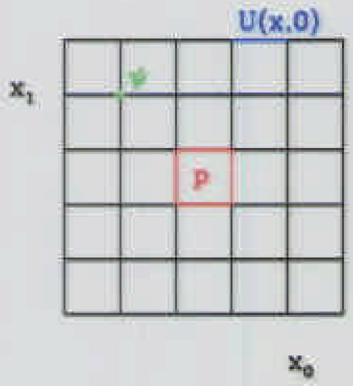
- quark bilinears, currents, $\Delta S = 2$ matrix elements
- $\alpha_{\rm s}(\mu)$
- quark masses ($\rightarrow m_{\rm s}$ essential for ϵ'/ϵ analyses)
- structure functions ($\rightarrow$ parton density operators)
- static current ($\rightarrow$ briefly touched at the end)

note: renormalization properties are of different nature

$\longrightarrow$ e.g. finite, scale and/or scheme dependencies

Lattice QCD: a short-cut

basic ingredients



lattice: $x = \{x_\mu \,|\, \mu = 1, \ldots, 4\} \in a\mathbb{Z}^4$

quarks: $\psi(x)$ living on lattice sites

gluons: $U(x,\mu) = \mathrm{e}^{iaA_\mu(x)} \in \mathrm{SU}(3)$

link variables

features and obstacles

- stochastic MC evaluation of $\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}; U]\, O\, \mathrm{e}^{-S}$ (importance sampling, $\Delta_O \propto 1/\sqrt{N_{\mathrm{meas}}} \propto 1/\sqrt{t_{\mathrm{CPU}}}$)
- continuum limit: lattice artifacts extrapolated away (improvement $\rightsquigarrow$ eliminate the $\mathrm{O}(a)$ cutoff effects)
- fermion dynamics very costly, so frequently used: quenched approximation = ignore all quark loops
- ambiguities in setting the scale in quenched QCD
- lattice discretization implies restriction $a \ll m_{\mathrm{q}}^{-1} \ll L$ (extrapolations in the quark mass m_{q} necessary but often difficult to control in practice)
- chiral symmetry violation (inherent in Wilson's fermion action) gives rise to additive & multiplicative renormalization constants

Intermediate renormalization schemes

representative example: quark masses through PCAC

$$F_K m_K^2 = (\overline{m}_u + \overline{m}_s)\langle 0|\overline{\psi}_u \gamma_5 \psi_s|K\rangle$$
$$(\overline{\psi}_u \gamma_5 \psi_s)_{\overline{MS}} = Z_P(g_0, a\mu)(\overline{\psi}_u \gamma_5 \psi_s)_{\text{lattice}}$$

+ Z_P relates the lattice results to the $\overline{\text{MS}}$ scheme
+ Z_P is scale and scheme dependent
+ computable in lattice perturbation theory
− uncertainties from (tadpole improved) bare couplings

task to match the low- and high-energy regimes reliably complicates the situation on the technical level:

one faces different scales to be covered simultaneously

- box size (typical extents $L \simeq 2$ fm)
- confinement scale $\simeq 0.2$ GeV
- $\mu \simeq 10$ GeV $\simeq 1/(0.02\text{ fm})$ to get contact with the perturbative approximation for $\beta(\overline{g}), \tau(\overline{g})$
- lattice spacing a as cutoff

$$L \gg 1/(0.2\,\text{GeV}) \gg 1/\mu \simeq 1/(10\,\text{GeV}) \gg a$$

suppression of a–effects implies prohibitively large L/a

idea to overcome this problem:
intermediate schemes (2 realizations in the context of non-perturbative renormalization on the market)

1. RI = Regularization Independent [Martinelli et al.]

impose Γ_{P} : amputated Green function of $(\overline{\psi}_{\mathrm{q}}\gamma_5\psi_{\mathrm{q}})_{\mathrm{lattice}}$ &

$$\frac{Z_{\mathrm{P}}^{\mathrm{MOM}}(g_0, a\mu)}{Z_2^{\mathrm{MOM}}(g_0, a\mu)} \times \Gamma_{\mathrm{P}}(ap)\Big|_{p^2=\mu^2} = 1$$

Z_2 : quark wave function renormalization

+ the matching MOM $\leftrightarrow$ $\overline{\mathrm{MS}}$ relies on renormalized *continuum* perturbation theory now

? applicability window $\Lambda_{\mathrm{QCD}} \ll \mu \ll a^{-1}$ enclose external momenta *range* to probe the scale dependence?

2. SF = Schrödinger Functional [Lüscher et al., ALPHA Collaboration]

basic strategy for a non-perturbative computation of short distance parameters on the lattice is a recourse to an intermediate finite-volume renormalization scheme: identify $\boxed{\mu = 1/L}$ $\rightsquigarrow$ $\alpha_{\mathrm{SF}}(1/L)$ and similarly $M/\overline{m}^{\mathrm{SF}}(1/L)$

$L_{\max} = C/F_\pi$: $\mathrm{O}(\frac{1}{2}\mathrm{fm})$ — hadronic $\hookrightarrow$ SF scheme $\longrightarrow$ $\alpha_{\mathrm{SF}}(\mu = 1/L_{\max})$

$\downarrow$

$\alpha_{\mathrm{SF}}(\mu = 2/L_{\max})$

$\downarrow$

•••

$\downarrow$

$\alpha_{\mathrm{SF}}(\mu = 2^n/L_{\max})$

PT $\downarrow$

results in: value for $\Lambda_{\mathrm{QCD}}/F_\pi$ $\xleftarrow{\mathrm{PT}}$ $\Lambda_{\mathrm{SF}} L_{\max}$

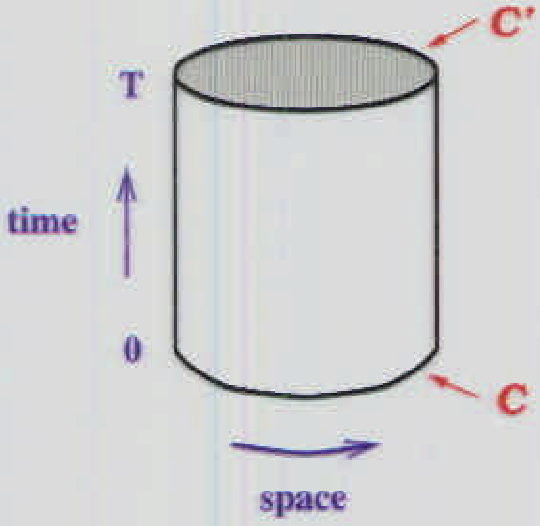
universal relations '$\rightarrow$' accessible in the continuum limit

Schrödinger functional

the Schrödinger functional (SF) is defined as the QCD partition function

$$\mathcal{Z} = \int_{\text{fields}} e^{-S} = e^{-\Gamma} \qquad \Gamma : \text{effective action}$$

where quark & gluon fields satisfy Dirichlet BCs in time:



a renormalized coupling in this scheme:

$\alpha_{\text{SF}}(\mu) = \bar{g}^2(L)/4\pi$ with $\bar{g} \equiv \bar{g}_{\text{SF}}$ being defined as the response to an infinitesimal variation of the BCs

reconstruction of the non-perturbative scale evolution of the renormalized 'running' coupling constant in the SF via the finite-size step scaling function (SSF) $\sigma(s, u)$:

$$\bar{g}^2(L) = u \qquad \bar{g}^2(sL) = u' \equiv \sigma(s, u)$$

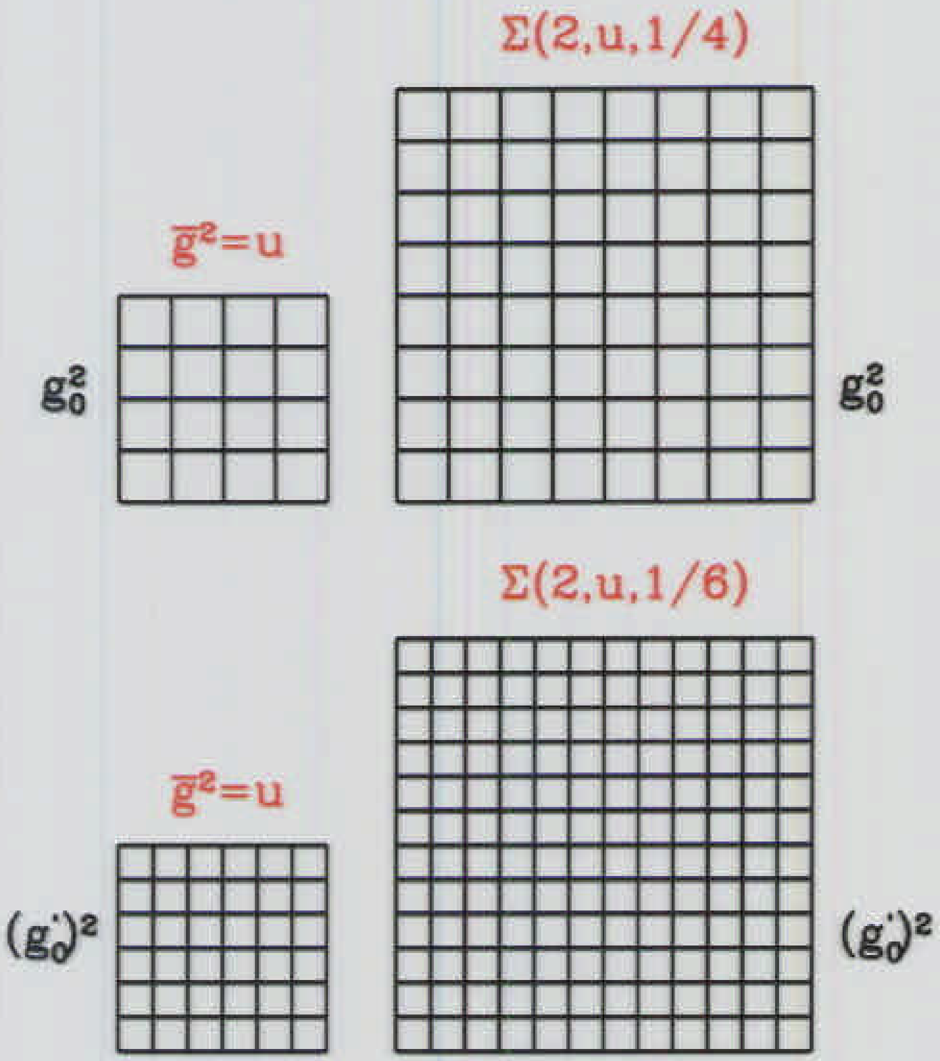
s: step size in change of the length scale

⇒ evident meaning of $\sigma(s, u)$: discrete beta function on the lattice: start with $\bar{g}^2(L) = u_0 \equiv 3.48$ at $L = L_{\max}$

$$\Sigma(u_{k-1}, a/L) \equiv u_k = \bar{g}^2(sL_{\max}) \quad s = 2^{-k} \quad k = 0, 1, 2, \ldots$$

$\rightsquigarrow$ calculable by solving the recursion $\sigma(u_{k+1}) = u_k$

$\rightsquigarrow$ at step $k \to k+1$ of the sequence: keep $g_0, L/a$ fixed



carries an additional dependence on the resolution a/L

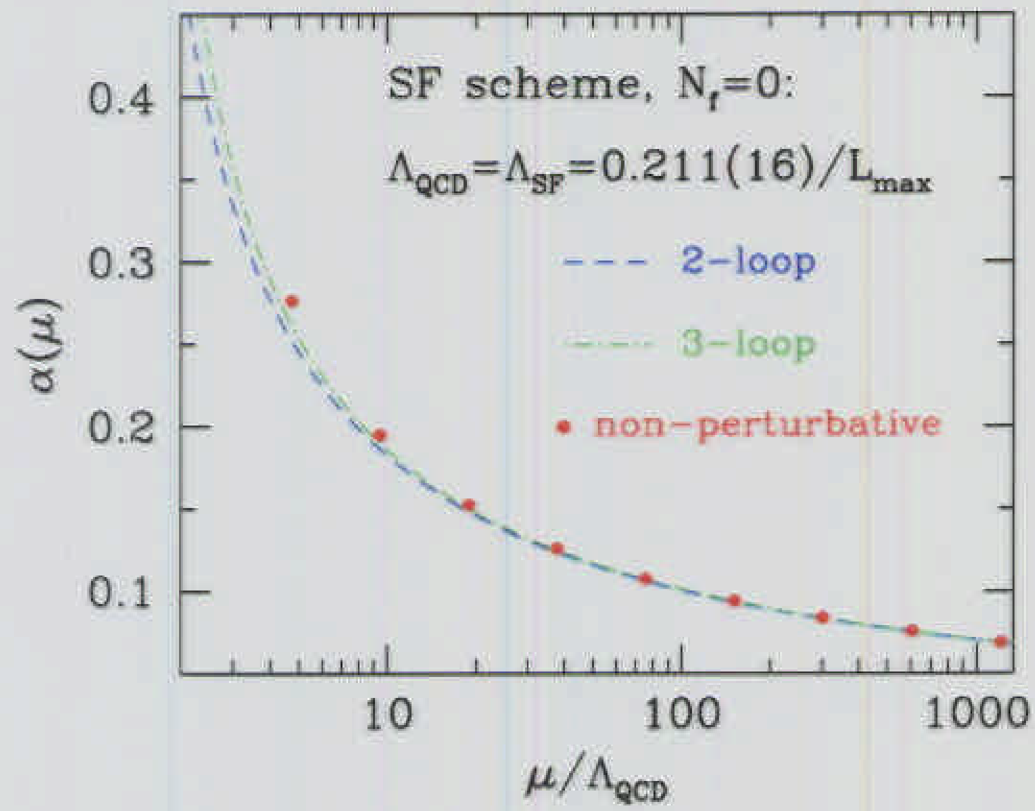
$$\Rightarrow \Sigma(u, a/L) = \sigma(u) + \mathrm{O}\left(\frac{a}{L}\right) \text{ in the continuum limit}$$

Λ_{QCD} and RGI quark masses [cf. NPB544(99)669]

successive application of the SSF σ yields $\bar{g}^2_{SF}(L_{max}/2^k)$ until contact can be made with the high-energy regime, which is well described by perturbative scaling:

$$\Lambda = \mu(b_0\bar{g}^2)^{-b_1/2b_0^2}\,e^{-1/2b_0\bar{g}^2}\exp\left\{-\int_0^{\bar{g}} dg\left[\frac{1}{\beta(g)}+\frac{1}{b_0g^3}-\frac{b_1}{b_0^2 g}\right]\right\}$$

result in the quenched approximation:

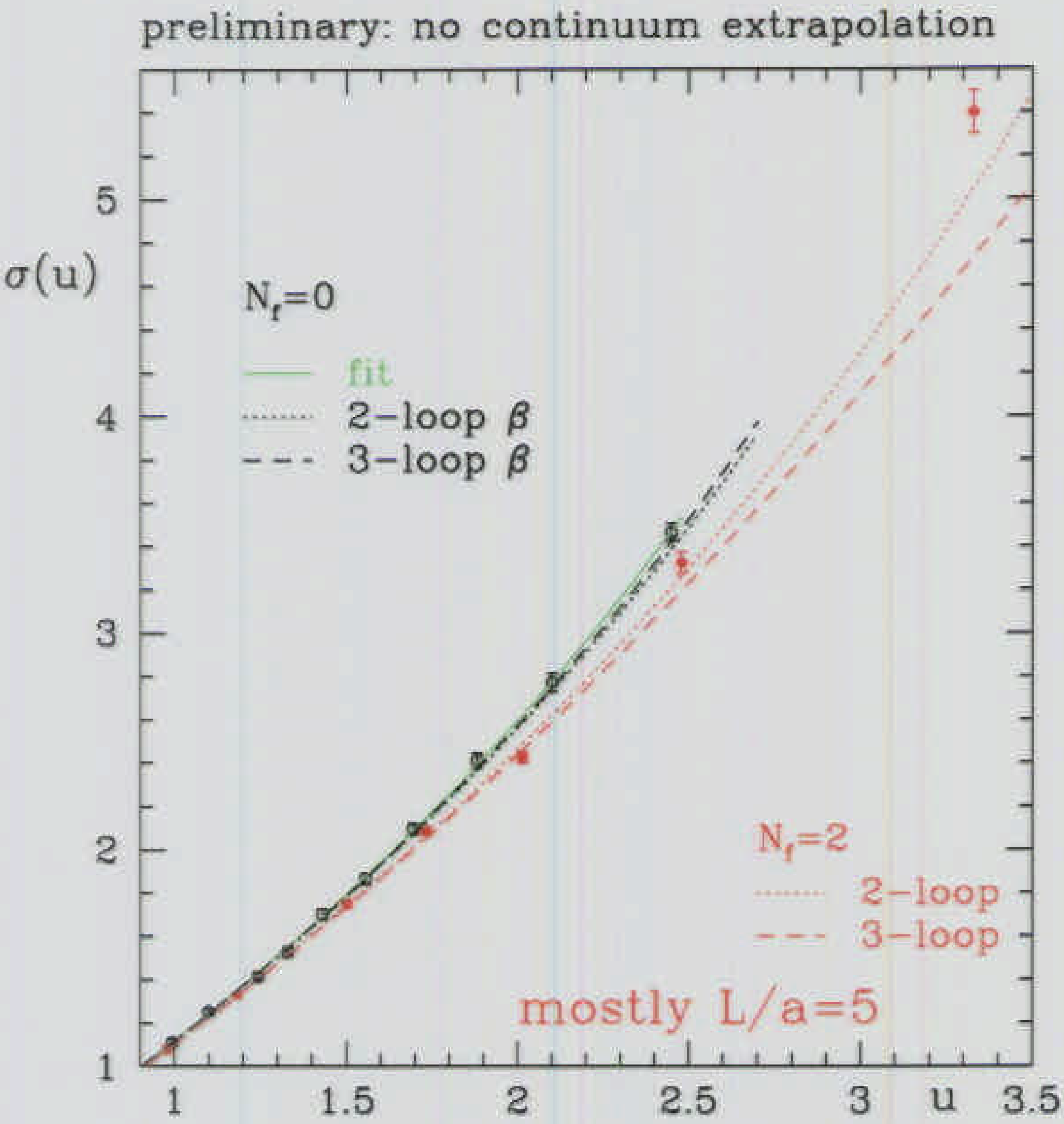


matching to a hadronic scheme: express L_{max} in units of the 'radius' $r_0 \simeq 0.5$ fm (from force between static quarks)

$\Rightarrow$ matching scale $\mu = \dfrac{1}{L_{max}} \simeq 550$ MeV

conversion SF $\to$ convenient $\overline{MS}$ scheme leads to

$\Rightarrow$ $\Lambda^{(0)}_{\overline{MS}} = 238(19)$ MeV status of the corresponding project by ALPHA Collaboration with $N_f = 2$ massless dynamical quarks:

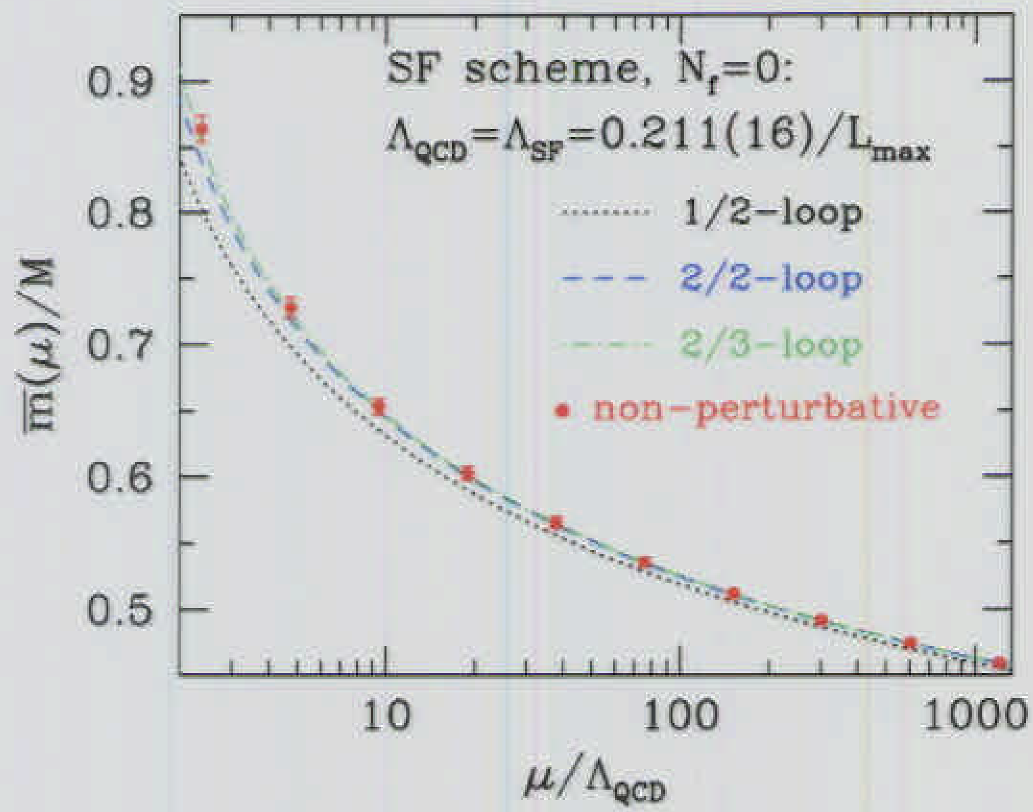


in the same way: obtain the scale evolution of the renormalized PCAC (current) quark mass $\overline{m}$, viz.

$$\overline{m} \equiv \frac{Z_{\rm A}(g_0)}{Z_{\rm P}(g_0, a\mu)}\, m \qquad \text{SSF}: \quad \frac{Z_{\rm P}(2^{-k} \times 2L_{\rm max})}{Z_{\rm P}(2L_{\rm max})} \hookrightarrow \sigma_{\rm P}(u)$$

to extract the *scheme* and *scale independent* (!) Renormalization Group Invariant quark masses

$$M = \overline{m}\,(2b_0\bar{g}^2)^{-d_0/2b_0} \exp\left\{-\int_0^{\bar{g}} {\rm d}g \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}$$



$\Rightarrow \quad M/\overline{m}^{\rm SF}(\mu) = 1.157(12) \quad @ \quad \mu = 1/2L_{\rm max} \simeq 275$ MeV

finally the total renormalization factor $Z_{\rm M}(g_0)$ is known non-perturbatively for the O(a) improved theory:

$$M = \underbrace{Z_{\rm M}(g_0)}_{\text{flavour independent}} \times \underbrace{m(g_0)}_{\text{current quark mass}} + {\rm O}(a^2)$$

$$Z_{\rm M} = \underbrace{M/\overline{m}(\mu)}_{\text{universal}} \times \underbrace{\overline{m}(\mu)/m(g_0)}_{=Z_{\rm A}/Z_{\rm P}}$$

The strange quark's mass

general strategy for computation of light quark masses

[ALPHA & UKQCD: Garden, H., Sommer, Wittig, NPB 571 (2000) 237]

- starting point: chiral perturbation theory in full QCD

  $M_u/M_d = 0.55(4) \quad M_s/\hat{M} = 24.4(1.5) \quad \hat{M} = \frac{1}{2}(M_u + M_d)$

- now we implicitly define an *not necessarily physical* reference quark mass (here: mass degenerate case)

$$m_{PS}^2(M_{ref})r_0^2 = (m_K r_0)^2 = 1.5736$$

$$\chi PT : \boxed{2M_{ref} \simeq M_s + \hat{M}}$$

$$\text{input}: m_K^2 = \frac{1}{2}(m_{K^+}^2 + m_{K^0}^2)\Big|_{\text{pure QCD}} = (495 \text{ MeV})^2$$

$\Rightarrow$ goal: calculate M_{ref} through Lattice QCD

MC simulations in $V = (1.5\,\text{fm})^3 \times 3\,\text{fm}$: evaluate correlation function to determine PS hadronic matrix elements

$$\left.\begin{array}{r} \langle 0|\overline{\psi}_u \gamma_0\gamma_5 \psi_s|K\rangle = m_K F_K \\ \langle 0|\overline{\psi}_u \gamma_5 \psi_s|K\rangle = G_K \end{array}\right\} \overset{\text{PCAC}}{\Longrightarrow} R \equiv \frac{F_K}{G_K} = \frac{2M_{ref}}{m_K^2}$$

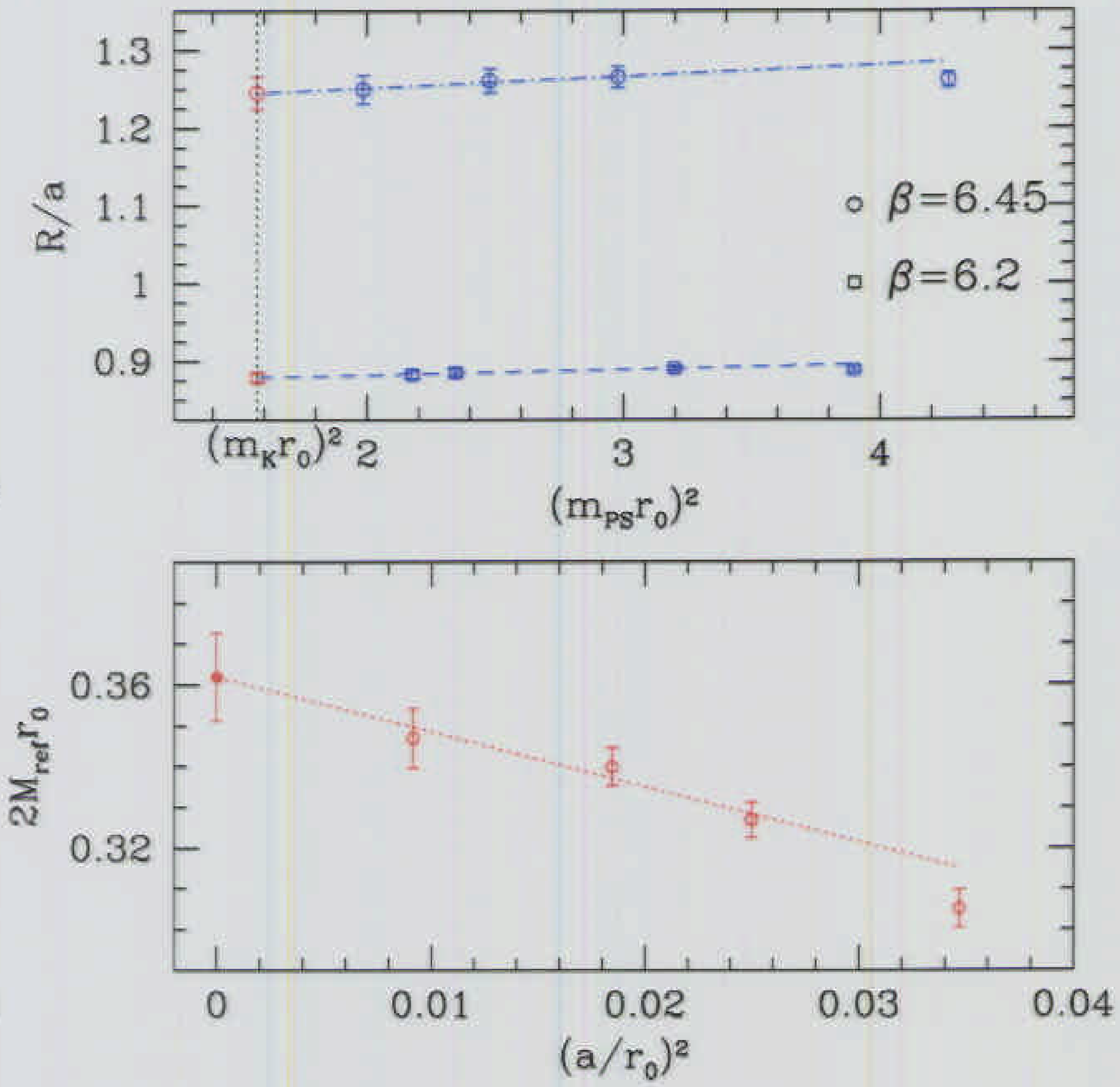
including renormalization and $O(a)$ improvement:

$$2r_0 M_{ref} = Z_M \frac{R\big|_{r_0^2 m_{PS}^2 = 1.5736}}{r_0} \times 1.5736 = (2r_0 M_{ref})\big|_{a=0} + O\left(\frac{a^2}{r_0^2}\right)$$

perturbative conversion to $\overline{\text{MS}}$ using 4–loop running

$\Rightarrow \quad \boxed{\overline{m}_s^{\overline{\text{MS}}}(2\,\text{GeV}) = 97(4)\ \text{MeV}}$ $\ni$ all errors except quenching examples from the extrapolations:

to the kaon mass scale (slightly !) and to the continuum

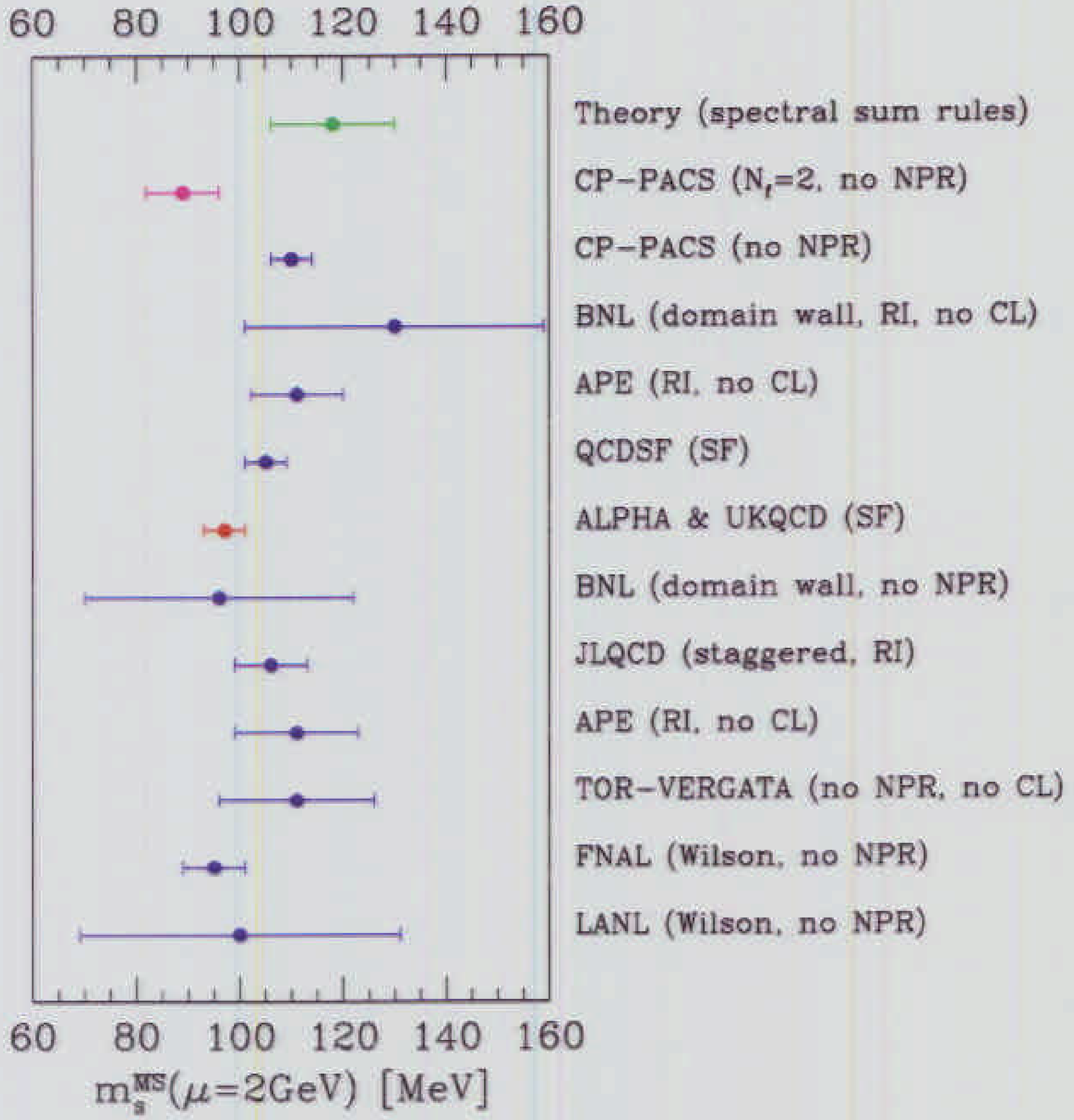


- stable continuum extrapolation but significant slope
- solid quenched result: $2M_{\rm ref} = 143(5)$ MeV RGI (!)
- quenched approximation: $\simeq 10\%$ scale ambiguity
- alternatively $2M_{\rm ref}/(F_{\rm K})_{\rm R} = (M/\overline{m}) \times (Z_{\rm P} r_0^2 G_{\rm PS})^{-1} (m_{\rm PS} r_0)^2$

  $\Rightarrow$ complete consistency !

comparison with quenched results of other groups:

(NPR = non-pert. renormalization, CL = continuum limit)



- mostly through (axial and/or vector) Ward identities
- caveat: no uniform estimation of systematic errors and differences when translating into physical units

Static axial current renormalization

- motivation: consider leptonic B-decays $\mathrm{B} \to \mathrm{l}\nu_\mathrm{l}$

$$\langle 0|(A_\mathrm{R})_\mu|\mathrm{B}(p)\rangle = p_\mu F_\mathrm{B} \qquad (A_\mathrm{R})_\mu = Z_\mathrm{A}\overline{\psi}_\mathrm{b}\gamma_\mu\gamma_5\psi_\mathrm{d}$$

$m_\mathrm{b} \simeq 4\,\mathrm{GeV} \gg \Lambda_\mathrm{QCD} \rightsquigarrow \mathrm{O}\left((am_\mathrm{b})^2\right)$ discretization errors

$\Rightarrow$ direct treatment is difficult on the lattice

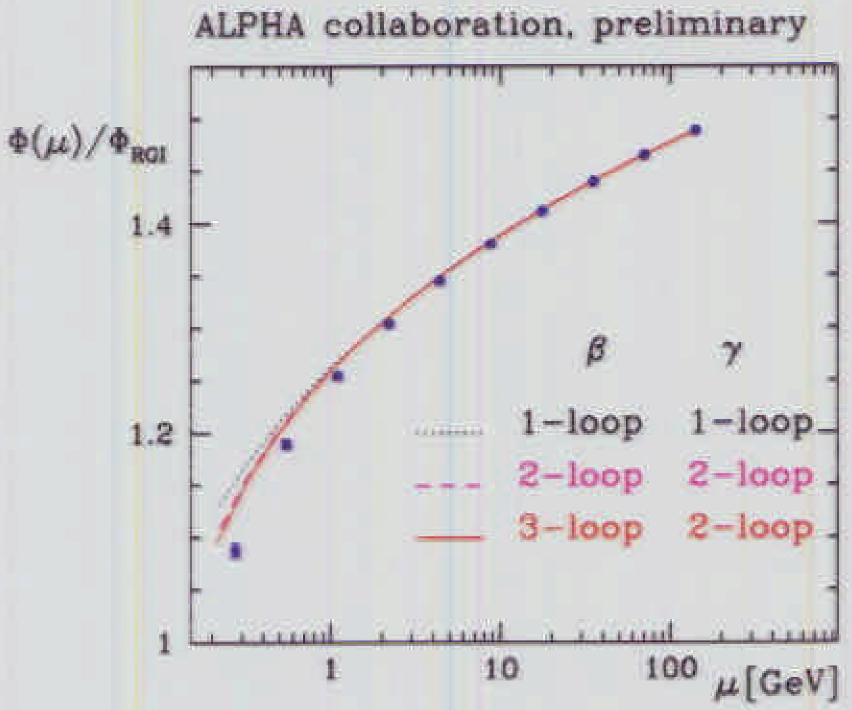
- but in an effective field theory: static approximation

$$F_\mathrm{B}\sqrt{m_\mathrm{B}} \equiv \Phi^\mathrm{stat}(\mu) + \mathrm{O}\left(\frac{\Lambda_\mathrm{QCD}}{m_\mathrm{b}}\right)$$

relevant scale: $\mu = m_\mathrm{b}$
'static': $m_\mathrm{heavy} \to \infty$

$\Rightarrow$ μ dependence and $Z_\mathrm{A}^\mathrm{stat}(\mu) \hookrightarrow \Phi^\mathrm{stat}(\mu)$ to predict F_B

analogous steps as for $\overline{m}(\mu)/M$ [J. H., M. Kurth, R. Sommer]



$$\Phi_\mathrm{RGI}^\mathrm{stat} = (2b_0\overline{g}^2)^{-\gamma_0/2b_0}\exp\left\{-\int_0^{\overline{g}}\mathrm{d}g\left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}\Phi^\mathrm{stat}(\mu)$$

for any matrix element Φ^stat of the static axial current

Summary

- large potential for Lattice QCD to address non-perturbative renormalization problems:
  - ✓ statistical, discretization & systematic uncertainties under good control
  - ✓ non-perturbative coupling and quark mass renormalization performed with confidence
  - ✓ high precision already achieved in the quenched approximation: e.g. 3% for $\overline{m}_s^{\overline{\rm MS}}(2\,{\rm GeV})$
    → dynamical quark effects under investigation
  - ✓ concept of combining Lattice QCD with χPT to determine light quark masses valuable for full QCD
- Schrödinger functional offers a clean & flexible approach to deal with accompanying scale differences:
  - ✓ technology is well established in quenched QCD
  - ✓ simulations much easier than in large volume
  - ✓ promising progress beyond α_s and M:
    RGI matrix elements in the static approximation
    → phenomenology of heavy flavour physics
- substantial work still to be done, but optimistic perspectives also owing to increasing computer power!