The effect of a small mixing angle in the atmospheric neutrinos

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1 Introduction

Asumption :

the existance of only three kind of convensional neutrinos

$$\nu_e, \nu_\mu, \text{ and } \nu_\tau$$

The results of Kamiokande and SuperKamiokande experiments suggest

$$P(\nu_\mu \to \nu_\mu) + P(\nu_\mu \to \nu_\tau) \simeq 1,$$

which means that the matrix elements of U defined by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} u_{e1} & u_{e2} & u_{e3} \\ u_{\mu 1} & u_{\mu 2} & u_{\mu 3} \\ u_{\tau 1} & u_{\tau 2} & u_{e\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

should be constrianed by the condition

$$u_{e3} \simeq 0.$$

c.f. CHOOZ experiment $\sin^2(2\theta_3) < 0.2$, *i.e.* $\theta_3 < 13°$

The parametrization of U:

$$\begin{aligned} U &= \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 \\ -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3 \end{pmatrix} \\ &= e^{i\theta_2 \lambda_7} e^{i\theta_3 \lambda_5} e^{-i\theta_1 \lambda_2} \end{aligned}$$

where

$$s_i = \sin\theta_i, \; c_i = \cos\theta_i \quad \text{for } i = 1, 2, \text{ and } 3$$

and

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

θ_1 : solar neutrino exp.

θ_2 : ongoing atmospheric neutrino exp.

The next issue is how to determine θ_3.

2 Neutrino Masses

$$\text{Neutrino masses}: m_1, m_2, \text{ and } m_3$$

$$\text{Mass differences}: \delta m^2_{ji} \equiv m^2_j - m^2_i$$

Asuumption :

$$\delta m^2_{21} \lesssim 10^{-5} eV^2$$

and

$$\delta m^2_{21} \ll \delta m^2_{32} \simeq \delta m^2_{31} \equiv \Delta m^2 \sim \mathcal{O}(10^{-3})\ eV^2,$$

which means that

$$\delta m^2_{21} = 0$$

to a good approximation.

3 Matter Oscillations in the Earth

The remaining parameters

- two mixing angles: θ_2, and θ_3
- one mass parameter: $\Delta m^2 \equiv \delta m^2_{32} \equiv \delta m^2_{31}$

Resonance Conditions

$$P(\nu_e \rightarrow \nu_e) = 0$$

The Prominent Feature of the One mass Dominant Model

The resonace condition in the earth can be determined by the the three parameters, namely,

1. θ_3 (lepton mixing angle which determines u_{e3})
2. $\Delta m^2/E$ where E is the neutrino energy
3. z (zenith angle)

- $P(\nu_e \rightarrow \nu_e)$ is completely determined by the above three parameters.
- The other probabilities such as $P(\nu_e \rightarrow \nu_\mu)$, $P(\nu_e \rightarrow \nu_\tau)$, $P(\nu_\mu \rightarrow \nu_\mu)$, $P(\nu_\mu \rightarrow \nu_\tau)$, and $P(\nu_\tau \rightarrow \nu_\tau)$ are also depend on θ_2.

4 Earth Model and the Calculations

We approximate the earth density as 20 constant layers.

The evolution matrix of the n layer model is given by

$$e^{-i\theta_2\lambda_7} S e^{i\theta_2\lambda_7} = e^{-i\theta_2\lambda_7} S_{n-1} S_{n-2} \cdots S_k \cdots S_1 S_0 S_1 \cdots S_k \cdots S_{n-2} S_{n-1} e^{i\theta_2\lambda_7}$$

Let

$$\begin{aligned}
&e^{-i\theta_2\lambda_7} S^{(k)} e^{i\theta_2\lambda_7} \\
&= e^{-i\theta_2\lambda_7} S_k \cdots S_1 S_0 S_1 \cdots S_k e^{i\theta_2\lambda_7} \\
&= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{i\Sigma_k} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} f_k - i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} g_k + i \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} h_k.
\end{aligned}$$

The coefficients f_k, g_k, and h_k satisfy the following recoursion relations.

$$\begin{aligned}
f_{k+1} &= f_k \cos(2\varphi_k) - g_k \sin(2\varphi_k)\sin(2\theta_k) - h_k \sin(2\varphi_k)\cos(2\theta_k) \\
g_{k+1} &= f_k \sin(2\varphi_k)\sin(2\theta_k) + g_k[\cos^2\varphi_k + \sin^2\varphi_k \cos(4\theta_k)] - h_k \sin^2\varphi_k \sin(4\theta_k) \\
h_{k+1} &= f_k \sin(2\varphi_k)\cos(2\theta_k) - g_k \sin^2\varphi_k \sin(4\theta_k) + h_k[\cos^2\varphi_k - \sin^2\varphi_k cos(4\theta_k)]
\end{aligned}$$

where

$$\Sigma_k = \sigma_0 + 2\sum_{i=1}^{k} \sigma_i$$

and the phases σ_i, φ_k, θ_k are determined by the nature of the k-th layer.

The calculation of the recoursion formula is faster than solving the differential equations.

The height of the neutrino production : 20km

Figures

5 Deformation of Neutrino Fluxes

Further Assumptions

- $\Delta m^2 = 3 \times 10^{-3} eV^2$
- $\theta_2 = 45^\circ$

Examples

Resonance Condition

$\cos z$	θ_3	$\Delta m^2/E(eV^2/GeV)$	$E(GeV)$
-0.90	6.4°	6.1×10^{-4}	$E = 5$
-0.837	8.3°	4.1×10^{-4}	$E = 7.3$
-0.60	12.7°	3.8×10^{-4}	$E = 7.9$

The Effect of the Earth Crossing

- Excess of upward-going ν_e fluxes.

Up-Down Asymmetry of Neutrino Fluxs

$$\text{definition} : A(\cos z) = 2\frac{\text{Up}(\cos z) - \text{Down}(\cos z)}{\text{Up}(\cos z) + \text{Down}(\cos z)}$$

Figures

6 Summary

- Investigation in the wide range of parameters the resonance conditions of neutrino oscillations in the earth.

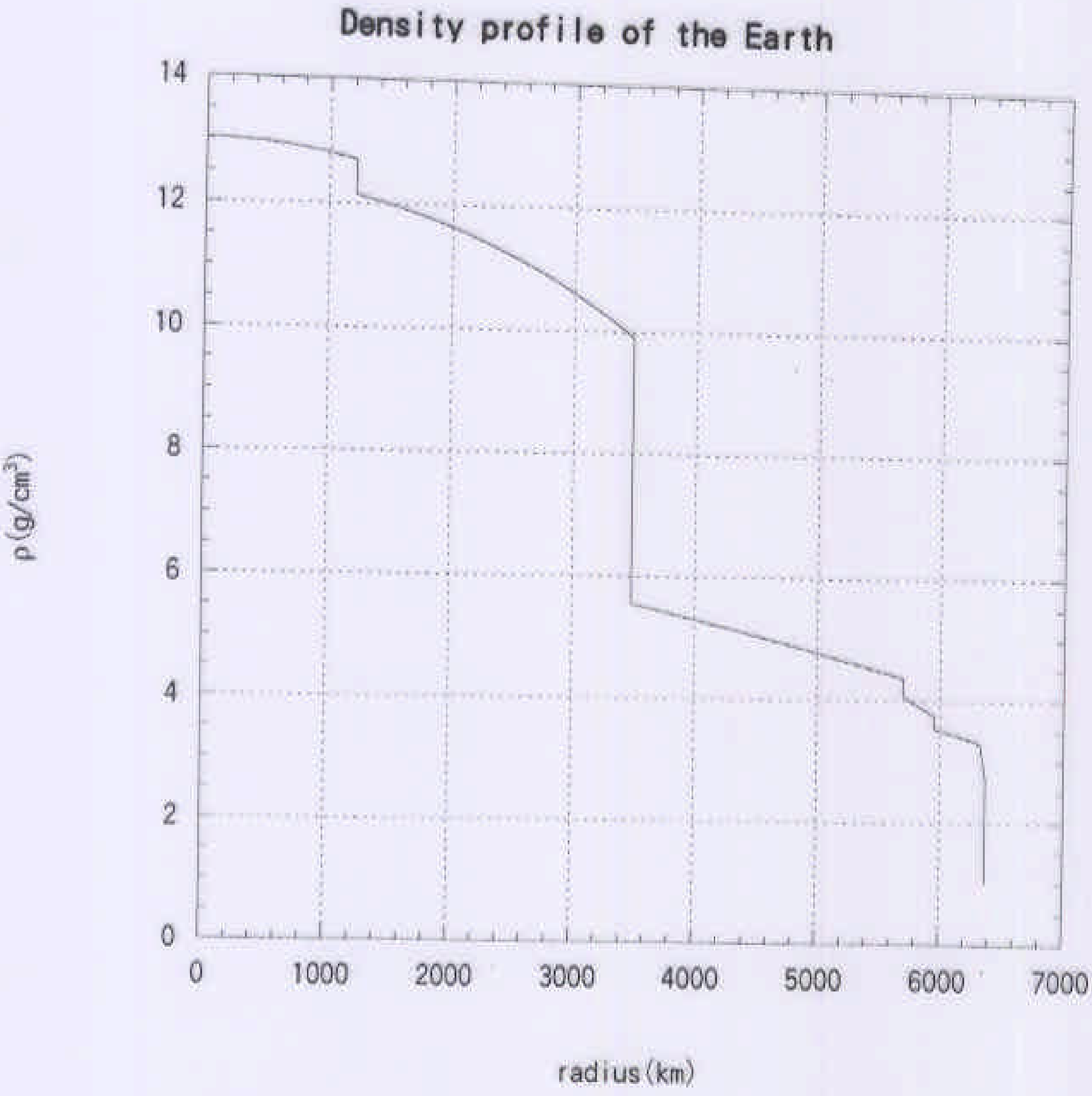
- The excess of the upward-going ν_e fluxes due to the matter enhanced oscillations.

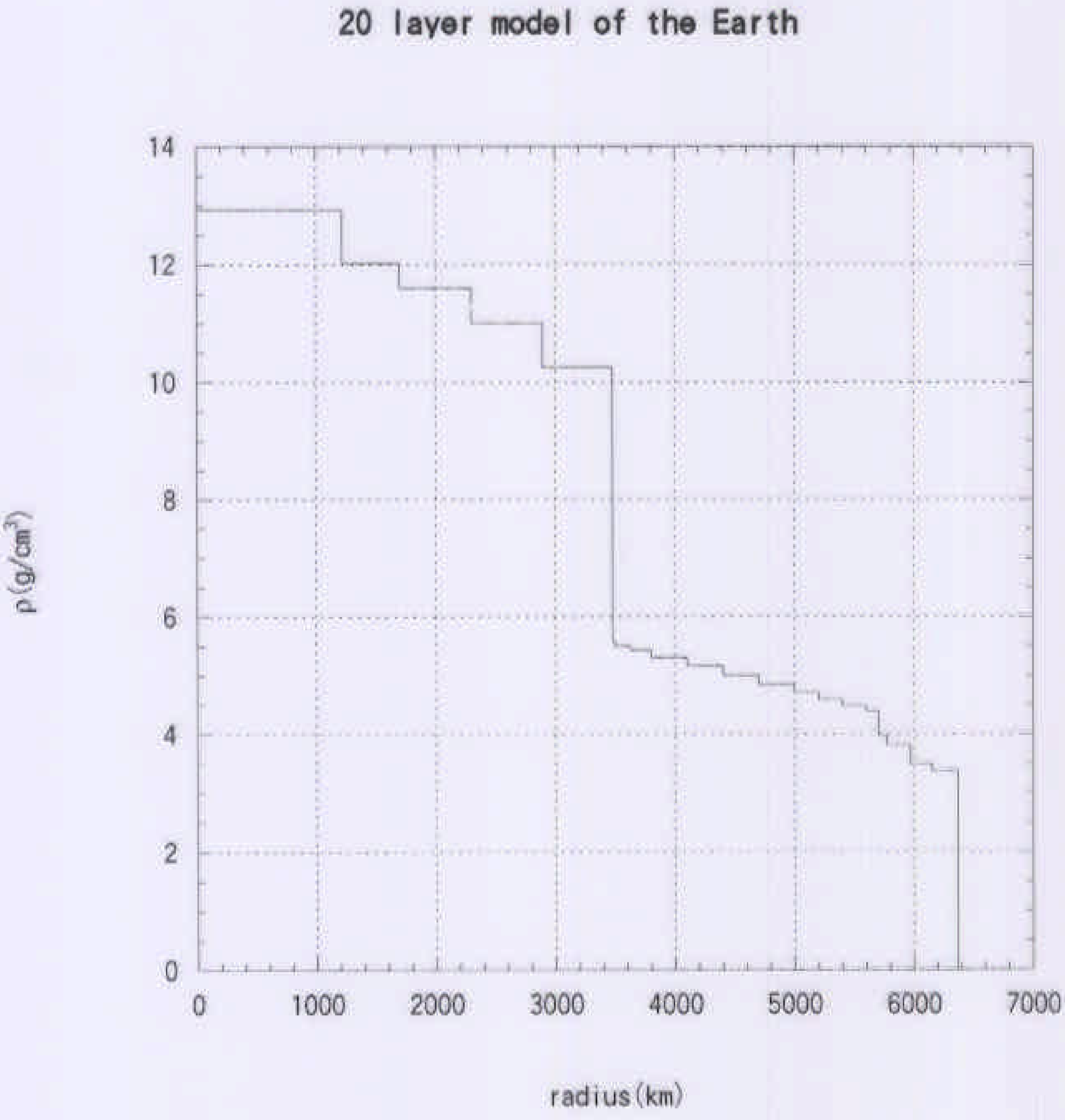
- The deficiency of the upward-going ν_μ fluxes.

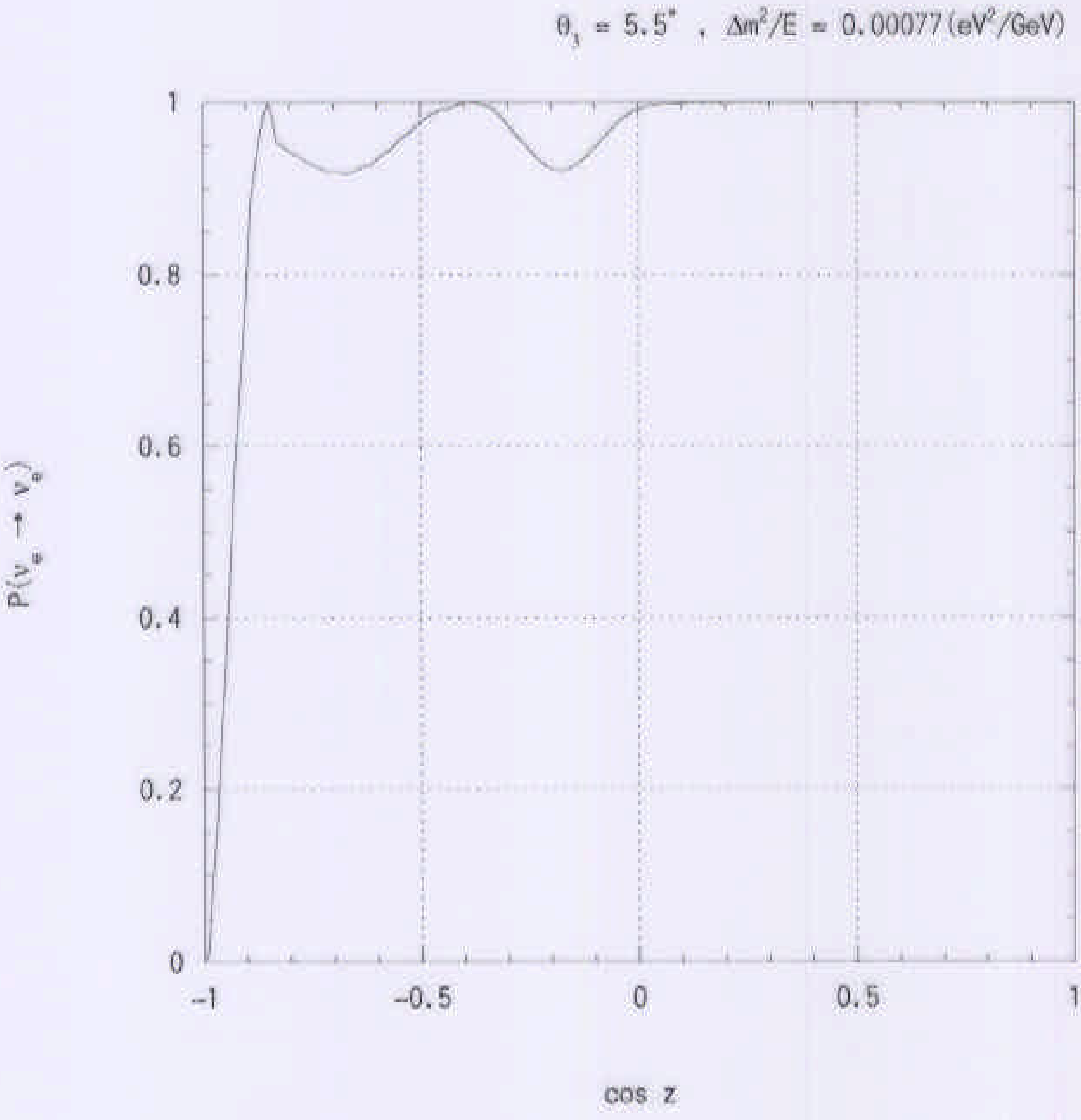
- Possiblity to determine the lepton mixing angle θ_3 in the wide range of $0° \leq \theta_3 \leq 40°$ by observing the zenith angle dependence of the atmospheric neutrinos in the energy range of $4\,GeV \lesssim E \lesssim 8\,GeV$.

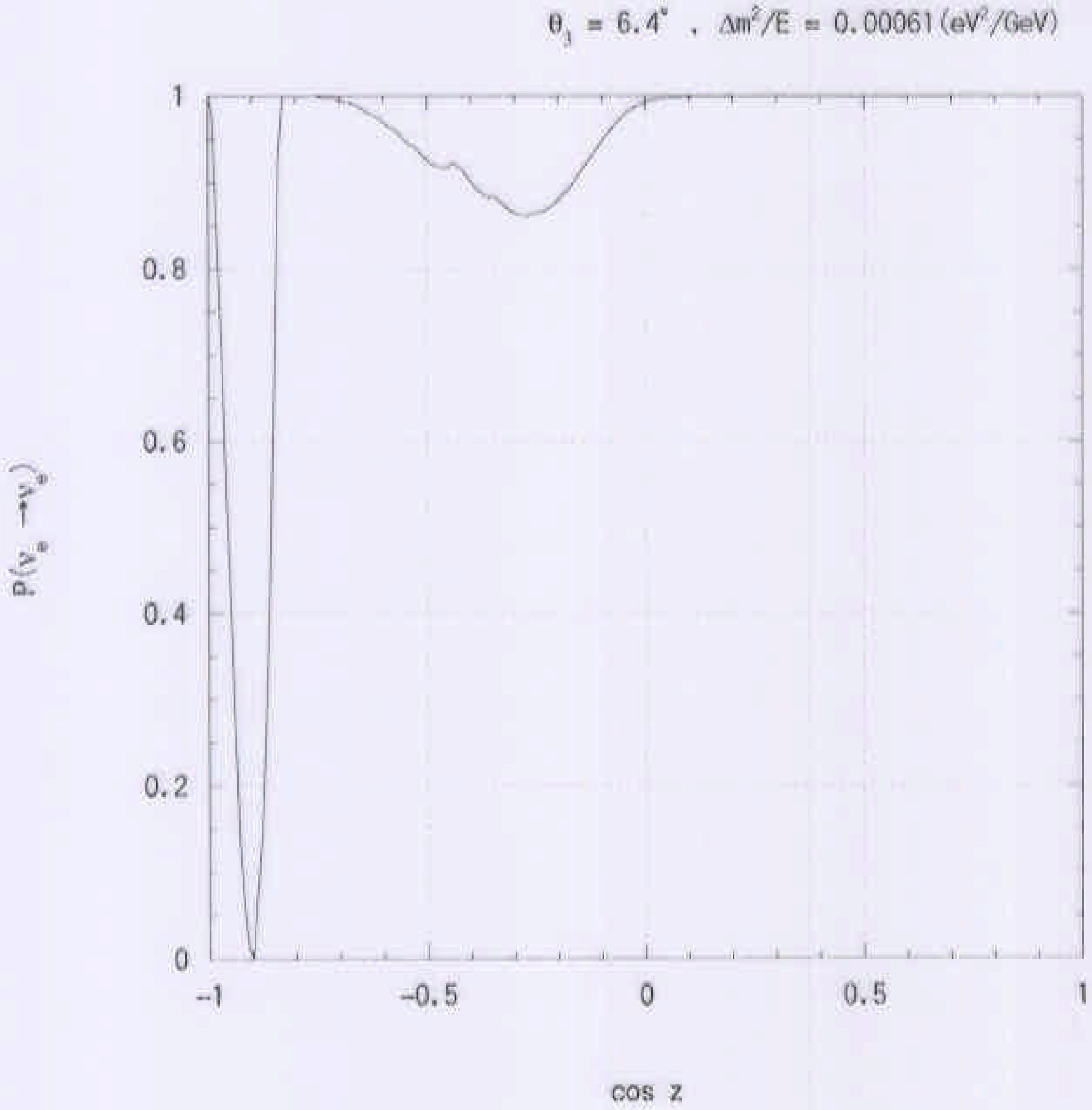
- The up-down asymmetry of $\nu_e + \bar{\nu}_e$ is nearly equal to 0.5 in the resonance region even in the domain of parameters allowed by the CHOOZ bound.

- Mega-(Hyper-, or Ultra-)Kamiokande would be the best place to study θ_3.


Density profile of the Earth
14
12
10
8
6
4
2
0
ρ(g/cm³)
0
1000
2000
3000
4000
5000
6000
7000
radius(km)

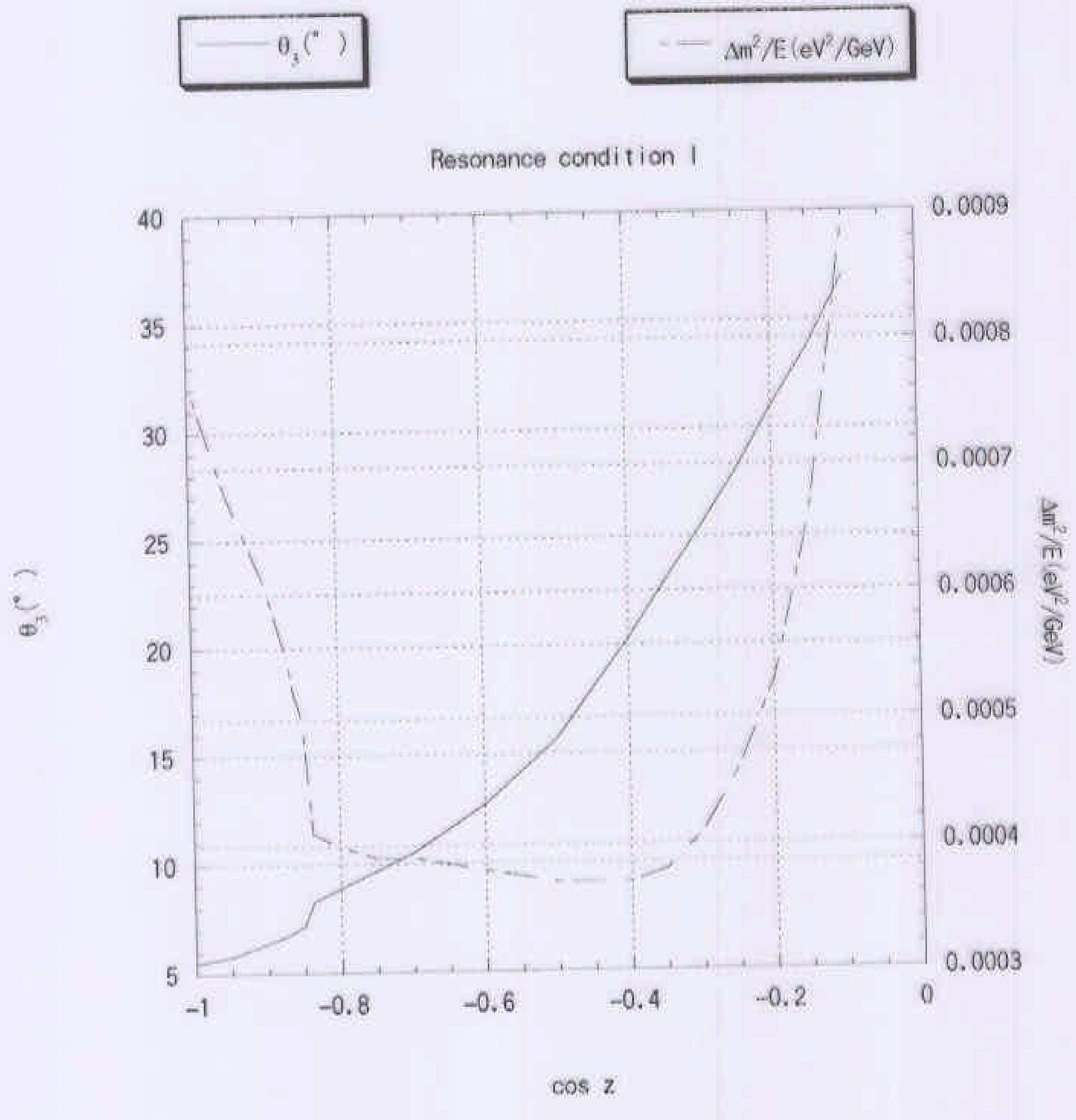

20 layer model of the Earth
14
12
10
8
6
4
2
0
0
1000
2000
3000
4000
5000
6000
7000
radius(km)
ρ(g/cm³)

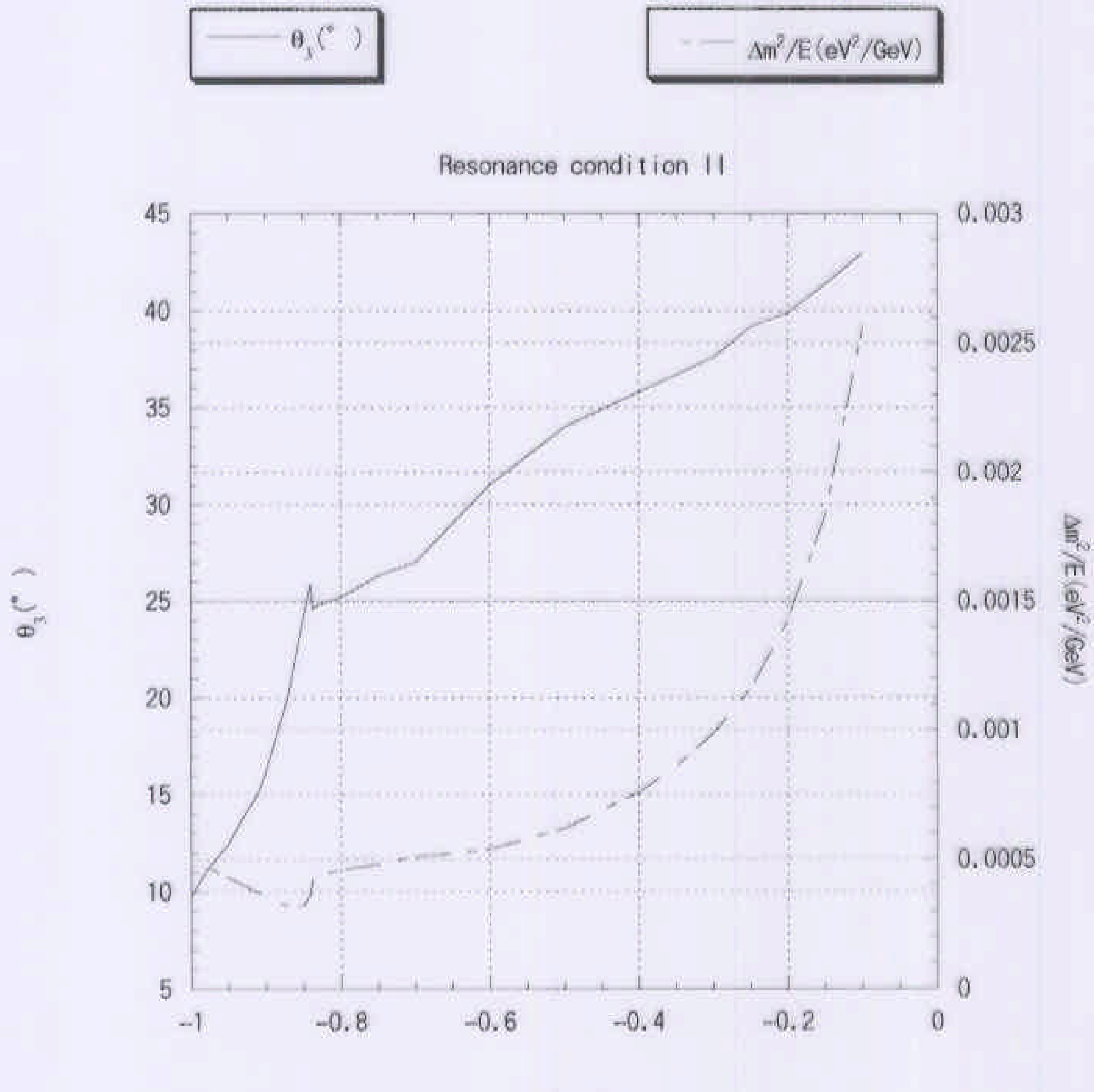

θ3 = 5.5° , Δm²/E = 0.00077 (eV²/GeV)
P(νe → νe)
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
cos z

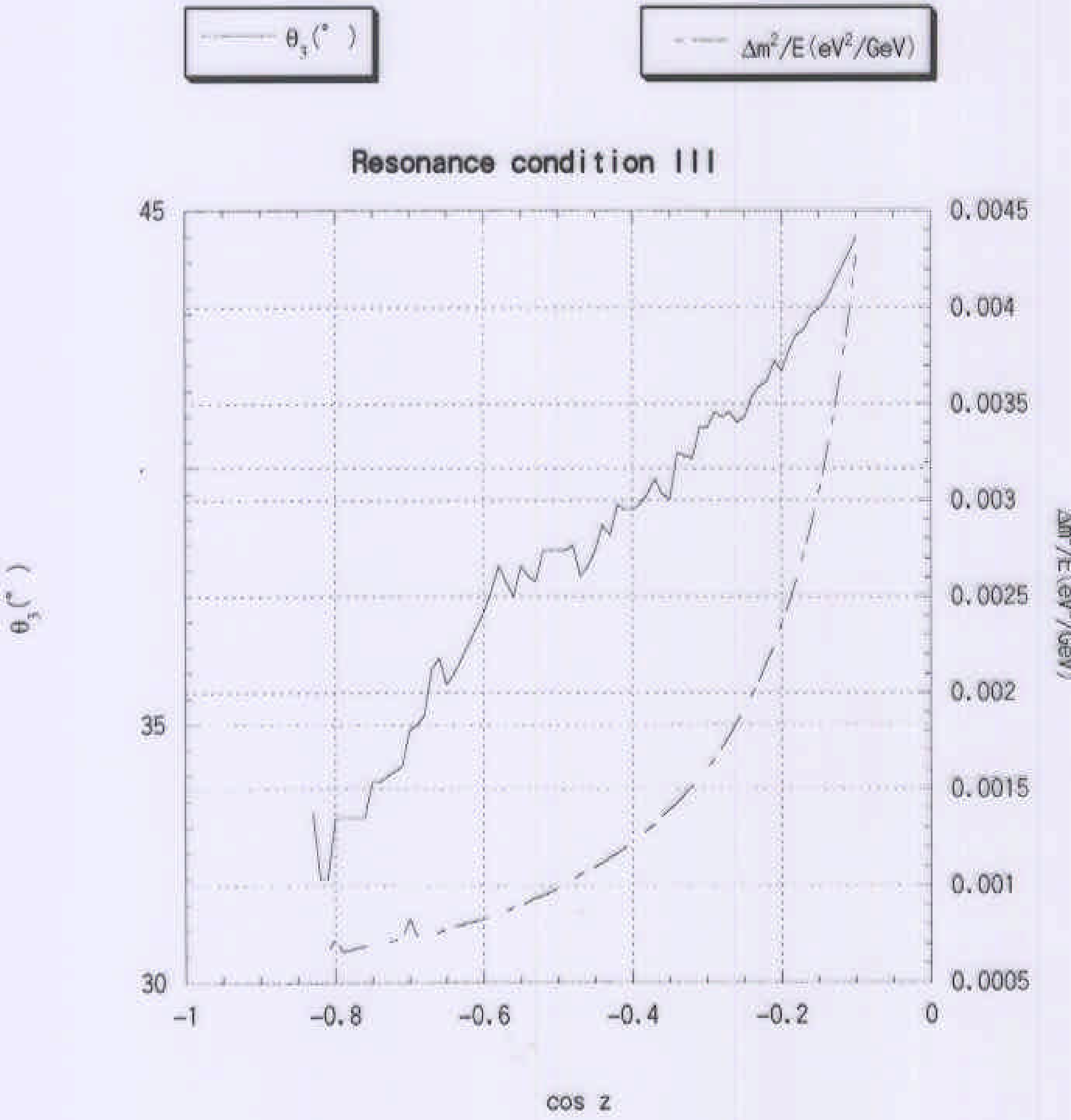


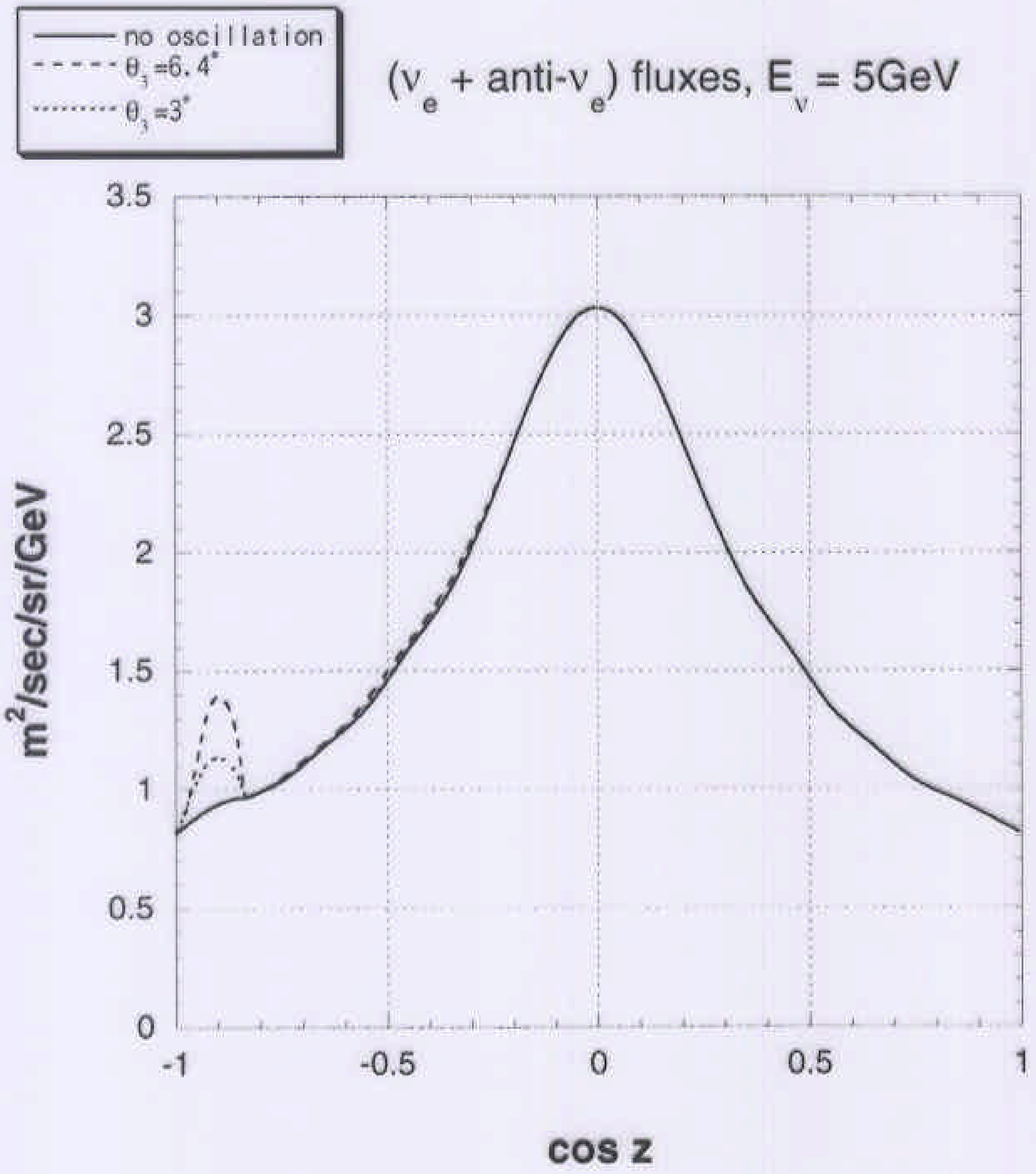
θ_3 = 6.4° , $\Delta m^2/E$ = 0.00061 (eV^2/GeV)
$P(\nu_e \to \nu_e)$
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
cos z

5

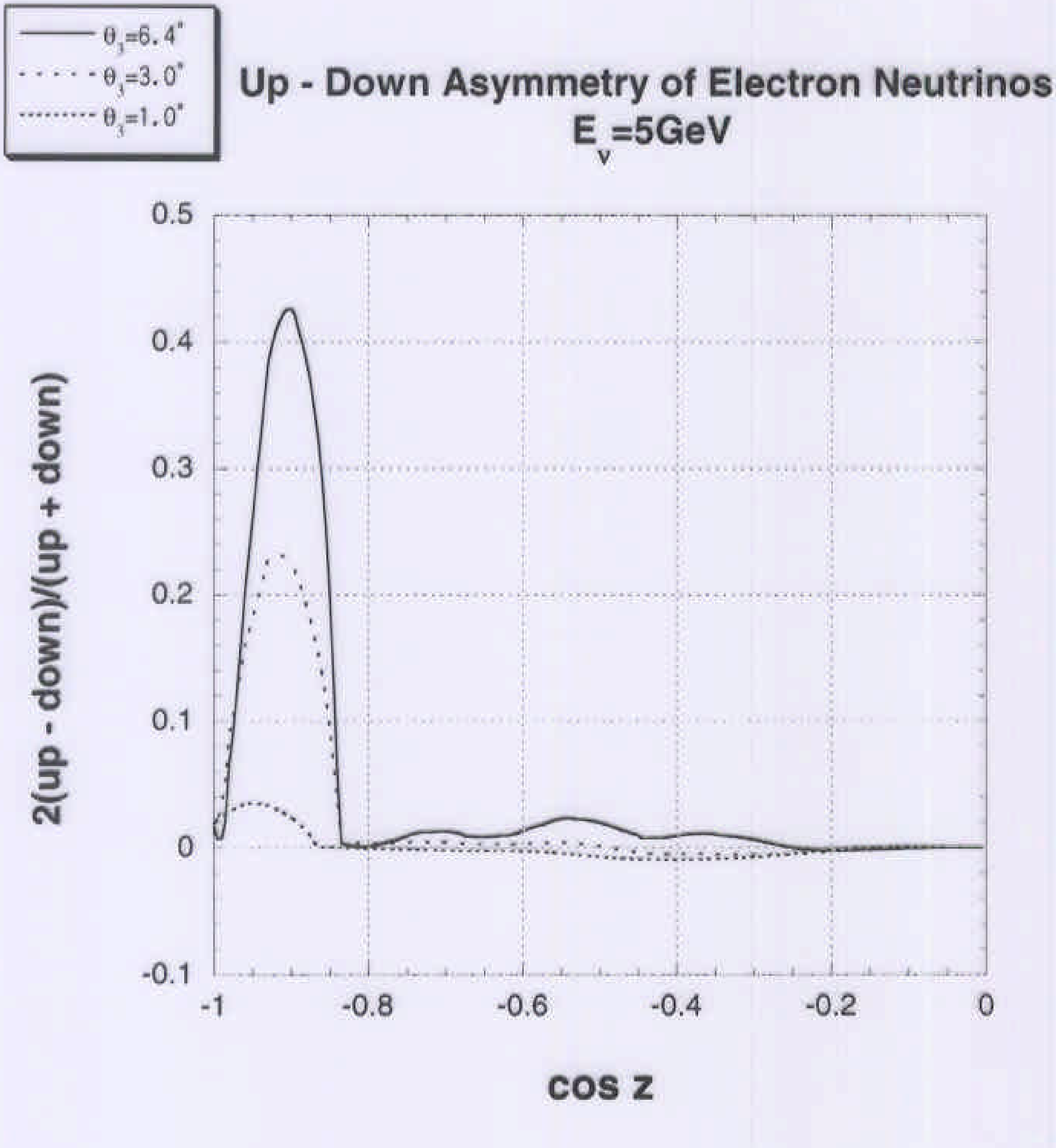

θ3(°)
Δm²/E(eV²/GeV)
Resonance condition I
40
35
30
25
20
15
10
5
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
-1
-0.8
-0.6
-0.4
-0.2
0
θ3(°)
Δm²/E(eV²/GeV)
cos z


θ3(°)
Δm²/E(eV²/GeV)
Resonance condition II
45
40
35
30
25
20
15
10
5
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
-1
-0.8
-0.6
-0.4
-0.2
0
cos z
θ3(°)
Δm²/E(eV²/GeV)

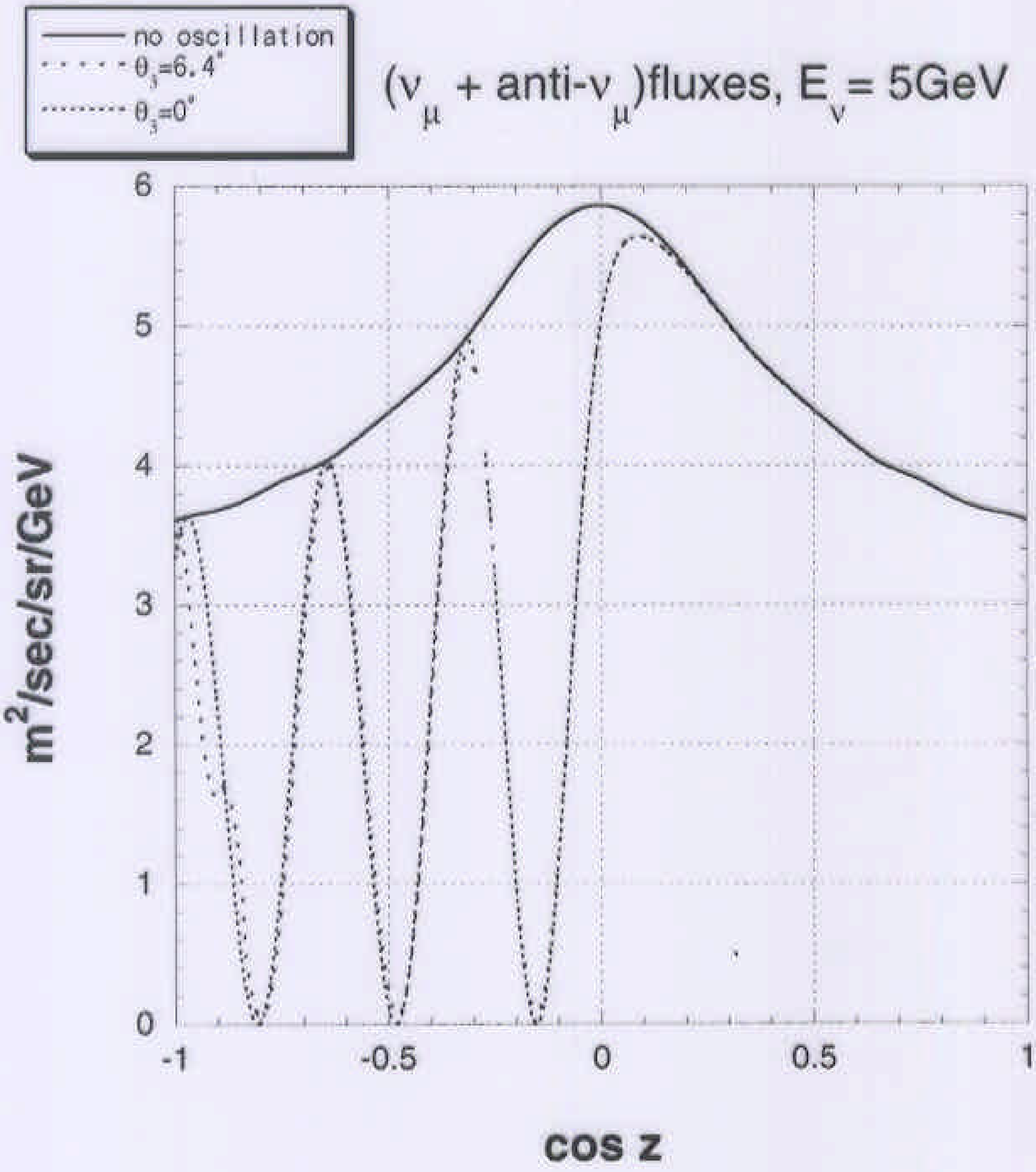

θ₃(°)
Δm²/E(eV²/GeV)
Resonance condition III
45
35
30
0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
-1
-0.8
-0.6
-0.4
-0.2
0
cos z
θ₃(°)
Δm²/E(eV²/GeV)



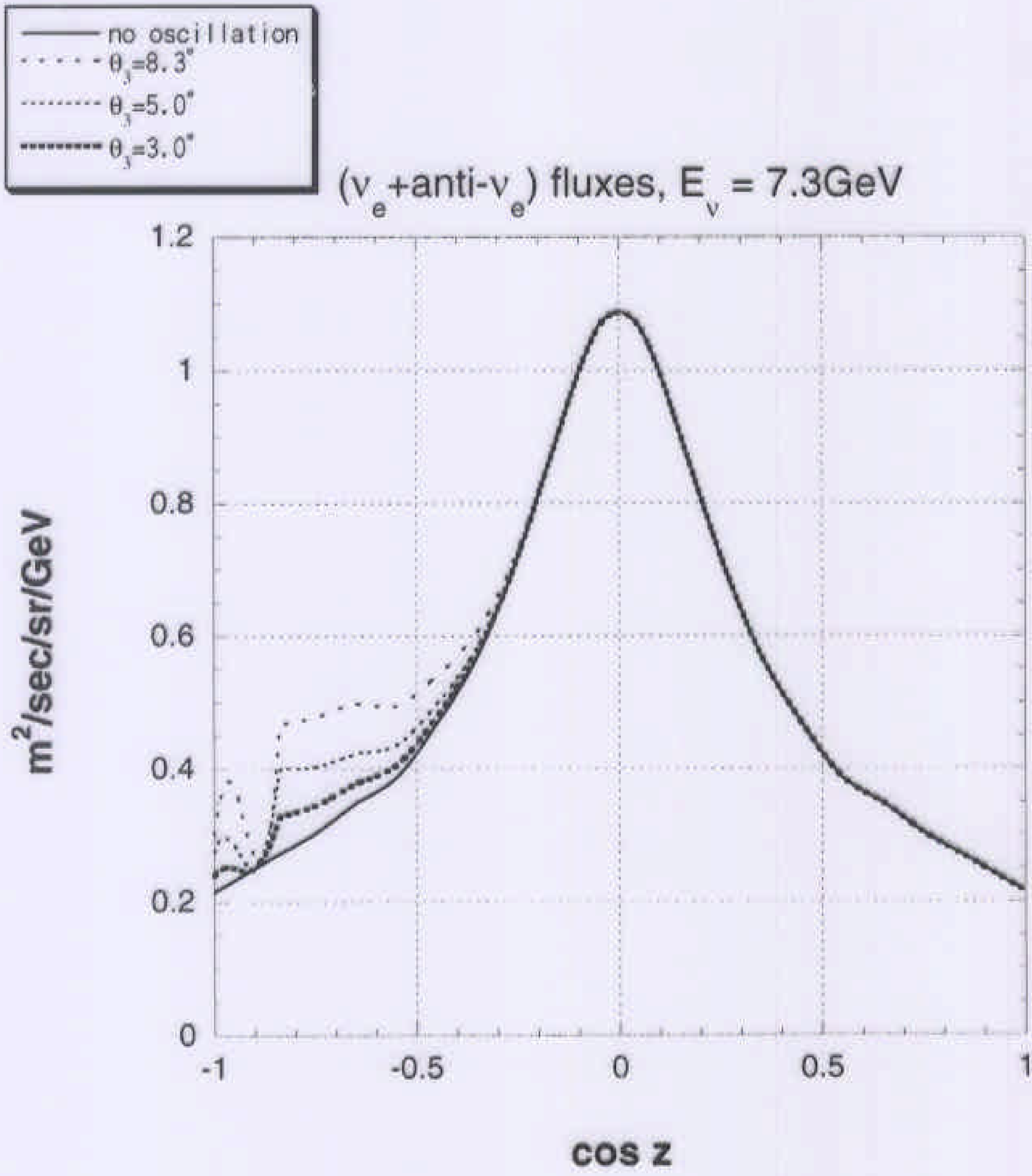
no oscillation
θ3 =6.4°
θ3 =3°
(νe + anti-νe) fluxes, Eν = 5GeV
m²/sec/sr/GeV
3.5
3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
cos z



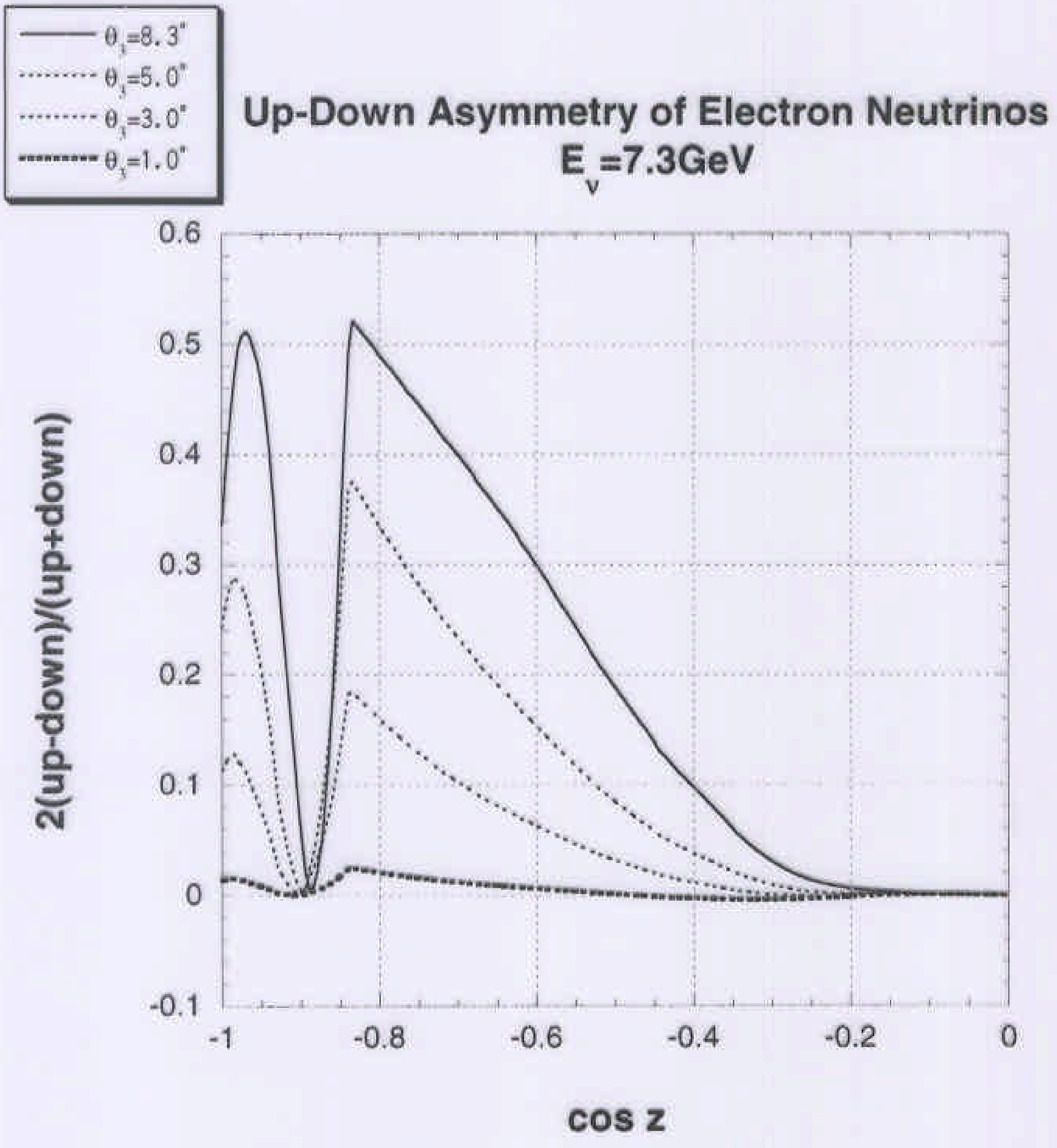
θ3=6.4°
θ3=3.0°
θ3=1.0°
Up - Down Asymmetry of Electron Neutrinos
Eν=5GeV
0.5
0.4
0.3
0.2
0.1
0
-0.1
2(up - down)/(up + down)
-1
-0.8
-0.6
-0.4
-0.2
0
cos Z

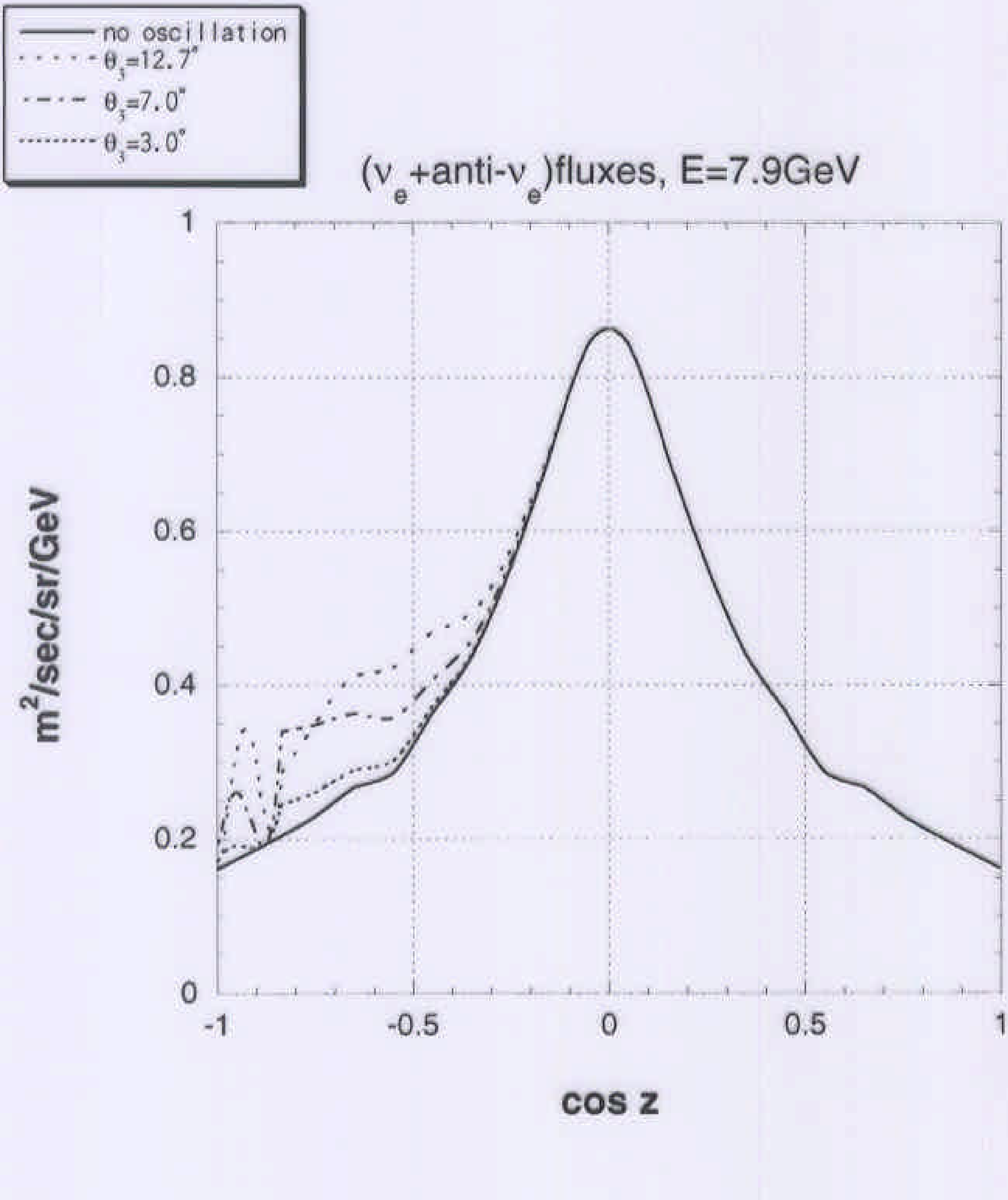


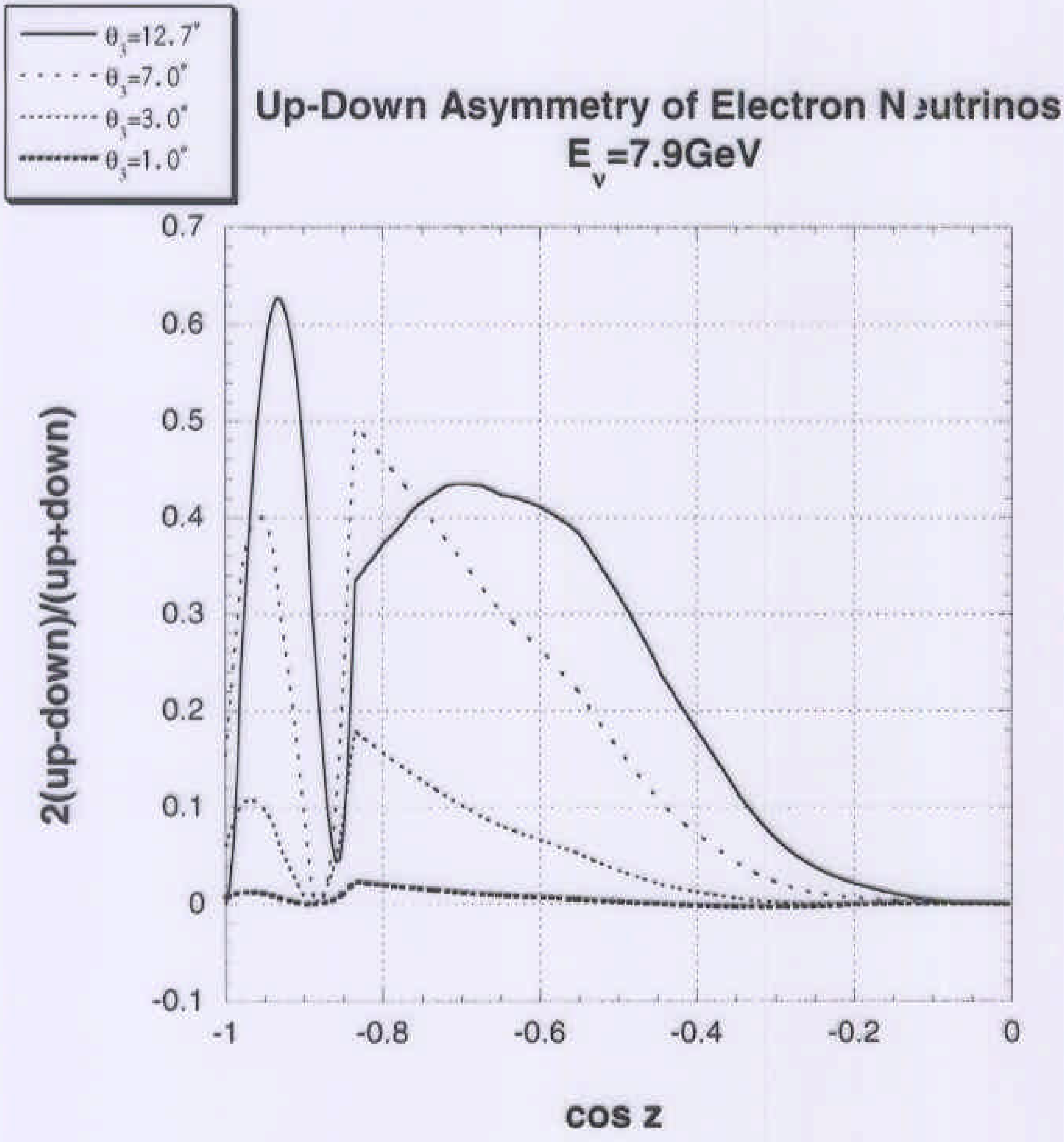
no oscillation
θ3=6.4°
θ3=0°
(νμ + anti-νμ)fluxes, Eν = 5GeV
m2/sec/sr/GeV
0
1
2
3
4
5
6
-1
-0.5
0
0.5
1
cos z



no oscillation
θ_3=8.3°
θ_3=5.0°
θ_3=3.0°
(ν_e+anti-ν_e) fluxes, E_ν = 7.3GeV
1.2
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
m^2/sec/sr/GeV
cos z


θ3=8.3°
θ3=5.0°
θ3=3.0°
θ3=1.0°
Up-Down Asymmetry of Electron Neutrinos
Eν=7.3GeV
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
2(up-down)/(up+down)
-1
-0.8
-0.6
-0.4
-0.2
0
cos z


no oscillation
θ3=12.7°
θ3=7.0°
θ3=3.0°
(νe+anti-νe)fluxes, E=7.9GeV
m2/sec/sr/GeV
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
cos z


Up-Down Asymmetry of Electron Neutrinos
E_ν=7.9GeV
θ_3=12.7°
θ_3=7.0°
θ_3=3.0°
θ_3=1.0°
2(up-down)/(up+down)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-1
-0.8
-0.6
-0.4
-0.2
0
cos z