

Kaons ,  $K_L$   $K_S$   $K^\pm$

CP Violation :  $\eta$

$$\left(\frac{\epsilon'}{\epsilon}\right)_{2\pi}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu} \quad (\pi^0 e^+ e^-, \pi^0 \mu^+ \mu^-)$$

$$K_L \rightarrow \pi^+ \pi^- e^+ e^-$$

$\rho$  :

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \mu^+ \mu^- \gamma \quad [\pi^0 \text{ factory}]$$

$\chi$  :

$$K_L \rightarrow \pi^0 \mu^\pm e^\mp$$

$$K^+ \rightarrow \pi^+ \mu^+ e^-$$

$$K_L \rightarrow \mu^\pm e^\mp$$

"EM" decays of neutral kaons:

02

$$K_L \rightarrow \gamma^* \gamma \quad \text{and} \quad K_L \rightarrow \gamma^* \gamma^*$$

$$K_L \rightarrow e^+ e^- \gamma$$

$$K_L \rightarrow \mu^+ \mu^- \gamma$$

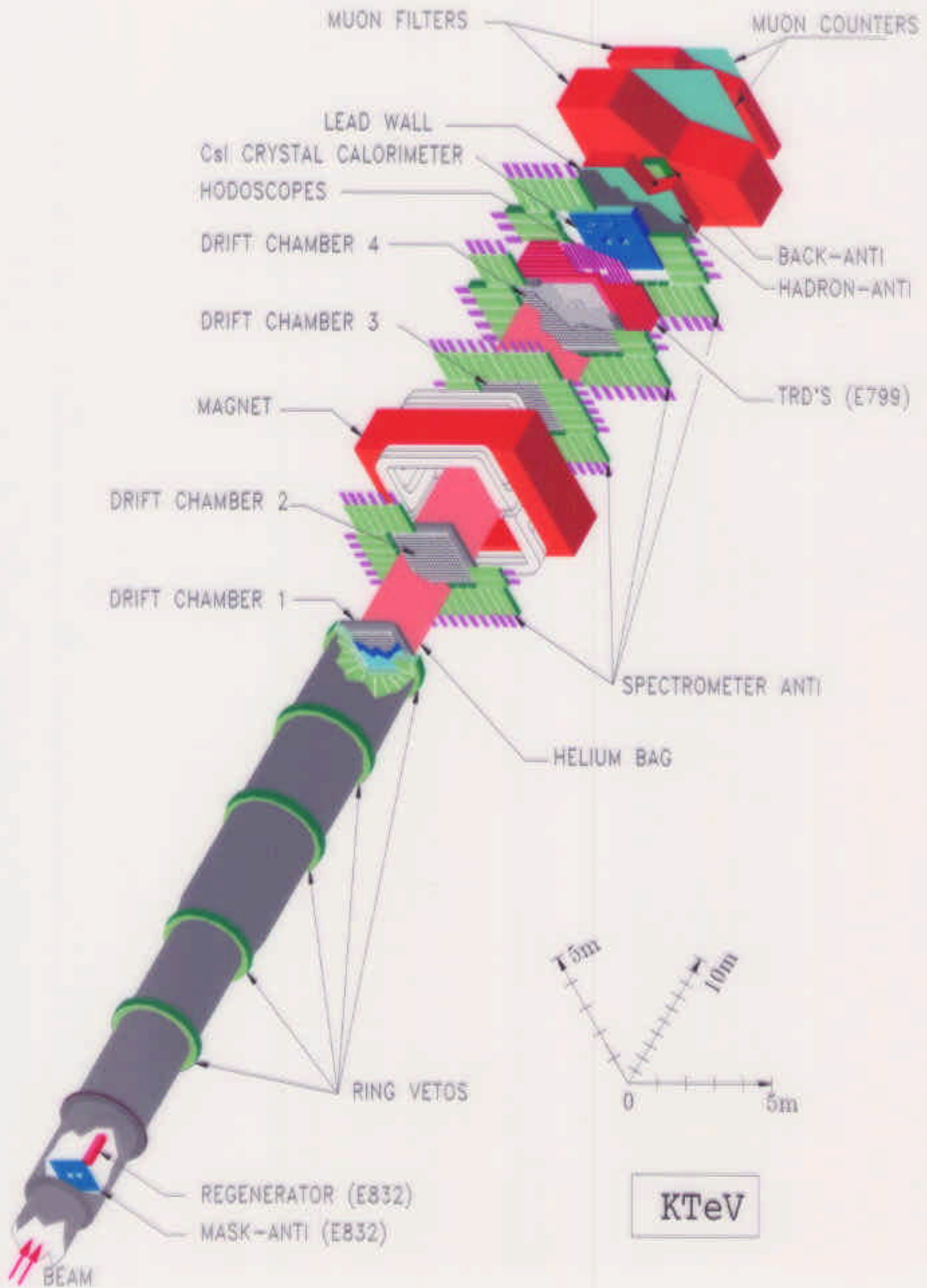
$$K_L \rightarrow e^+ e^- e^+ e^-$$

$$K_L \rightarrow \mu^+ \mu^- e^+ e^-$$

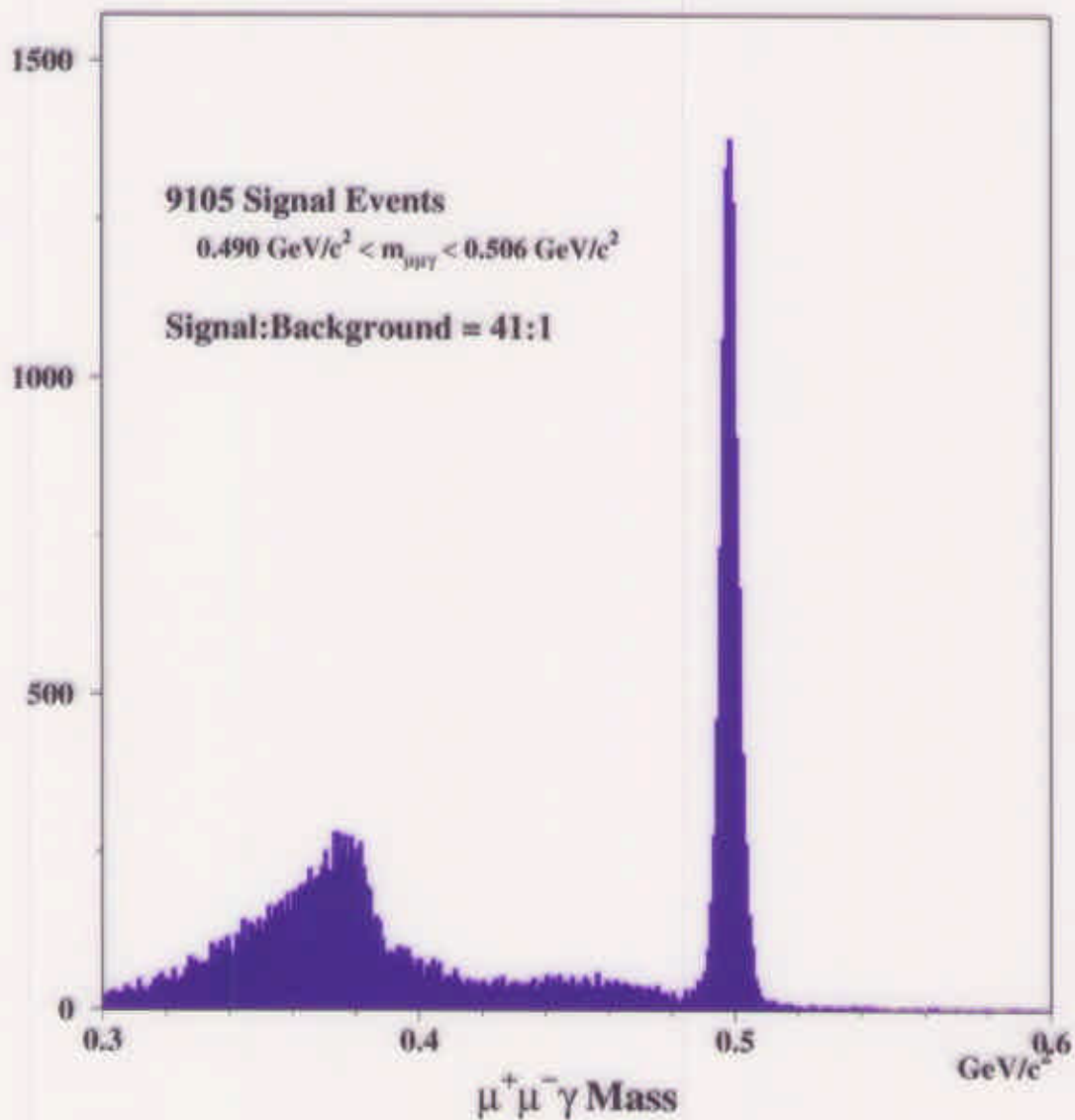
$$K_L \rightarrow \mu^+ \mu^+ \mu^- \mu^-$$

# KTeV Collaboration

- Arizona
- UCLA
- UCSD
- CHICAGO
- Colorado
- Elmhurst
- FNAL
- Osaka
- Rice
- Rutgers
- Virginia
- Wisconsin
- Campinas
- Sao Paulo



# $K_L \rightarrow \mu^+ \mu^- \gamma$ Branching Ratio Measurement



$$BR(K_L \rightarrow \mu^+ \mu^- \gamma) = (3.70 \pm 0.04_{stat} \pm 0.07_{sys}) \times 10^{-7}$$

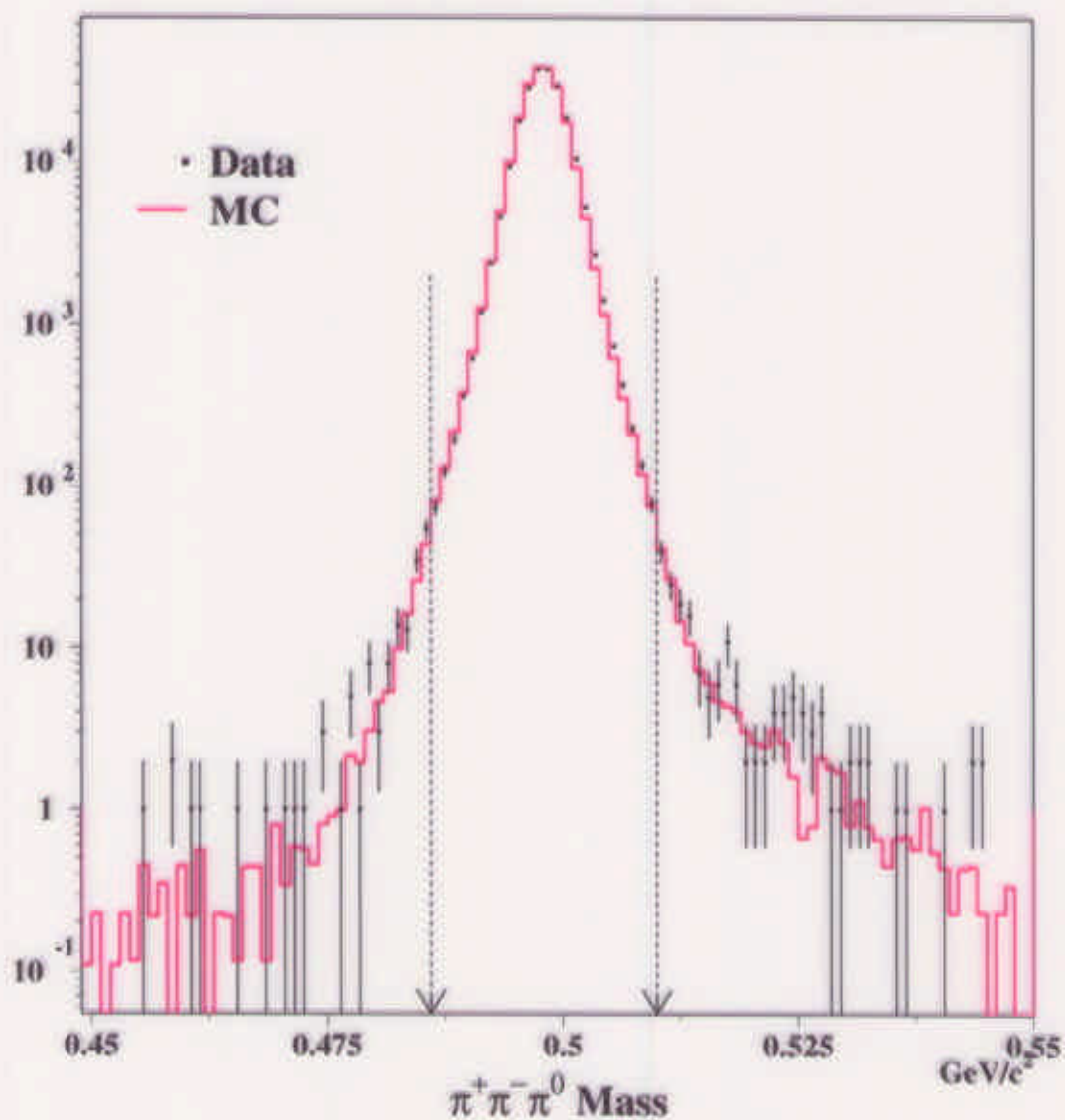
$$BR_{winter} = 3.69 \times 10^{-7}$$

$$BR_{summer} = 3.71 \times 10^{-7}$$



## $\pi^+\pi^-\pi^0$ Normalization Sample

Background-free normalization sample of 210,660  
 $K_L \rightarrow \pi^+\pi^-\pi^0$  events



## Backgrounds to $K_L \rightarrow \mu^+ \mu^- \gamma$

All backgrounds involve charged pions, and require the pions to decay or punch-through the filter steel. All MC background samples except  $K_{\mu 3}$ 's are scaled to the measured flux.

- $K_L \rightarrow \pi^+ \pi^- \pi^0$ 
  - Generally very low mass due to lost photon, but has large BR
- $K_L \rightarrow \pi^+ \pi^- \gamma$ 
  - Includes Direct Emission and Inner Brem
  - Near in mass, but low BR
- $K_L \rightarrow \pi^+ \pi^-$ 
  - Includes External Brem, and accidental photons

## Backgrounds to $K_L \rightarrow \mu^+ \mu^- \gamma$

- $K_L \rightarrow \pi^\pm \mu^\mp \nu$ 
  - Requires an accidental photon  $\rightarrow$  acceptance harder to understand
  - MC is normalized to sidebands in the data
  - Largest background
- $K_L \rightarrow \pi^\pm \mu^\mp \nu \gamma$ 
  - Recently measured by NA48
  - MC uses  $K_{e3}$  matrix element, with  $m_\mu$  and  $K_{\mu 3}$  form factor parameters
  - MC is normalized to sidebands in the data
- $K_L \rightarrow \pi^0 \pi^\pm \mu^\mp \nu$ 
  - No existing measurement of this mode
  - BR estimated from  $K_L \rightarrow \pi^0 \pi^\pm e^\mp \nu$  and similar  $K^+$  modes
  - Negligible



# $\mu^+ \mu^- \gamma$ Background Components

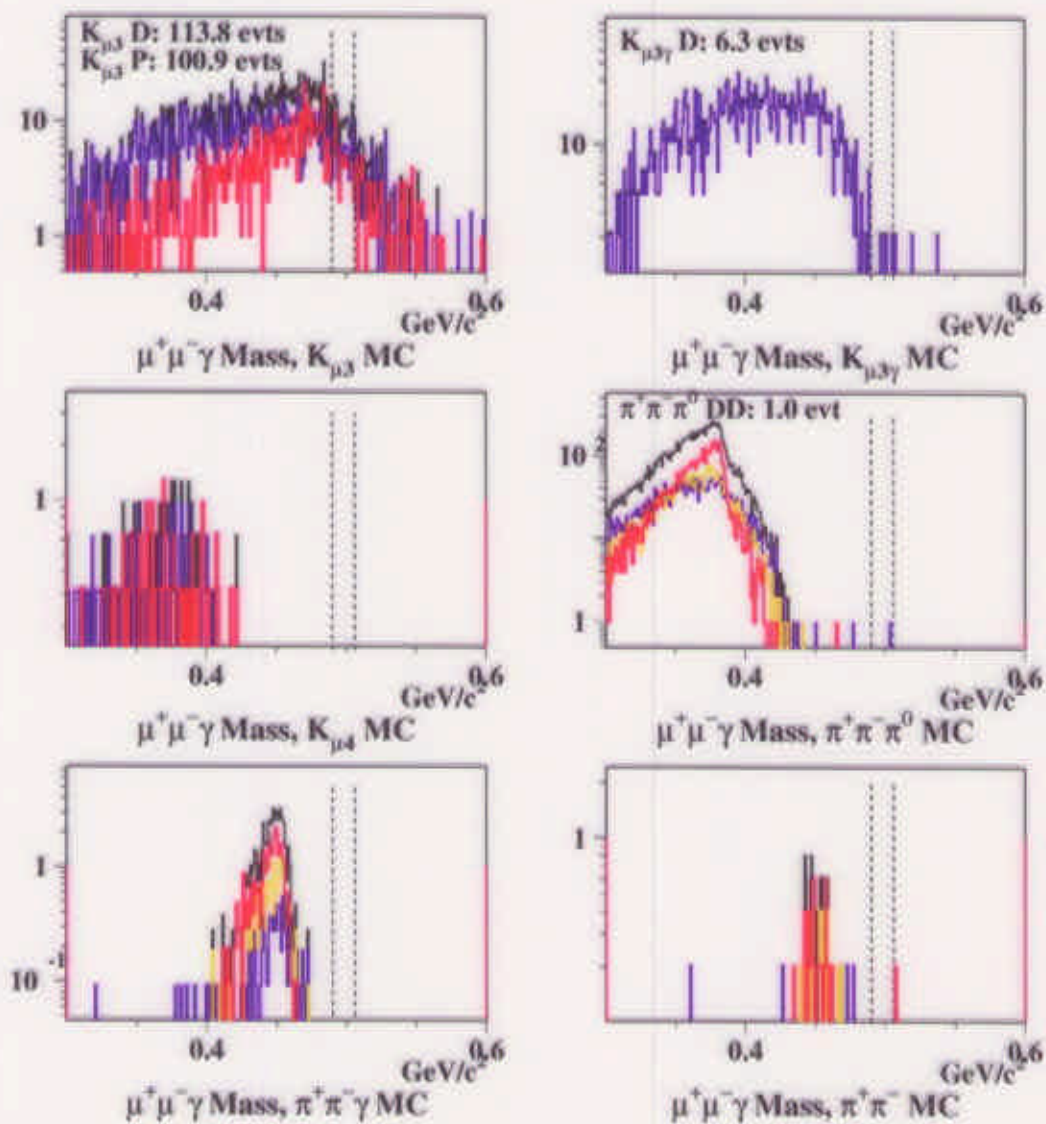
All components are normalized to the KTeV data set

**Blue:**  $\pi$  decay

**Red:**  $\pi$  punch-through

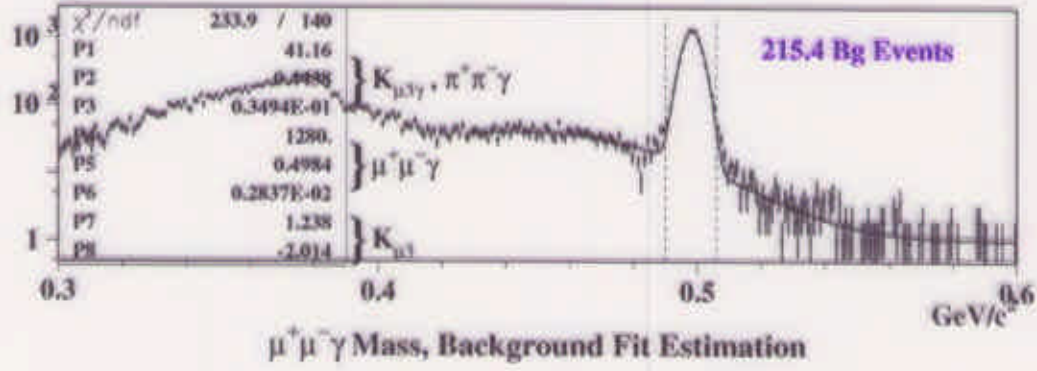
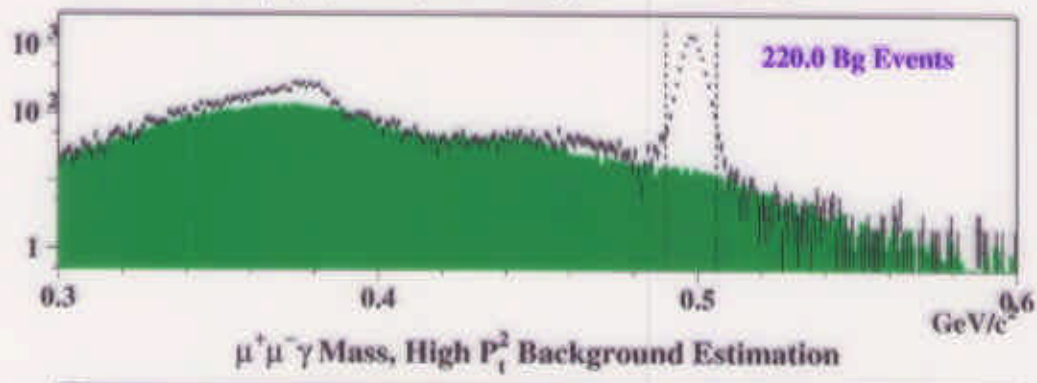
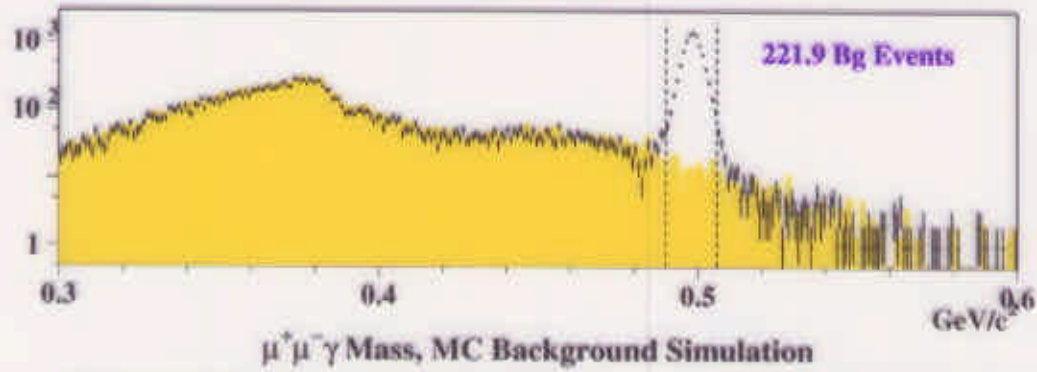
**Yellow:** one  $\pi$  decay, one  $\pi$  punch-through

**Black:** total



# Estimation of Background to $K_L \rightarrow \mu^+ \mu^- \gamma$

- MC is normalized to data
- $0.0004(\text{GeV}/c)^2 < P_t^2 < 0.001(\text{GeV}/c)^2$  data is scaled to signal region
- Data is fit to two gaussians and an exponential

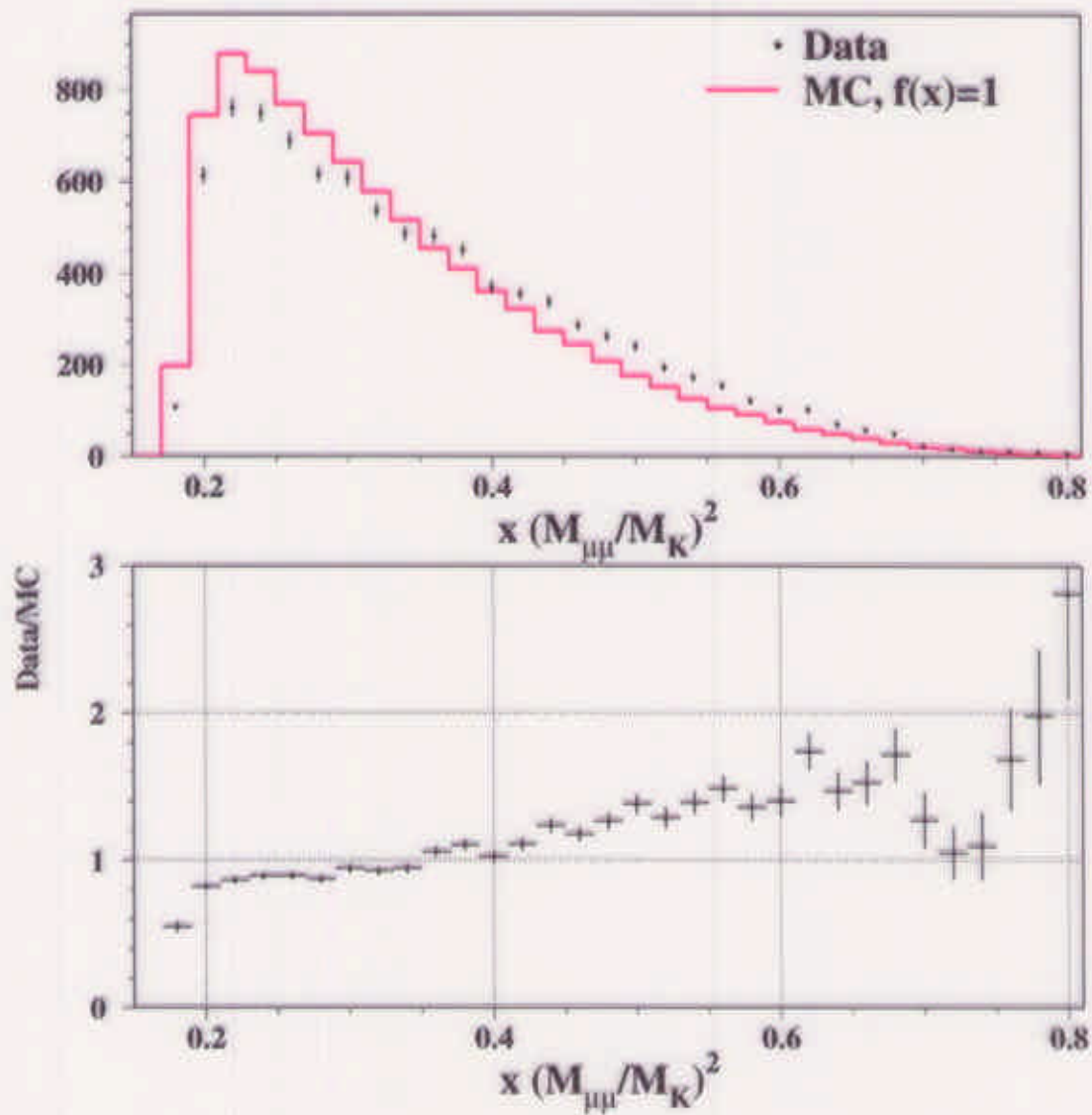


## $K_L \rightarrow \mu^+ \mu^- \gamma$ Systematic Errors

Source of Error	BR Error
MU3 Crack Simulation	0.5%
$\mu$ Filter Thickness	0.4%
DC Simulation (MAPS,Illum,Z)	0.5%
$\gamma$ Energy Uncertainty	0.1%
$P_t^2$ Cut	0.6%
Track Momentum Cut	0.2%
Background Uncertainty	0.2%
MC Statistics	0.2%
MC form factor	0.4%
Total from internal syst. err.	1.1%
$BR(K_L \rightarrow \pi^+ \pi^- \pi^0)$	1.6%
Total Systematic Error	1.9%

## Dimuon Mass Distribution

- The Data/MC ratio is the measured value of the form factor,  $|f(x)|^2$
- Completely model-independent





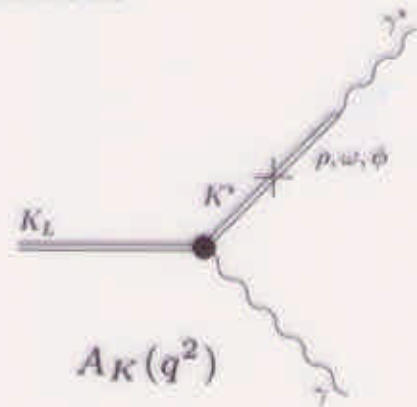
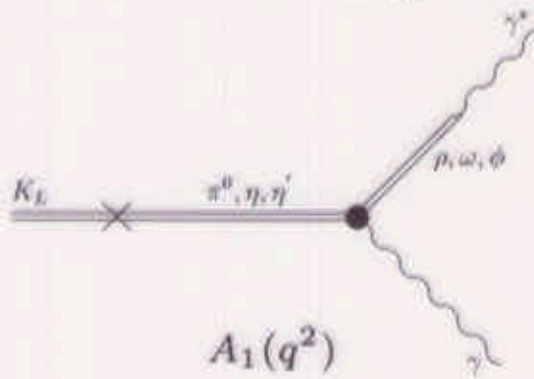
$$K_L \rightarrow l^+ l^- \gamma$$

$$K_L \rightarrow \gamma^* \gamma \rightarrow l^+ l^- \gamma$$

$$\frac{1}{\Gamma_{\gamma\gamma}} \frac{d\Gamma(K_L \rightarrow l^+ l^- \gamma)}{d(q^2)} = \frac{2\alpha_{EM}}{3\pi} \frac{|f(q^2)|^2}{q^2} \left(1 - \frac{q^2}{m_K^2}\right)^3 \left(1 + \frac{2m_l^2}{q^2}\right) \times \left(1 - \frac{4m_l^2}{q^2}\right)^{\frac{1}{2}}$$

### Bergström, Massó, Singer

2 long distance contributions



$$f(q^2) \propto A_1(q^2) + \alpha_{K^*} A_K(q^2)$$



# Measurement of $\alpha_{K^*}$

## Branching Ratio

Integrating the differential decay rate with the BMS form factor

$$f(x) = \frac{1}{1 - 0.418x} + \frac{2.3\alpha_{K^*}}{1 - 0.311x} \left( \frac{4}{3} - \frac{1}{1 - 0.418x} - \frac{1}{9(1 - 0.405x)} - \frac{2}{9(1 - 0.238x)} \right)$$

yields the expression

$$\frac{BR(K_L \rightarrow \mu^+ \mu^- \gamma)}{BR(K_L \rightarrow \gamma\gamma)} = (14.33\alpha_{K^*}^2 - 51.91\alpha_{K^*} + 55.52) \times 10^{-5}$$

With our measurement of  $BR(K_L \rightarrow \mu^+ \mu^- \gamma)$  and the PDG value for  $BR(K_L \rightarrow \gamma\gamma)$ ,  $\alpha_{K^*}$  is calculated to be

$$\alpha_{K^*}^{BR} = -0.130^{+0.038}_{-0.037}$$

## Dimuon Mass Shape

An additional systematic error must be added to the shape analysis result of  $\alpha_{K^*} = -0.193 \pm 0.035$ .  $\alpha_{K^*}$  varies slightly with the cut on track momentum. Fluctuations are all more negative and the largest is -0.034. With this systematic error included,

$$\alpha_{K^*}^{Shape} = -0.193^{+0.035}_{-0.049}$$

## Combined

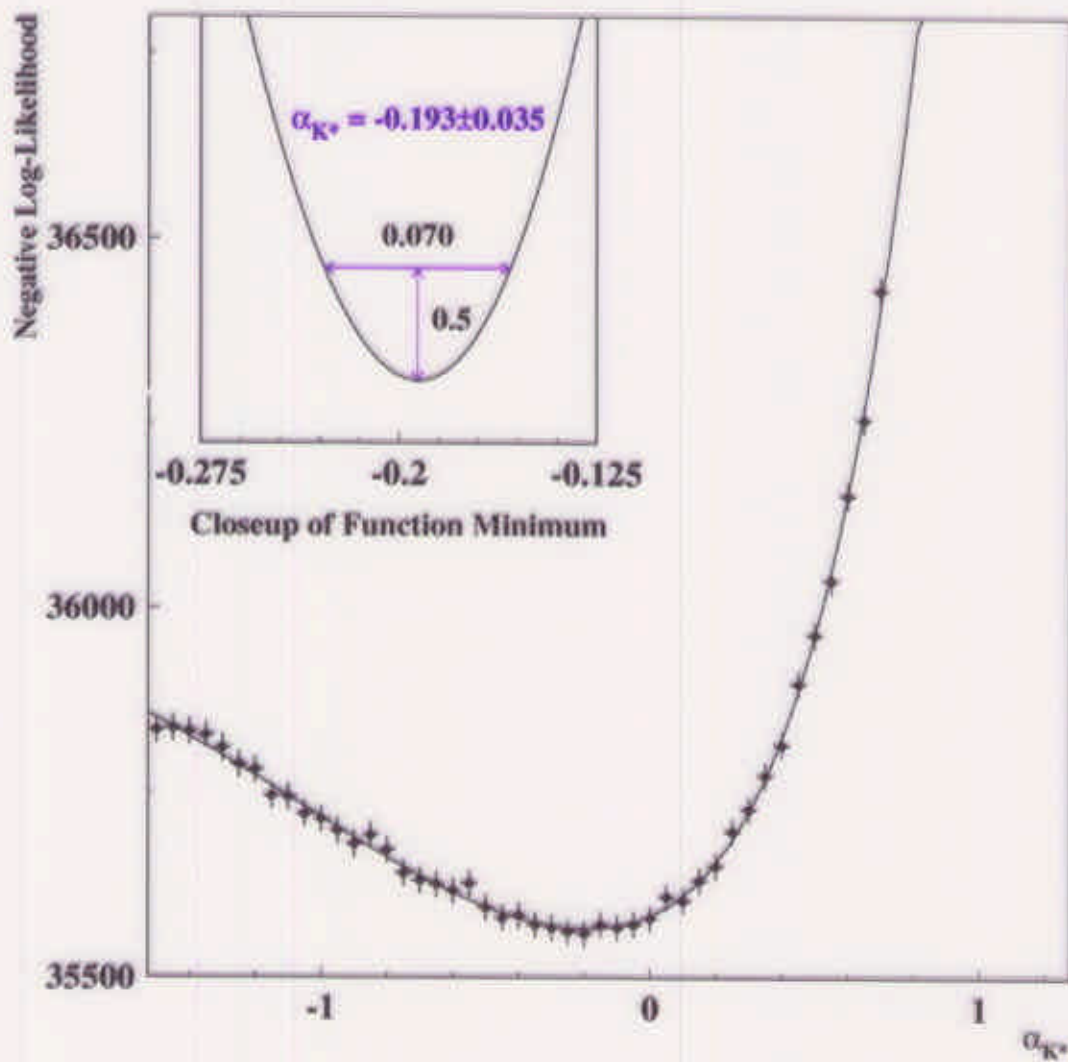
Combining these two measurements we find

$$\alpha_{K^*} = -0.163^{+0.026}_{-0.027}$$

## Measurement of $\alpha_{K^*}$

$\alpha_{K^*}$  is determined by calculating the NLL of the data being consistent with MC distributions generated at various values of  $\alpha_{K^*}$ .

10000 MC 'Data' sets are generated with the measured  $\alpha_{K^*}$ . The NLL method is used to measure  $\alpha_{K^*}$  for each of these sets, and the width of that distribution is a check on the error.



$$K_L \rightarrow l^+ l^- \gamma$$

### D'Ambrosio, Isidori, Portolés

$K_L \rightarrow \gamma^* \gamma^*$  form factor:

$$f(q_1^2, q_2^2) = 1 + \alpha \left( \frac{q_1^2}{q_1^2 - m_\rho^2} + \frac{q_2^2}{q_2^2 - m_\rho^2} \right) + \beta \frac{q_1^2 q_2^2}{(q_1^2 - m_\rho^2)(q_2^2 - m_\rho^2)}$$

- Most general parameterization compatible with a Chiral Perturbation Theory expansion
- Includes characteristic long distance pole behavior
- No specific diagrams of long distance processes are assumed
- Reduces to  $K_L \rightarrow \gamma^* \gamma$  form factor by setting  $q_2^2 = 0$

Dispersive long distance contribution is given by

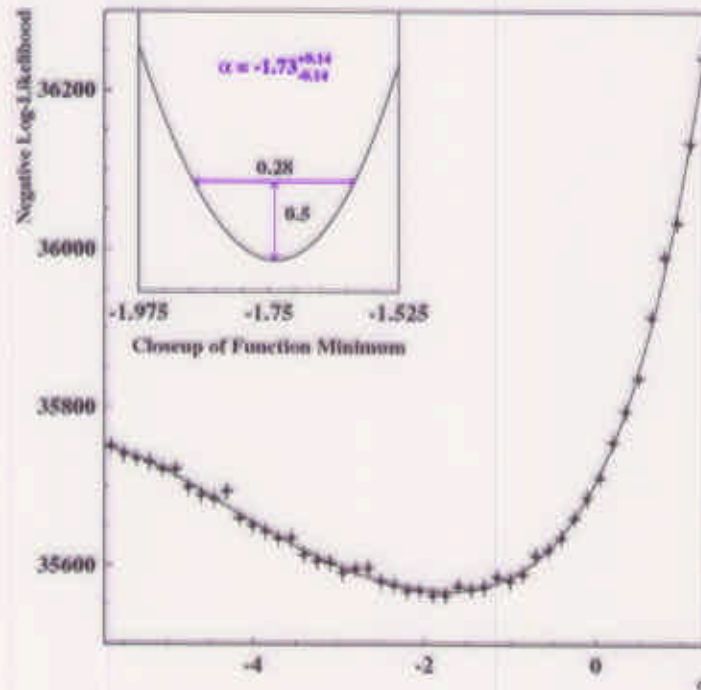
$$|\text{Re}A_{LD}| = \left[ \frac{2\alpha_{EM} \beta_\mu BR(K_L \rightarrow \gamma\gamma)}{\pi^2 m_K^2} \right]^{1/2} \times \left| 5.25 + 3.47\alpha + 3(1 + 2\alpha + \beta) \ln \frac{\Lambda}{m_\beta} \right|$$

where  $\Lambda$  is an ultraviolet cutoff, and perturbative QCD limits the last term to

$$|(1 + 2\alpha + \beta)| \ln \frac{\Lambda}{m_\beta} < 0.4$$

$\alpha$  is directly accessible through  $K_L \rightarrow l^+ l^- \gamma$ .

## Measurement of $\alpha$



### Dimuon Mass Shape

With the track momentum cut systematic error included,

$$\alpha^{Shape} = -1.73^{+0.14}_{-0.18}$$

### Branching Ratio

Integrating the differential decay rate with the DIP form factor

$$f(x) = 1 + \alpha \left( \frac{x}{x - 2.40} \right),$$

and using our measurement of  $BR(K_L \rightarrow \mu^+ \mu^- \gamma)$  and the PDG value for  $BR(K_L \rightarrow \gamma \gamma)$ ,  $\alpha$  is calculated to be

$$\alpha^{BR} = -1.42 \pm 0.12.$$

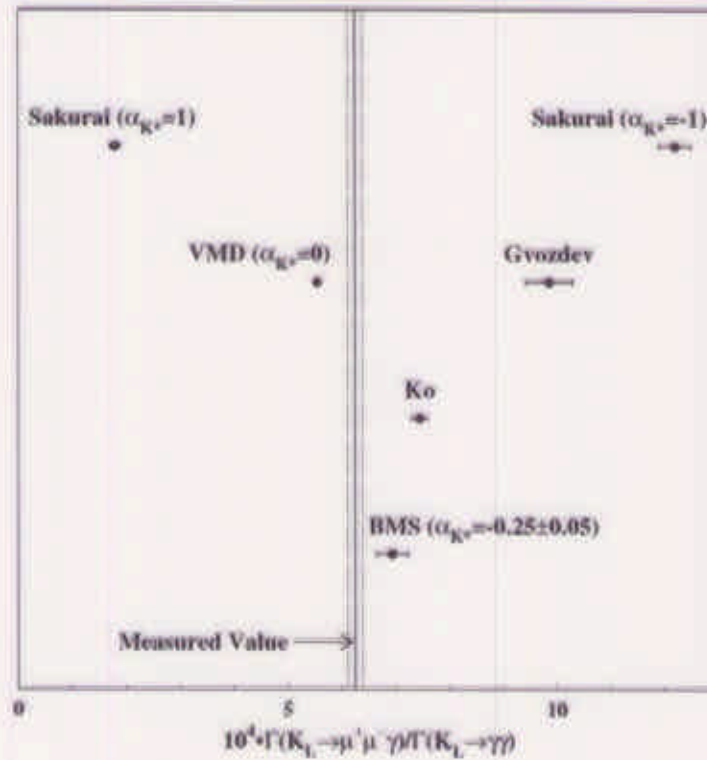
### Combined

Combining these two measurements we find

$$\alpha = -1.55 \pm 0.09$$



# Model Comparisons



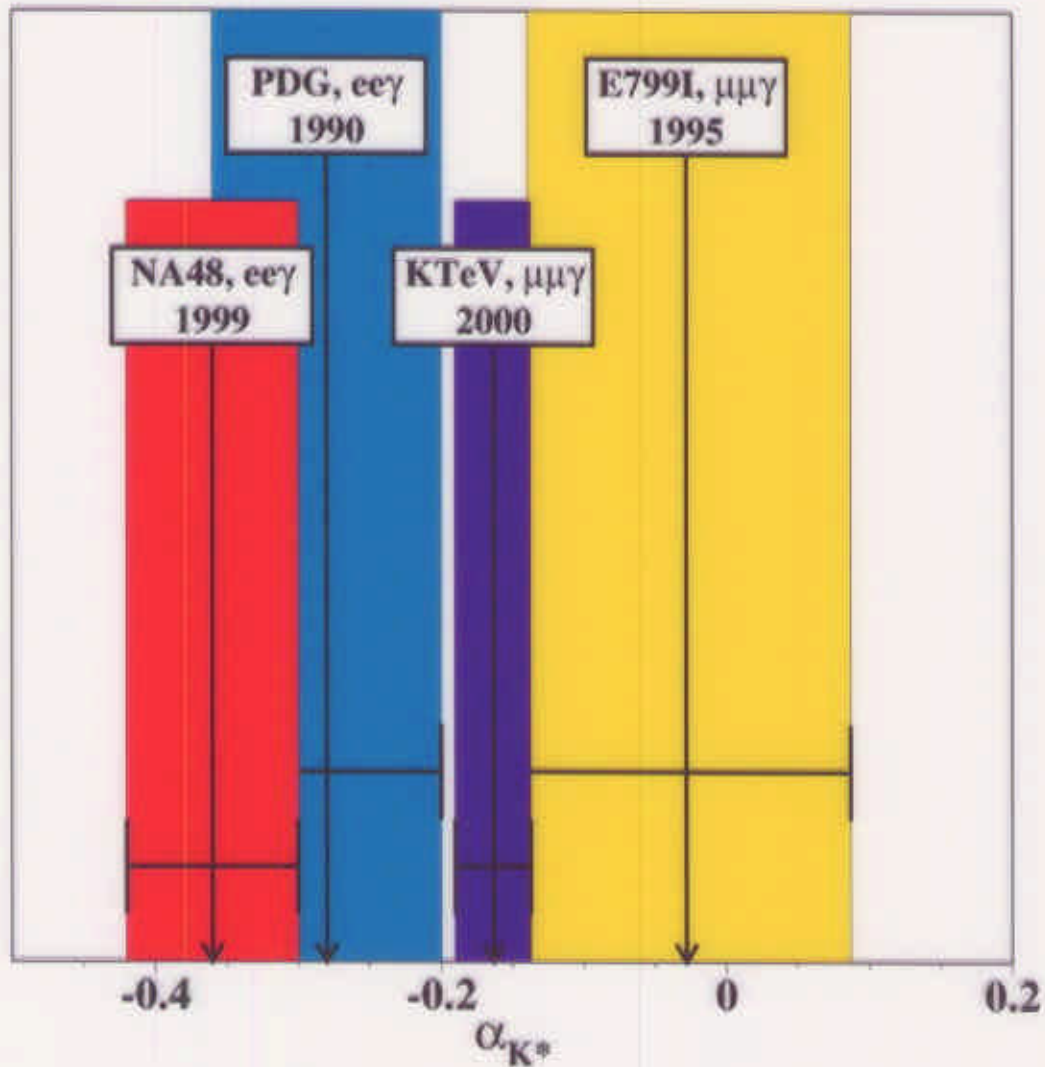
- DIP does not make a prediction for  $\alpha$
- No model predicts sign of  $\alpha_{K^*}$  (negative value for BMS is unambiguously preferred based on proximity to measured value)
- Ko includes additional pole diagrams
- Gvozdev uses a quark model calculation

## Self-consistency of BMS and DIP Models

Form Factor Model	Parameter Measurement		Significance of Difference
	Shape	BR	
$\alpha_{K^*}$ (BMS)	$-0.193^{+0.035}_{-0.049}$	$-0.130^{+0.038}_{-0.037}$	$1.2\sigma$
$\alpha$ (DIP)	$-1.73^{+0.14}_{-0.18}$	$-1.42 \pm 0.12$	$1.6\sigma$



## Comparison with Previous Experiments



- Twice the precision of other experiments
- Difference between  $K_L \rightarrow \mu^+ \mu^- \gamma$  and  $K_L \rightarrow e^+ e^- \gamma$  established at the  $3\sigma$  level

Limit on  $\bar{\rho}$ :

$$B(K_L \rightarrow \mu^+ \mu^-) = |\text{Re } A|^2 + |A_{LD}|^2$$

dispersive

absorptive

$$\text{Re } A = A_{SD} + A_{LD}$$

"unitarity bound"

$$= (7.07 \pm 0.18) \times 10^{-9}$$

$$|A_{SD}|^2 = 2 \text{Re}(V_{td}^* V_{ts})$$

$$= [(0.852 \pm 0.273) \times 10^{-9}] (1 - \bar{\rho})^2$$



$$\bar{\rho} = 1.2 - [(3.43 \pm 0.55) \times 10^9] \left| |\text{Re } A_{\text{exp}}| \pm |\text{Re } A_{LD}| \right|$$

$$\uparrow B(K_L \rightarrow \mu\mu) = (7.18 \pm 0.17) \times 10^{-9}$$

$|\text{Re } A_{LD}|$  ??

BMS :

$$(-5.1 - 11.0\alpha_{k*}) \times 10^{-5} < \text{Re } A_{LD} < (-2.9 - 10.2\alpha_{k*}) \times 10^{-5}$$

$$\Rightarrow \rho > -1.0$$

DIP :

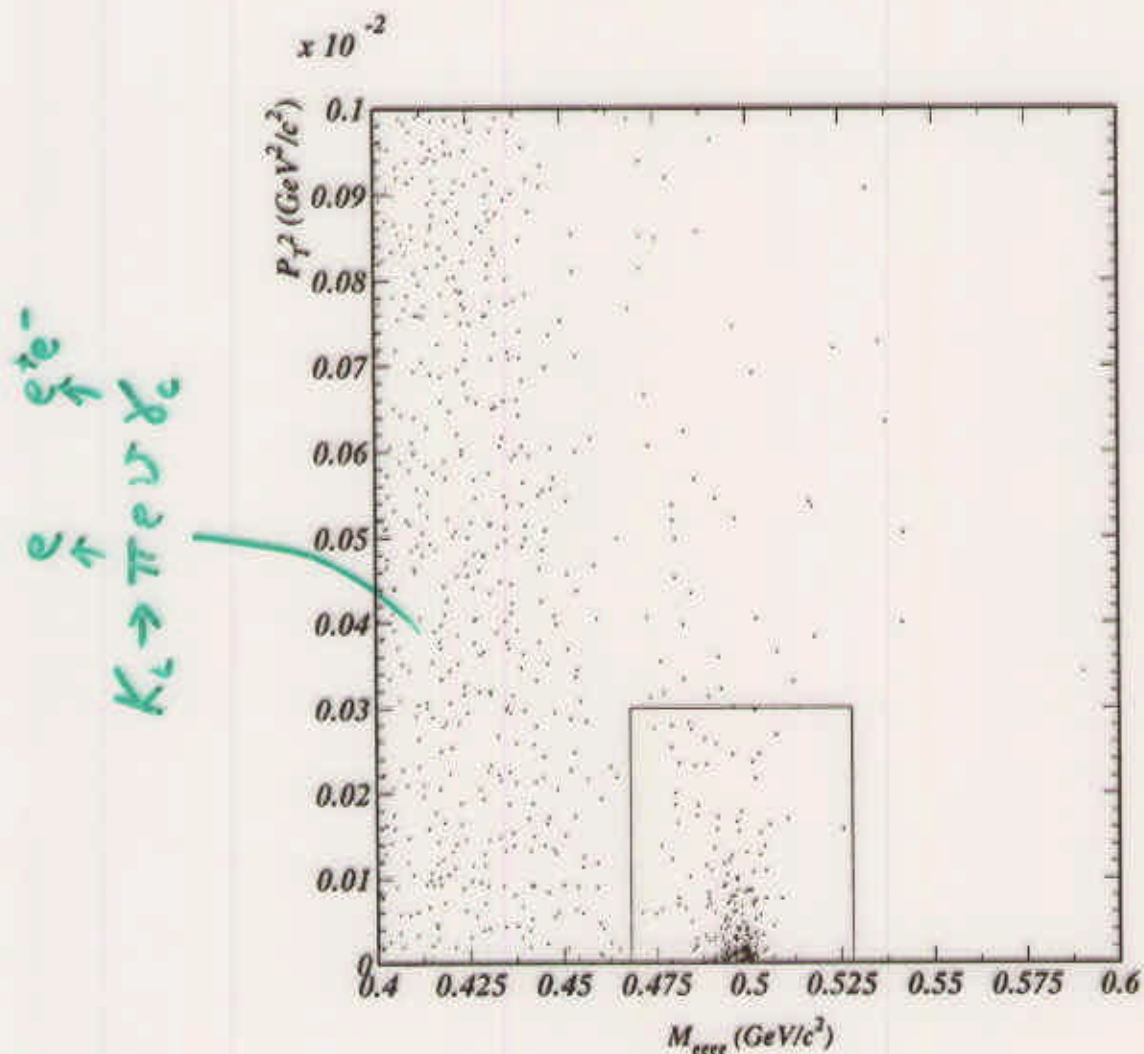
$$|\text{Re } A_{LD}| = 1.61 \times 10^{-5} |5.25 + 3.47\alpha + 3(1 + 2\alpha + \beta) \ln \frac{\Delta}{m_p}|$$

$$|(1 + 2\alpha + \beta) \ln \frac{\Delta}{m_p}| < 0.4$$

$$\Rightarrow \rho > 0.2$$

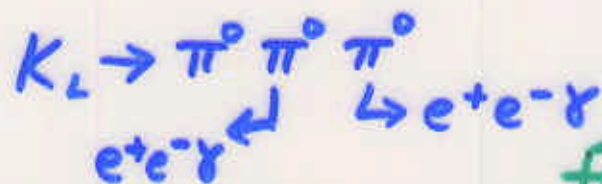
Note that this reg'  $\alpha \rightarrow \rho$ , so need more validations on theories.

$$K_L \rightarrow e^+ e^- e^+ e^-$$

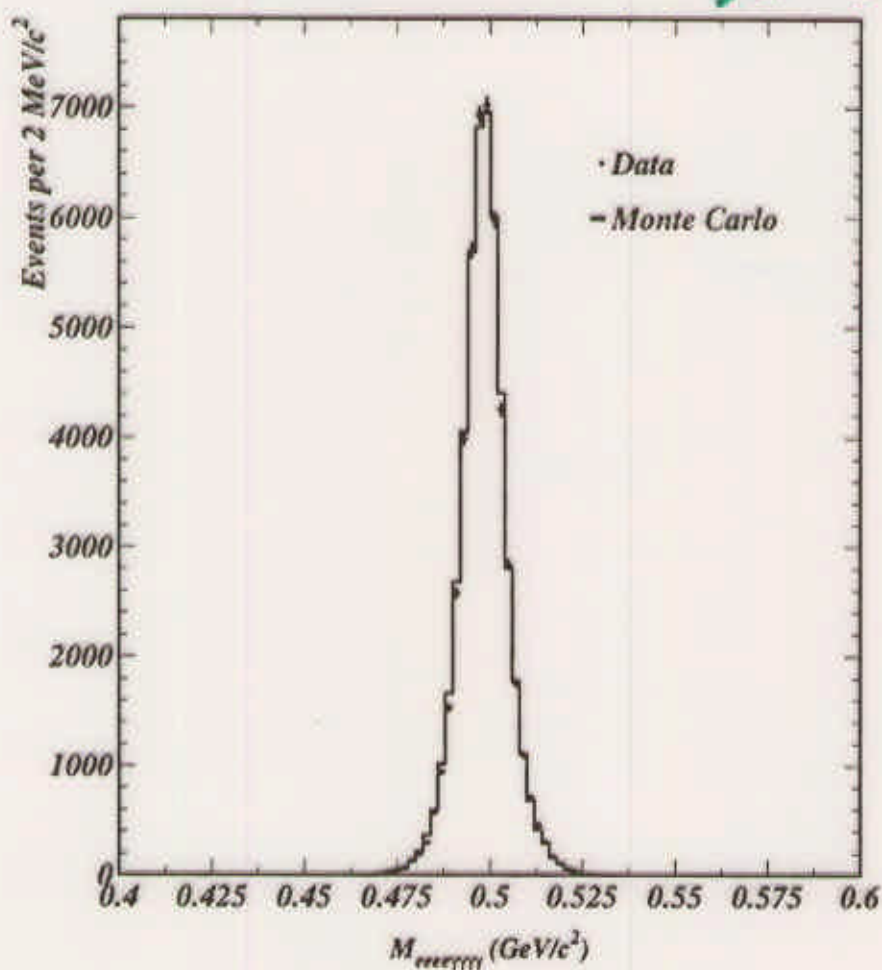


436 evts in the box

Bkgs:  $K_L \rightarrow e^+ e^- \gamma$  - track separation  $\sim 3$  evts  
 $K_L \rightarrow \pi e \gamma$  - "  $\sim 0.5$  evts



for normalization

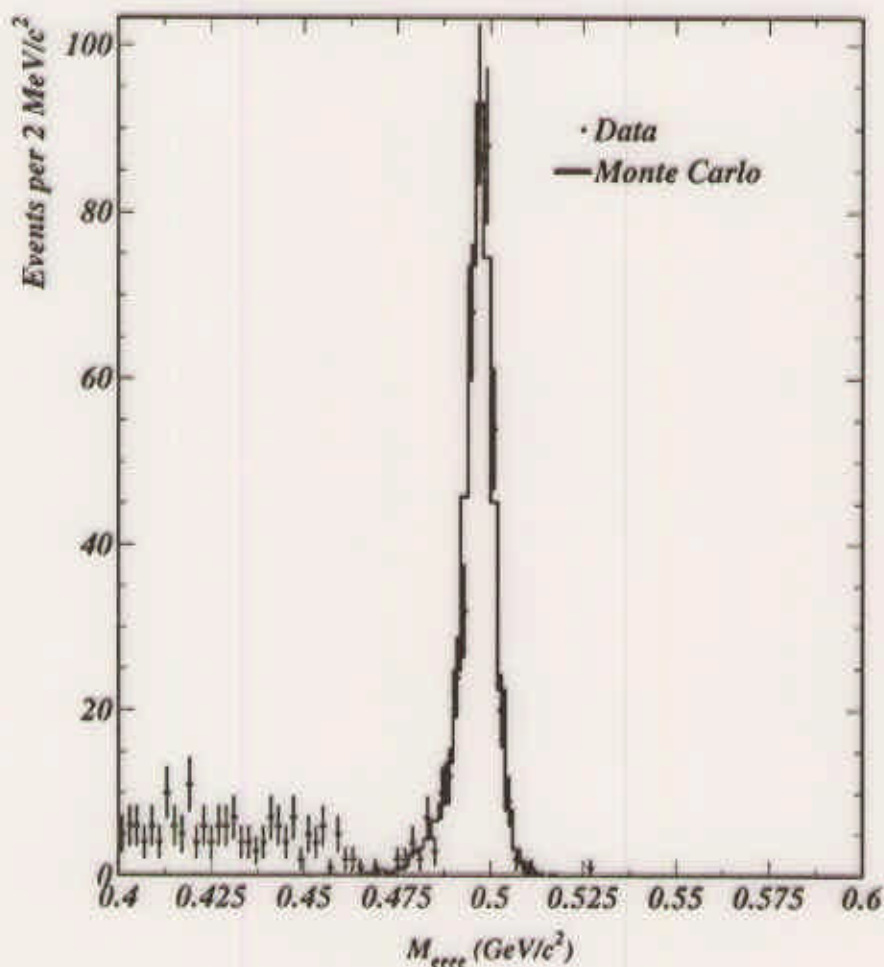


$$N_K (K_L \rightarrow \pi^0 \pi^0 \pi^0) = (2.60 \pm 0.01 \pm 0.14) \times 10^{11}$$

↑  
from  $\pi^0$  br!



$$K_L \rightarrow e^+ e^- e^+ e^-$$



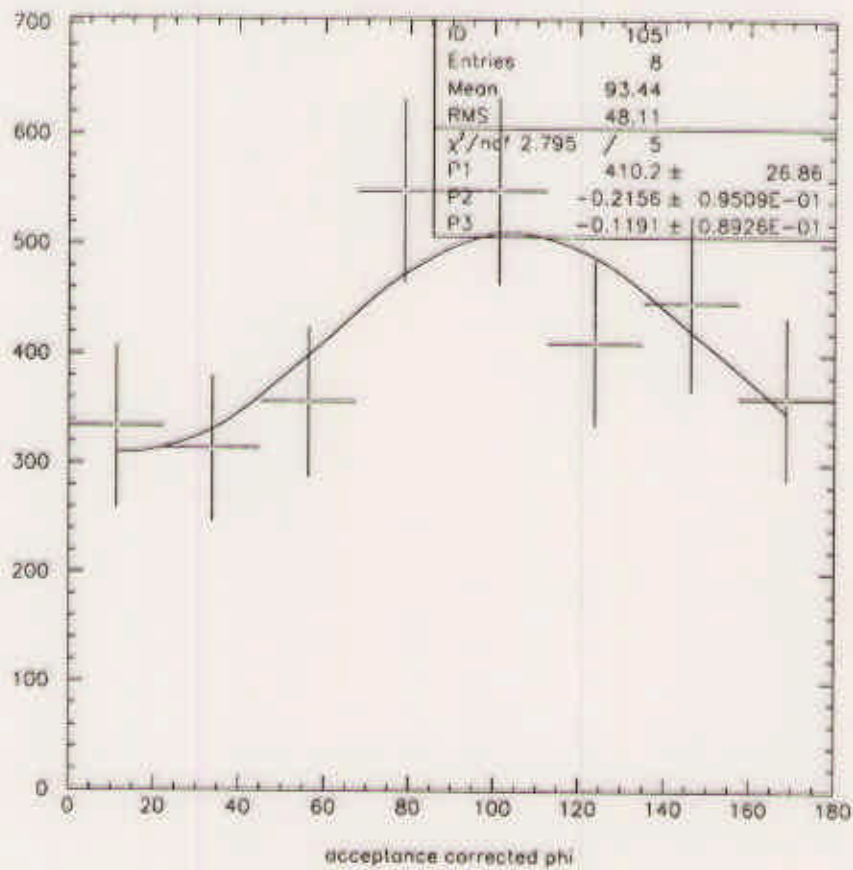
S : 436 evts with 3.5 evts bkgs,  $A = 9.46\%$

N : 47497 evts with 684 evts bkgs,  
 $A = 0.20\%$

$$BR(K_L \rightarrow e^+ e^- e^+ e^-) = (3.77 \pm 0.18 \pm 0.13 \pm 0.21) \times 10^{-8}$$

Stat                      syst                      PDG

$$K_L \rightarrow e^+ e^- e^+ e^-$$



$$\frac{d\Gamma}{d\phi} \sim 1 + \beta \cos 2\phi + \gamma \sin 2\phi ; \quad \beta = -0.212 \pm 0.091 \pm 0.02$$

$$\quad \quad \quad \uparrow$$

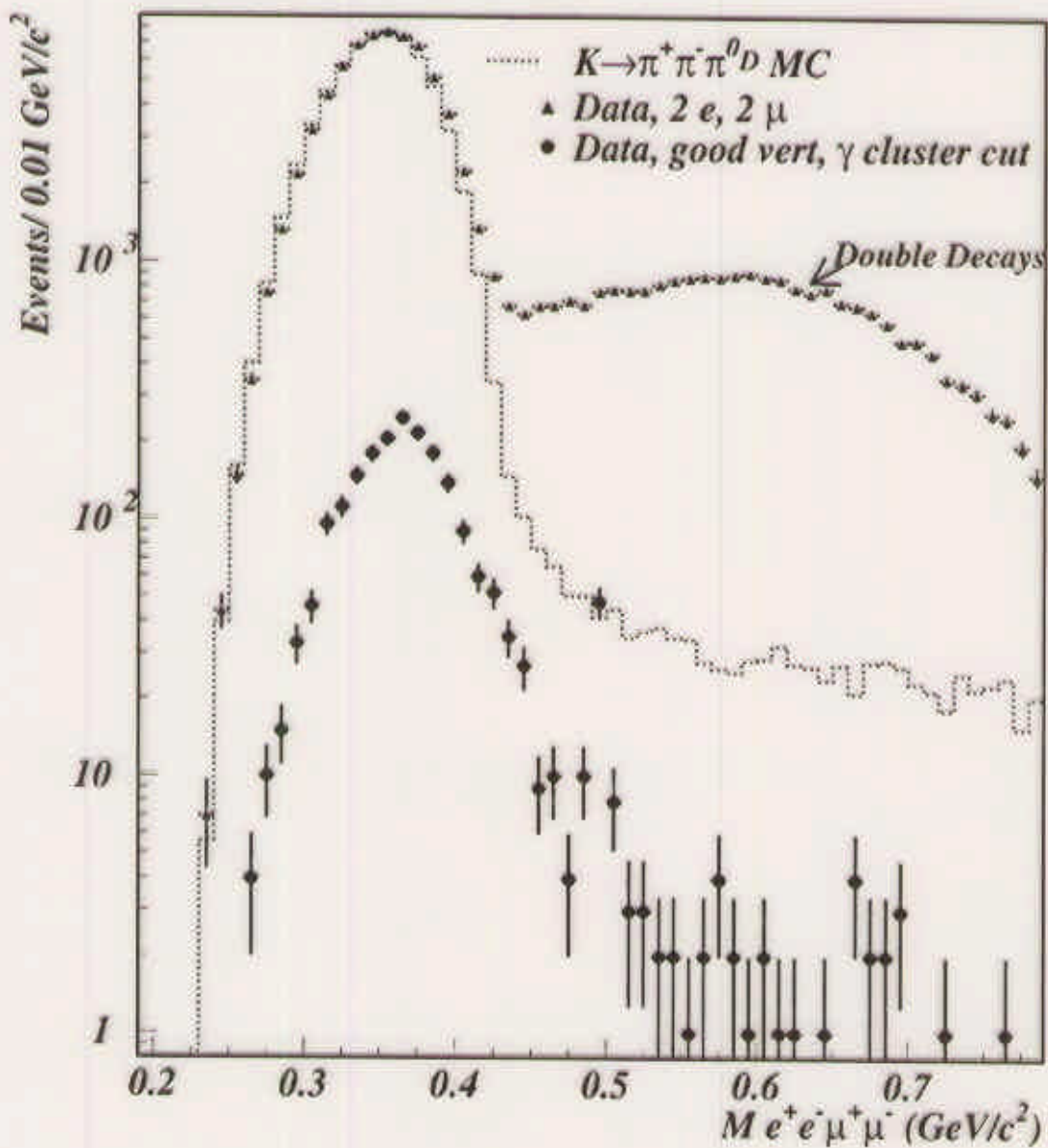
$$\quad \quad \quad -0.2$$

$$\quad \quad \quad \gamma = -0.119 \pm 0.085 \pm 0.02$$

$$\alpha_{K^*} (\text{BMS}) = 0.03 \pm 0.17 \pm 0.18$$

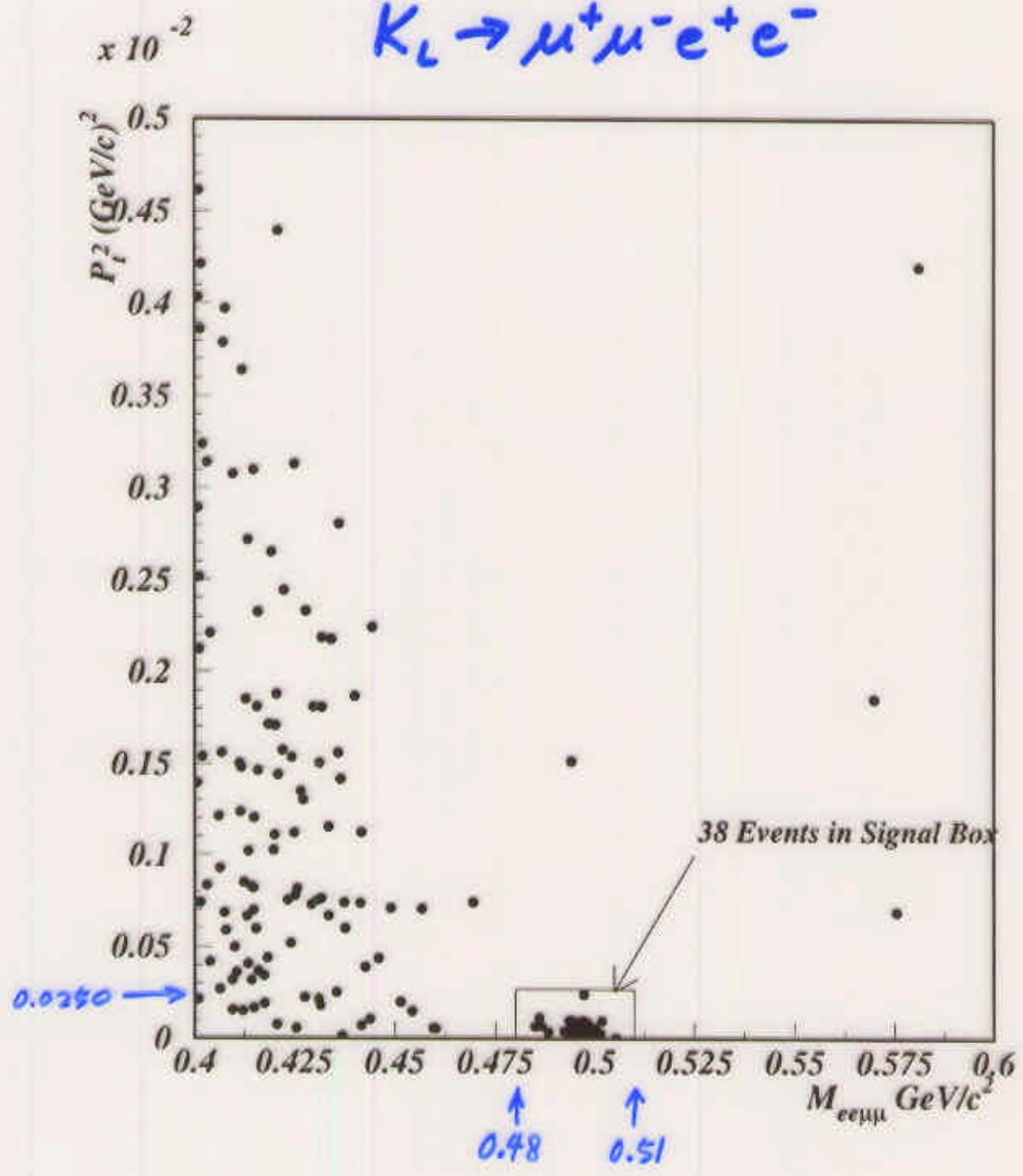
(stat) (syst)

$$K_L \rightarrow \mu^+ \mu^- e^+ e^-$$



- \* Good vertex eliminates double decays
- \* Extra γ cluster eliminates π<sup>+</sup>π<sup>-</sup>π<sup>0</sup><sub>D</sub>

# $K_L \rightarrow \mu^+ \mu^- e^+ e^-$

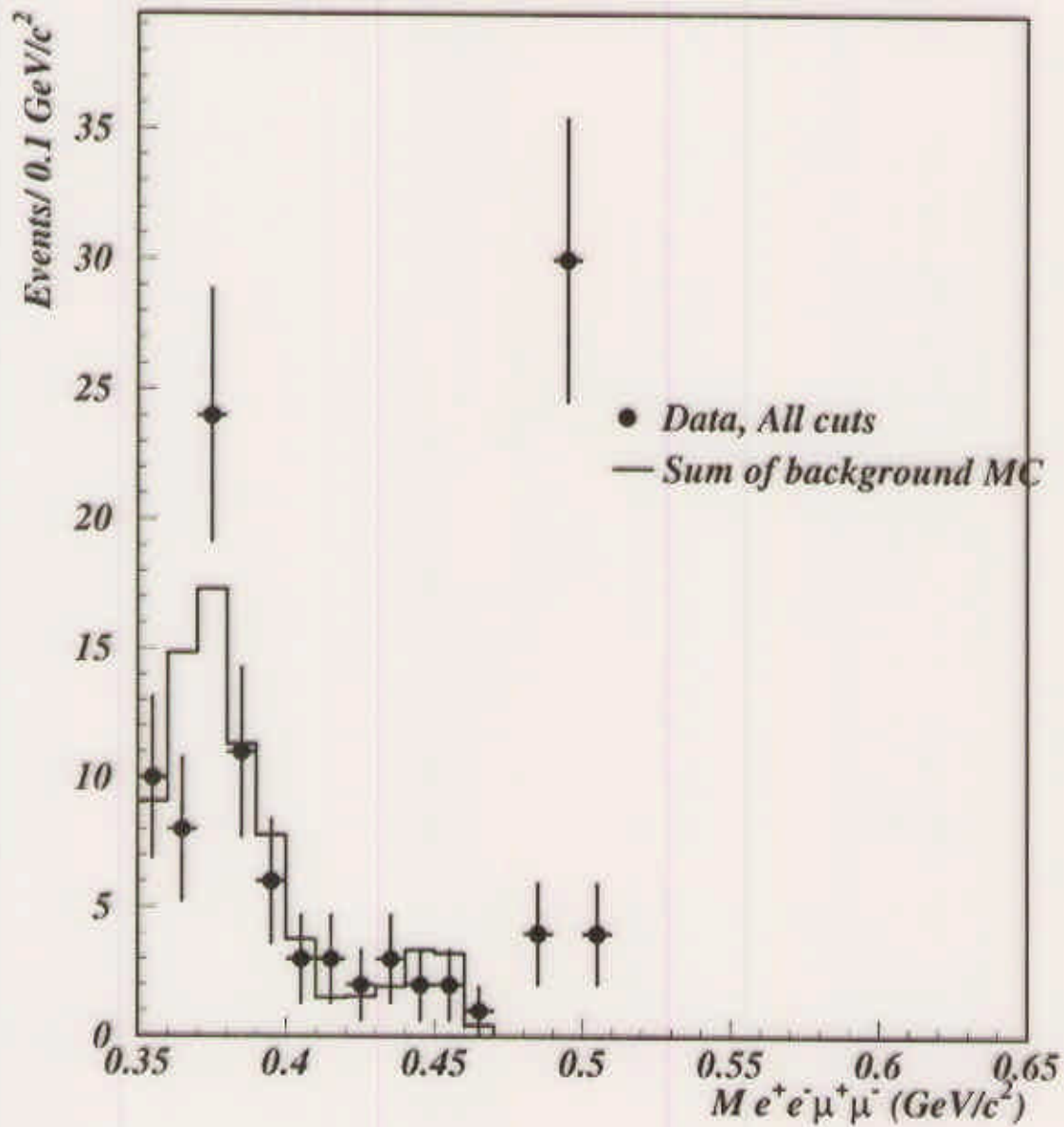


Bkg [ $\mu\mu\gamma_c$ ;  $\pi^+\pi^-\pi_D^0$ ;  $\pi^+\pi^-e^+e^-$ ] = 0.18 evts

Normalization  $K_L \rightarrow \pi^+\pi^-\pi_D^0$  from minimum bias data



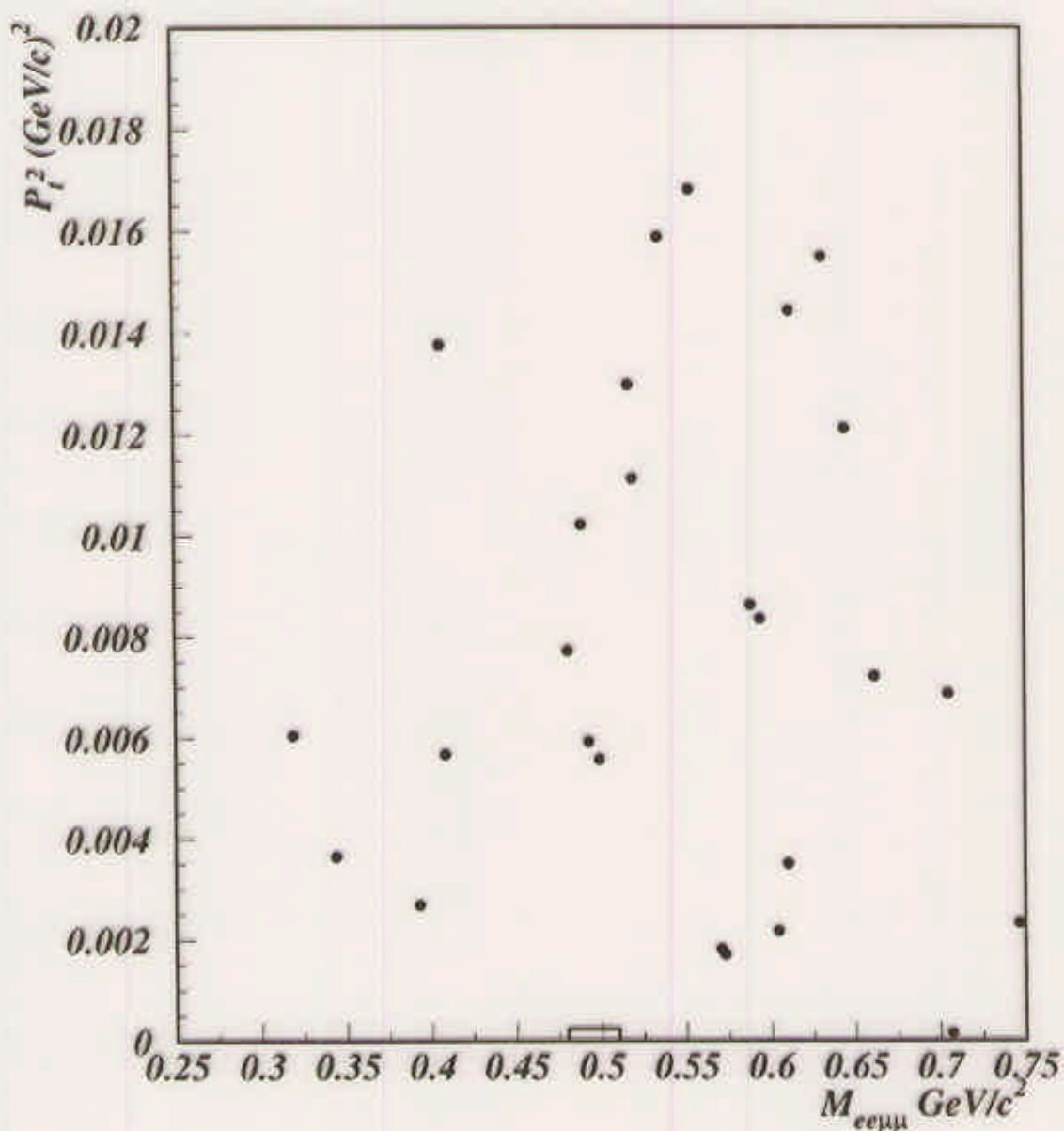
$$K_L \rightarrow \mu^+ \mu^- e^+ e^-$$



Acceptance  $\mu^+ \mu^- e^+ e^- \sim 5.6\%$

$$BR(K_L \rightarrow \mu^+ \mu^- e^+ e^-) = (2.50 \pm 0.41 \pm 0.15) \times 10^{-9}$$

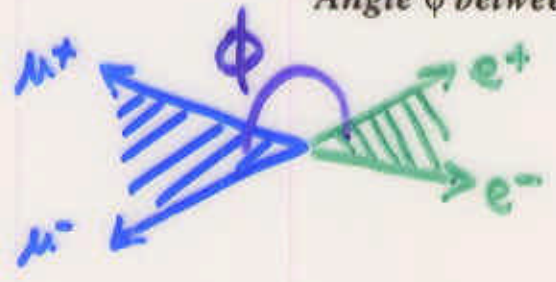
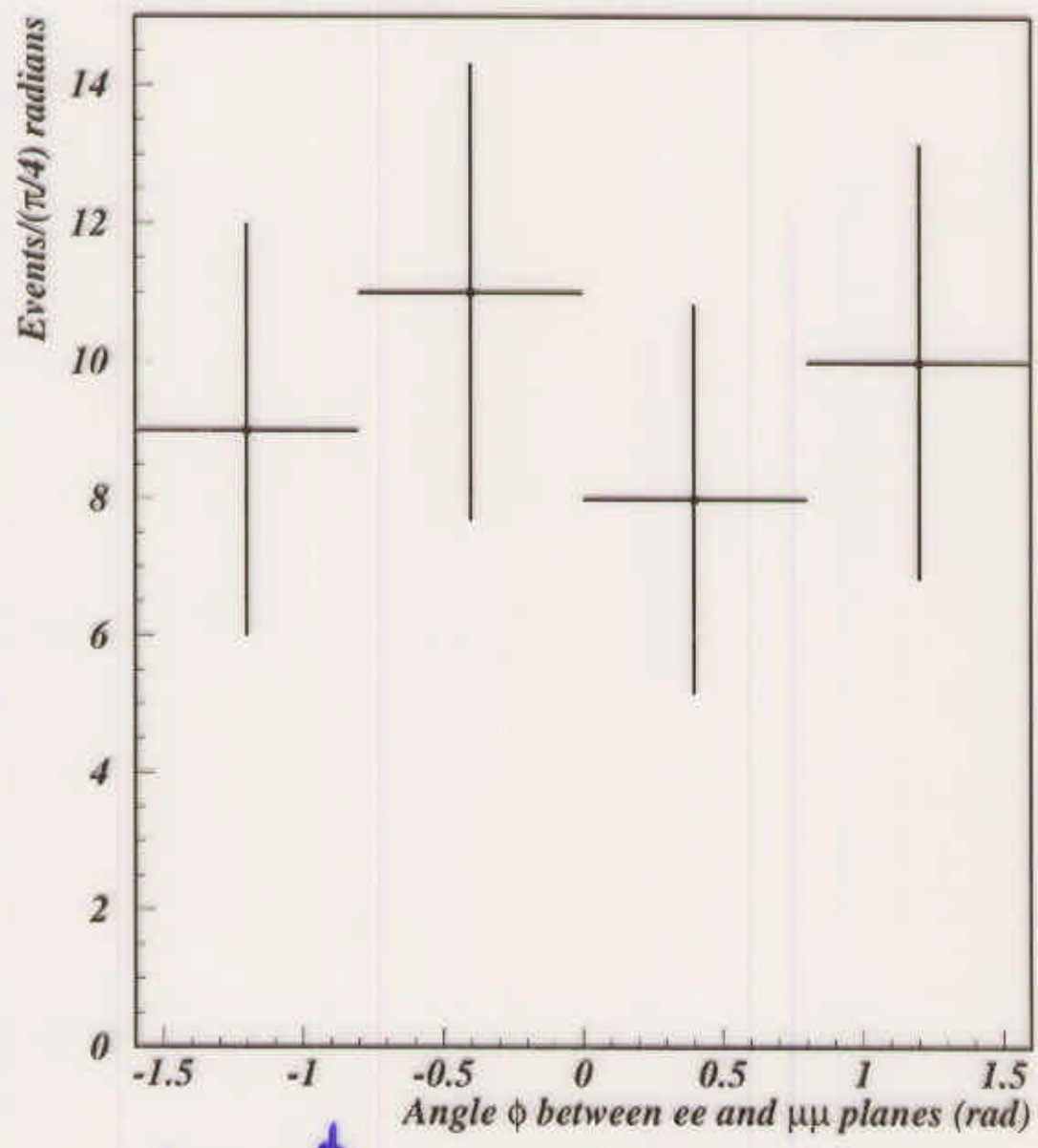
$$K_L \rightarrow \mu^\pm \mu^\pm e^\mp e^\mp$$



$$\text{Acceptance} = 6.65\%$$

$$\text{BR}(K_L \rightarrow e^\mp e^\mp \mu^\pm \mu^\pm) < 1.36 \times 10^{-10} \quad (90\% \text{ CL})$$

$$K_L \rightarrow \mu^+ \mu^- e^+ e^-$$



Small asymmetry in φ if CP violation.

Summary :

$$\text{BR}(K_L \rightarrow \mu^+ \mu^- \gamma) = (3.70 \pm 0.09 \pm 0.07) \times 10^{-7}$$

$$\alpha_{K^*} = -0.163^{+0.026}_{-0.027} \quad (\text{BMS})$$

$$\alpha = -1.55 \pm 0.09 ; \rho > 0.2 \quad (\text{DIP})$$

$$\text{BR}(K_L \rightarrow e^+ e^- e^+ e^-) = (3.77 \pm 0.18 \pm 0.27) \times 10^{-8}$$

$$\beta = -0.21 \pm 0.10 \pm 0.02$$

$$\gamma = -0.12 \pm 0.09 \pm 0.02$$

$$\alpha_{K^*} = 0.03 \pm 0.17 \pm 0.18$$

$$\text{BR}(K_L \rightarrow \mu^+ \mu^- e^+ e^-) = (2.50 \pm 0.41 \pm 0.15) \times 10^{-9}$$

$$\text{BR}(K_L \rightarrow e^{\mp} e^{\mp} \mu^{\pm} \mu^{\pm}) < 1.36 \times 10^{-10} \quad (90\% \text{ CL})$$