Additional Isospin-breaking effects in ϵ'/ϵ

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Isospin limit

$$A(K^0 \to \pi^+\pi^-) = A_0 e^{i\delta_0} + 1/\sqrt{2}\, A_2 e^{i\delta_2}$$

$$A(K^0 \to \pi^0\pi^0) = A_0 e^{i\delta_0} - \sqrt{2}\, A_2 e^{i\delta_2}$$

Data: $\frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} \equiv \omega \simeq \frac{1}{22}$

$$\left|\frac{\epsilon'}{\epsilon}\right| \simeq \frac{\omega}{\sqrt{2}\,\epsilon\, \mathrm{Re}\, A_0} \Big[\underset{\uparrow \atop \Delta I = 1/2}{\mathrm{Im}\, A_0} - \frac{1}{\omega} \underset{\uparrow \atop \Delta I = 3/2}{\mathrm{Im}\, A_2} \Big]$$

Isospin Violation

$$A_2 \quad (\Delta I = 3/2) \quad \begin{cases} \text{weak } \Delta I = 3/2 \\ \text{weak } \Delta I = 1/2 \otimes \Delta I = 1 \quad (m_d - m_u) \end{cases}$$

And: $\frac{\text{Weak } \Delta I = 3/2}{\text{Weak } \Delta I = 1/2} \sim \frac{1}{22} \sim 0.04$

whereas: $\frac{\text{Weak } \Delta I = 1/2 \times \frac{m_d - m_u}{m_s}}{\text{Weak } \Delta I = 1/2} \sim 0.025$

$\Big\}$ comparable

$$\text{Define: } \left|\frac{\epsilon'}{\epsilon}\right| = \frac{\omega}{\sqrt{2}\,\epsilon} \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \Big(1 - \underbrace{\frac{1}{\omega} \frac{\mathrm{Im}\, A_2^{IB}}{\mathrm{Im}\, A_0}}_{\equiv\, \Omega_{IB}} - \frac{1}{\omega} \frac{\mathrm{Im}\, A_2}{\mathrm{Im}\, A_0} \Big)$$

Calculating Ω_{IB}:

Look for $K^0 \to \pi^+\pi^- \neq K^0 \to \pi^0\pi^0$ $\sim (m_d - m_u)$

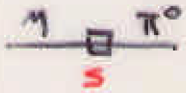
Leading order chiral perturbation theory (χPT)

Weak $\Delta I = 1/2$ chiral $\mathcal{L}$ gives

with no quark masses (except kin)

but also gives:

Strong chiral $\mathcal{L}$ gives



$\sim (m_d - m_u)$

So:

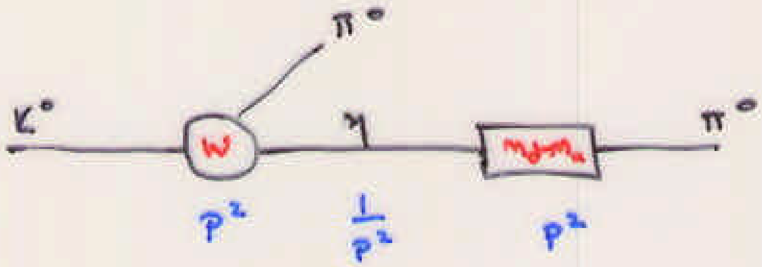
$$\Omega_\eta = \frac{1}{3\sqrt{2}\, w}\left(\frac{m_d - m_u}{m_s - \hat{m}}\right) \sim 0.13$$

$\hat{m} \equiv 1/2\,(m_d + m_u)$

- what happens beyond leading order χPT ?

(or: what is the error in Ω ?)

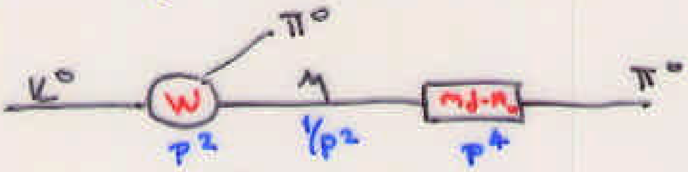
Leading (p^2)



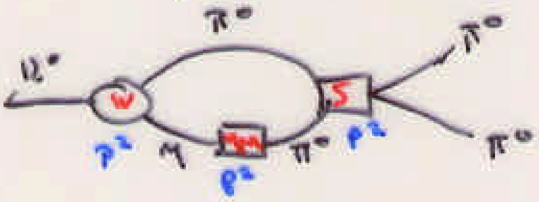
$\rightarrow p^2$

Beyond leading order (p^4)

A.

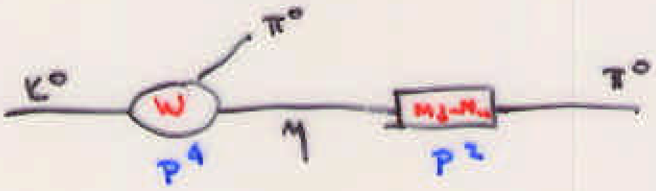


B.

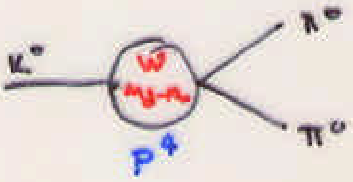


can be computed without unknowns

C.



D.



Contain UNKNOWN low energy constants

Two points:

1) A + B is known (Ecker et. al.) but INCOMPLETE

2) Will illustrate a case of large D $\Rightarrow$ large error in Ω

Pure χPT (diagrams 4)

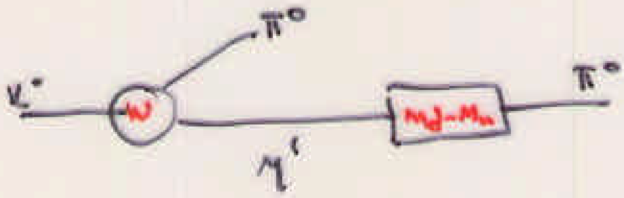
$$\Omega^{IB} = 0.12\ \text{GeV}^2\ \text{Im}\left(2\frac{E_1}{c_2} - 2\frac{E_3}{c_2} - 4\frac{E_4}{c_2} - \frac{E_{10}+E_{11}+E_{15}}{c_2} - 4\frac{E_{12}}{c_2}\right)$$

dim analysis: $E_i/c_2 \sim (1\ \text{GeV})^{-2}$

• "Natural" size of $\Omega^{(4)}$ is 0.12 : $\Omega = 0.13 \pm 0.12$ (?)

Models (beyond χPT)

– the η' $\begin{cases} \text{Donoghue, Golowich, Holstein, Trampetic} \\ \text{Buras, Gerard} \end{cases}$



models for vertices + η-η' mixing $\Rightarrow$

$$\Omega_{\eta\eta'} \simeq 0.28$$

Recall: $\Omega^{(2)}_{\pi} = 0.13$ $\Rightarrow$ Again see next-to-leading corrections as large as leading order

The "New" effect

Susan Gardner, G.V.

- Find an example of large p^4  $\sim (m_s - m_u)$

- Scalar Octet $\left\{ \begin{array}{l} a_0(980) \\ K_0^*(1430) \end{array} \right.$

- Dominant $(\Delta I = 1/2)$ contribution to $\mathrm{Im}\, A_0$ $\to$ penguin

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, c_6 \, (-8) \underbrace{(\bar{s}_L q_R)} \underbrace{(\bar{q}_R d_L)} + \text{h.c.}$$

- factorization: product of 2 scalar densities that I can get from strong Lagrangian

$$(\bar{s}_L q_R)(\bar{q}_R d_L) \sim \left(\frac{\delta \mathcal{L}_S}{\delta \chi^\dagger_{3i}}\right)\left(\frac{\delta \mathcal{L}_S}{\delta \chi_{i2}}\right)$$

- But: to get a weak chiral $\mathcal{L}$ at p^4

  need a strong chiral $\mathcal{L}$ at p^6 ...

constants are not known $\to$ resonance saturation

in particular: scalar octet

$$\mathcal{L} = \frac{1}{2}\langle D^\mu S D_\mu S - M_S^2 S^2 \rangle + c_d \langle \xi^+ S \xi u^2 \rangle + c_m \langle \xi^+ S \xi \chi_+ \rangle + d_m \frac{1}{2} \langle \xi^+ S^2 \xi \chi_+ \rangle$$

c_d $\sim$ $p_1^\mu p_{2\mu}$ pion momenta

c_m $\sim$ m_q $\begin{cases} m_u + m_d & I \checkmark \\ m_u - m_d & \cancel{I} \end{cases}$

d_m $\sim$ m_q

$\downarrow$

No pions $\rightarrow$ $m_q S^2$

$\rightarrow$ SU(3) breaking mass term in S octet

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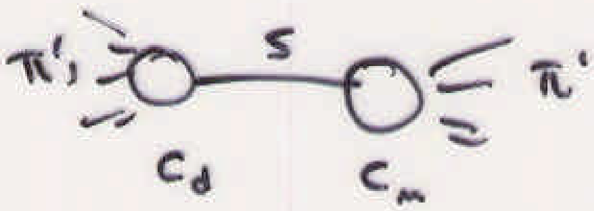
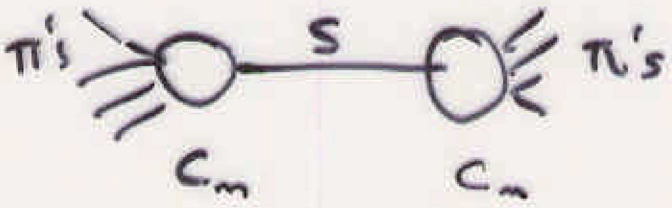
How do I determine the couplings?



$\sim \frac{c_d c_m}{M_S^2} \langle \chi_+ u^2 \rangle$

$\sim L_5$ measured



$\sim \frac{c_m^2}{M_S^2} \langle \chi_+^2 \rangle$

$\sim L_8$ measured

d_m From $M(K_0^*(1430)) - M(a_0(980))$

1) Physical masses $d_m = -2.4$

2) quark model $\bar{q}q$ masses $\rightarrow$ $d_m = -0.76$

$$d_m = \frac{M_{K_0^*}^2 - M_{a_0}^2}{2(M_\pi^2 - M_K^2)}$$

$d_m < 0$ From $M_{K0}^2 > M_{a_0}^2$

We Find:

$$\mathcal{L}_s^{(4)} = \frac{d_m c_m^2}{2 M_s^4} \langle \chi_+^3 \rangle + \frac{c_d c_m d_m}{M_s^4} \langle \chi_+^2 L^2 \rangle$$

+ others (not relevant)

Factorization of Q_6 normalized to c_2:

$$E_1/c_2 = \frac{3 d_m c_m^2}{2 M_s^4 L_5} \simeq -1.8\ \mathrm{GeV}^{-2} \quad [\text{or } -1.6\ \mathrm{GeV}^{-2}]$$

$-2.4 = d_m = -0.76$

$$E_{10}/c_2 = \frac{c_d c_m d_m}{M_s^4 L_5} \simeq -2.4\ \mathrm{GeV}^{-2} \quad [\text{or } -0.8\ \mathrm{GeV}^{-2}]$$

$$\Omega_p = 0.12\ \mathrm{GeV}^2 \frac{d_m c_m (3c_m - c_d)}{M_s^4 L_5} \sim (-0.85) \sim \underline{\underline{-0.27}}$$

- Recall N.D.A. for $E_i/c_2 \sim 1\ \mathrm{GeV}^{-2}$

⑨

Summary and Conclusion

- Isospin violation can significantly affect ϵ'/ϵ

  • Factors of 2 are possible depending on B_6, B_8

- Usual (pre-1999) number $\Omega_{IB} = 0.25 \pm 0.08$ underestimates the uncertainty

- $\Omega_{IB} = \begin{cases} 0.13 & \text{- leading } \pi\text{-}\eta \text{ mixing} \\ 0.16 \pm 0.03 & \text{- } \pi^0\text{-}\eta_8 \text{ at } p^6 \quad \text{(Ecker et al 99)} \\ 0.25 \pm ? & \text{- } \pi^0\text{-}\eta\text{-}\eta' \quad \text{\{Donoghue et al, Buras et al\}} \\ 0.13 \pm 0.12 & \text{- } \chi\text{PT}, p^4 \\ 0 \pm ? & \text{- } \pi^0\text{-}\eta\text{-}\eta'\text{-}\underline{\underline{S}} \quad \text{Gardner, G.V. 99} \\ (-0.6 \text{ to } 0.2) & \quad \hookrightarrow d_m = -0.76 \end{cases}$

• At this point I would probably use $\Omega = 0.13 \pm 0.12$

certainly <u>NOT</u> 0.16 ± 0.03 as some recent papers do.