ICHEP2000

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# WEAK MATRIX ELEMENTS from THE LATTICE

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0.6 T flop.

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OUTLINE

1. Introduction

2. Theory

3. Strategy for $\Delta S=1$ M.E of $K \to 2\pi$

4. PRELIMINARY RESULTS (NOT Yet for $\Delta S=1$)

5. SUMMARY + OUTLOOK

3

INTRODUCTION

$\Delta S=1$ WME STILL BEING RECOMPUTED & ARE NOT Yet COMPLETE

RECALL ~ 6 MONTHS AGO found ERRORS IN OUR PREVIOUS CALCULATION of $K \to 2\pi$ & $\epsilon'/\epsilon$.

SPENT ~ 4 MONTHS IN INTENSIVE STUDY of THEORETCAL UNDERPINNINGS of Domain WALL Quarks wrt $\chi S$... MAIN ISSUE IS Finite LENGTH IN $5^{th}$ DIM.

$\Delta S=1$ WME RECOMPUTATION IN PROGRESS for about ~~[illegible]~~ 2 MONTHS ...
NEED SOME MORE TIME.

Conventional DISCRETIZATIONS explicitly break $\chi$l sym to remove doublers. Only gets restored in the contm limit $a \to 0$

KOGUT-SUSSKIND (OR STAGGERED) $SU(N_f)_L \times SU(N_f)_R \to U(1)_A$ (non-singlet). $\chi$L is still $m_q \to 0$ But fl sym broken. ONLY 1 of $N_f^2 - 1$ pion; errors are $O(a^2)$.

WILSON Fermions $SU(N_f) \times SU(N_f) \to SU(N)_V$ ... $\chi$s explicitly broken .. $\chi$L is not $m_q \not\to 0$ .... Complicated fine tuning (op. mixing) ... Error $O(a)$ FIG.

Domain Wall fermions ... Remove doublers while preserve $SU(N_f)_L \times SU(N_f)_R$ even at $a \neq 0$ in the limit $N_s \to \infty$. $\chi$L and Contm Limit separated for the 1st time.

Practical viability for QCD Demonstrated ie. $N_s \sim 10\text{-}20$ sufficed

T. BLUM + A.S. PRD, PRL 97

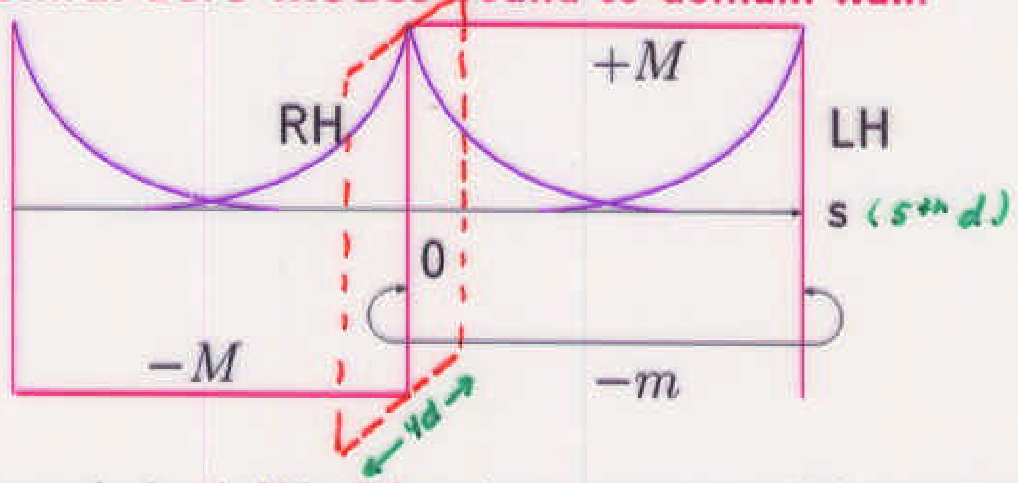
Error $O(a^2)$

Improved scaling anticipated also by
Y. KIKUKAWA, R. NARAYANAN & H. NEUBERGER
Phy. Lett. B. 399, 105 (97).

# DOMAIN WALL QUARKS

- **Kaplan**: Add extra *5th dimension* with mass defect, or domain wall. **4d Chiral zero modes** bound to domain wall.

$\boxed{N_5, M}$




- **Shamir**: couple both Weyl fermions to **same 4d gauge field** and simulate **vector** theory(QCD).

$$\boxed{\begin{aligned} q_x &= P_R \psi_{x,1} + P_L \psi_{x,2N} \\ \bar{q}_x &= \bar{\psi}_{x,2N} P_R + \bar{\psi}_{x,1} P_L \end{aligned}}$$

- **Chiral symmetry** is manifest, left-handed and right-handed quarks are globally separated in the 5th dimension. Explict breaking from quark mass $m$ same as in continuum.
- **5d spectrum**: $1 \, light$ (Dirac) fermion, $N_s - 1$ *heavy* fermions; Elegant flavor interpertation due Neuberger and Narayanan.
- **Overlap** induces exponentially small additive quark mass. Chiral limit: $N_s \to \infty$, $m_{quark} = mM(2 - M) \to 0$.

6

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# The FIVE DIMENSIONAL ACTION

$$S_q = \sum_{x,y,s,s'} \bar{\psi} \left( \not{D}_{x,y} \delta_{s,s'} + \not{D}_{s,s'} \delta_{x,y} \right) \psi,$$

Gauge Fields 4 dim.
$U_\mu(x,s) = U_\mu(x)$ $\mu = 1 \ldots 4$
$U_5(x,s) = 1$

$$S_W = \frac{a}{2} \frac{\partial^2}{\partial x^2} \to \sum_\mu \frac{\delta_{x+\hat{\mu},y} + \delta_{x-\hat{\mu},y} - 2\delta_{x,y}}{2a}$$

$$\not{D}_{x,y} = \sum_\mu \left( \frac{1+\gamma_\mu}{2} U_{x,\mu} \delta_{x+\hat{\mu},y} + \frac{1-\gamma_\mu}{2} U^\dagger_{y,\mu} \delta_{x-\hat{\mu},y} \right) + (M-4)\delta_{x,y}$$

WILSON MASS TERM
with opp sign.

$$\not{D}_{0,s'} \quad \tfrac{(1+\gamma_5)}{2}\delta_{1,s'} - m\tfrac{(1-\gamma_5)}{2}\delta_{N_s-1,s'} - \delta_{0,s'}$$

$$\not{D}_{s,s'} = \tfrac{(1+\gamma_5)}{2}\delta_{s+1,s'} + \tfrac{(1-\gamma_5)}{2}\delta_{s-1,s'} - \delta_{s,s'}$$

$$\not{D}_{N_s-1,s'} \quad -m\tfrac{(1+\gamma_5)}{2}\delta_{0,s'} + \tfrac{(1-\gamma_5)}{2}\delta_{N_s-2,s'} - \delta_{N_s-1,s'}$$

# Strategy for $K \to \pi\pi$ calculation

Two Key features:

1. Exploit *good chiral properties* of DWQ and use LO$\chi$PT:

   (Bernard, *et al.*, Phys. Rev. D '85) BDSPW

   IMPORTANT LIMITATION
   NO FSI

   $$\langle K|Q_i|\pi\pi\rangle = A_i\langle K^+|Q_i|\pi^+\rangle + B_i\langle K^0|Q_i|VAC\rangle$$

   coeff given by CPTh.

   Note $\langle K|Q|VAC\rangle$ is a non-perturbative subraction corresponding to unphysical $\bar{s}$-$d$ mixing.

2. Need to find proper counterparts of continuum operators:

   Since the Lattice & contm are diff. reg. & ren. schms.

   $$Q_i^{cont} = Z_{ij}Q_j^{\text{latt}}$$

   CORRESPONDING Lattice perturbation th. calc by S. AOKI & Y. KURAMASHI

   Use Nonperturbative Renormalization (NPR) method of Martinelli, Pittori, Sachrajda, Testa and Vladikas, NPB 445,81,1995 to calculate $Z_{ij}$.

   *Method has never been used for full $\Delta S = 1$ Hamiltonian* INVOLVES 2 NON-TRIVIAL DIFFICULTIES

# LATTICE CONTRACTIONS

Figure 8


Eye


Subtraction

# $\Delta S = 1$ FOUR QUARK OPERATORS

$SU(3)_L \times SU(3)_R$

(8,1)

$$Q_1(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\beta$$

$$Q_2(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\alpha$$

ORIG. 4 F

$$Q_{3,5}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\beta$$

$$Q_{4,6}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\alpha$$

QCD-Peng.

(8,8)

$$Q_{7,9}(\mu) = \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\beta$$

$$Q_{8,10}(\mu) = \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\alpha$$

EWP

$$Q_{1c}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\beta$$

$$Q_{2c}(\mu) = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\alpha$$

NOTE: *active* charm and charm *integrated out* cases are being investigated.

(10) (32)

# NONPERTURBATIVE RENORMALIZATION

- OPE: operators get renormalized, depend on energy scale $\mu$, $O^{ren}(\mu) = Z_{ij}(\mu) O^{bare}_{ij}$
- Operator Mixing: present in continuum and on the lattice **restricted by symmetries**, more is better: advantage for DWF
- Use the *Regularization Independent* Scheme (Martinelli et al NPB445,81,1995) require Green functions calculated on the lattice between *off-shell* quark and gluon states equal their tree level values.

  $Z^{-1}(\mu)\left\langle \bar{q}(p=\mu)|O^{Latt}|q(p=\mu)\right\rangle = \langle \bar{q}(p=\mu)|O^{tree}|q(p=\mu)\rangle$
- Construct amputated vertex from lattice fourier transformed propagator, project out desired spin, color, flavor: $Z^{-1}(\mu)\mathrm{tr}(P\Lambda) = 1$

  $a^{-1} >> \mu >> \Lambda_{QCD}$
- $p^2 = \mu^2 >> 0$ to match on to perturbation theory("window"). Procedure is *not gauge invariant. i.e.*, $Z$'s depend on gauge and external states. After matching to continuum scheme ($\overline{\mathrm{MS}}$), $O^{ren}(\mu) = Z^{-1}_{mat}(\mu,\lambda) Z^{-1}_{RI}(a\mu,\lambda) O^{Latt}(a)$ is *gauge invariant.*

# Renormalization of Lattice Ops (4q) (11) ~~(33)~~

1-loop lattice pert Theory

$\Delta S=2$ $O_{LL}, B_K$

+ ......  Martinelli '84 (WILSON)

$\Delta S=1$

+ ......

+ ...  Bernard, Draper, A.S. '87 (WILSON)

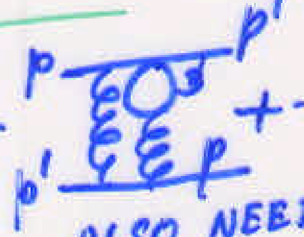
Lattice pert theory poor convergence...
Boosted ... a la Mackenzie & Lepage Helps

NPR a la Martinelli et al NPB 95  Forgoes Latt Pert Theory Altogether! 2

$\Delta S=2$

------ quark hop from a single pt. source suffices
Cheap   Used with Wilson (by APE

+ ....




+ ....

$\Delta S=1$ Much Harder
DONE For the 1st TIME

ALSO NEEDS Prop with momentum sources

I $(\text{Much})^2$ Noisier

$(\bar{s}\,\Gamma\,d)^2 \leftrightarrow \bar{s}d$
$\leftrightarrow \bar{s}\not{D}d$

+ Mixing with LDO  Extremely nasty due $a^{-n}$

XS IS CRITICAL For Handling These

12

~~19~~
~~21~~
12

LATTICE '88



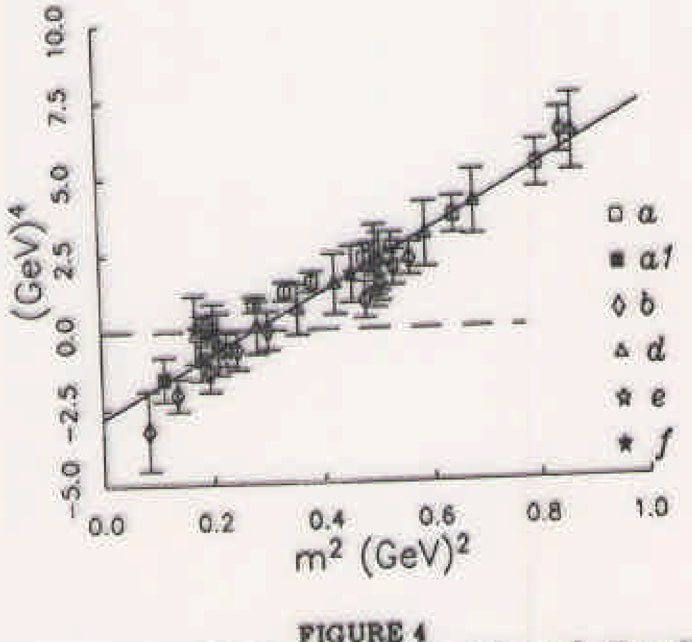



FIGURE 4
The amplitude $\langle \bar{K}^0 | (\Delta s = 2)_{LL} | K^0 \rangle \times 10^3$ vs. $m^2$. The solid line is a naive (uncorrelated) fit to the data.

$\langle \bar{K}^0 | (\Delta s = 2)_{LL} | K^0 \rangle$ with Wilson fermions has been proposed in Ref. 32. One starts by writing the CPTh form for the matrix elements of the continuum (physical) operator and for its Wilson lattice counterpart:

$$
\begin{aligned}
\langle \bar{K}^0 | (\Delta s = 2)_{LL} | K^0 \rangle^{cont} &= \gamma (p_K \cdot p_{\bar{K}}) + \cdots \\
\langle \bar{K}^0 | (\Delta s = 2)_{LL} | K^0 \rangle^{latt} &= \alpha + \beta m^2 + \gamma' (p_K \cdot p_{\bar{K}}) + \cdots ,
\end{aligned}
\tag{8}
$$

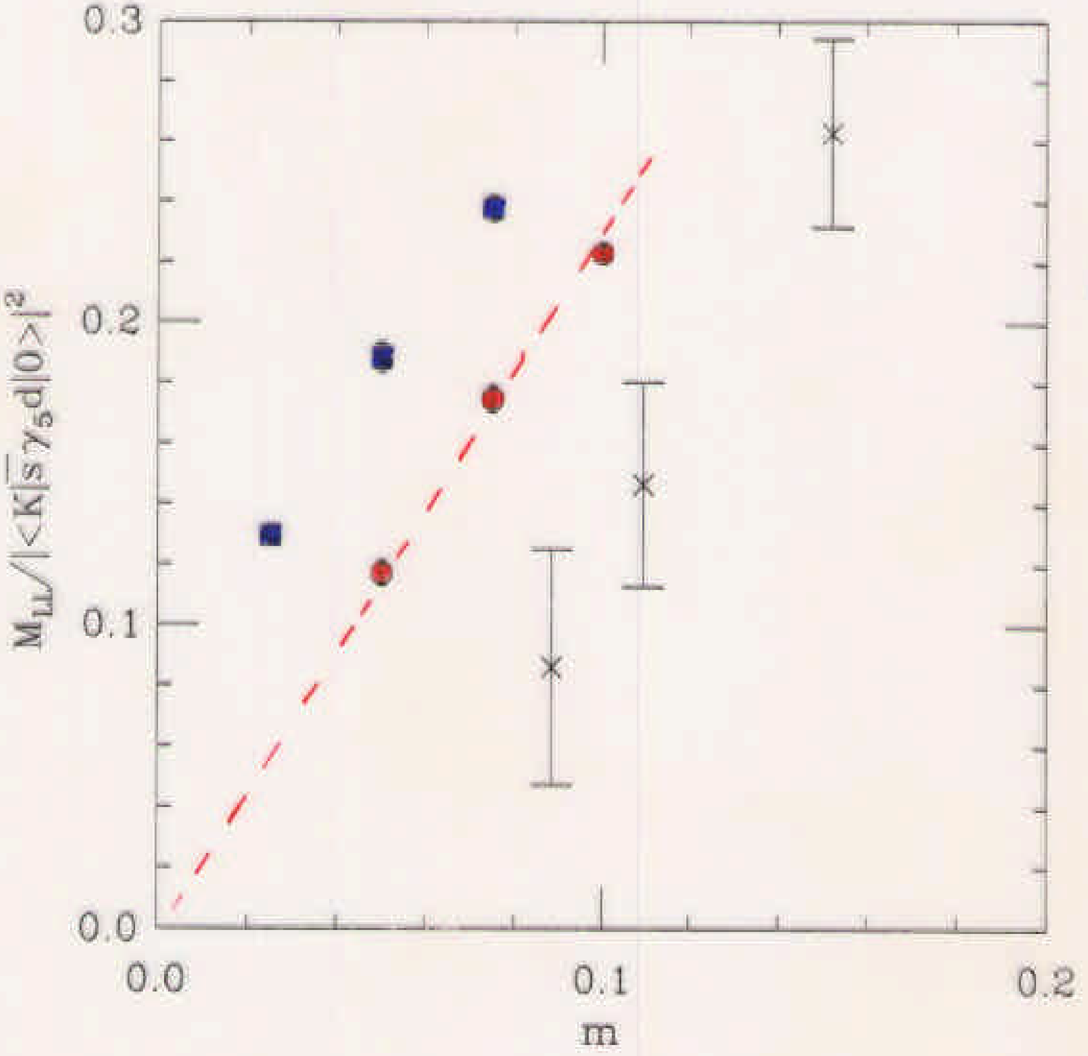
where the $\alpha$ and $\beta$ terms in the lattice amplitude (and the change from $\gamma$ to $\gamma'$) are due to "bad" chirality operators such as $O'_{\pm}$ which have not been correctly removed by perturbation theory. Note that for $K$, $\bar{K}$ at rest, $p_K \cdot p_{\bar{K}} = m^2$; while for the crossed amplitude $\langle \bar{K}^0 \bar{K}^0 | (\Delta s = 2)_{LL} | 0 \rangle$, $p_K \cdot p_{\bar{K}} = -m^2$. Both the original $K^0 - \bar{K}^0$ amplitude and the crossed amplitude are then computed at rest on the lattice for various values of $m$, and the $\gamma'$ term is extracted by a fit to the data. Finally, with the assumption $\gamma \simeq \gamma'$ (see below for a critique), the order $m^2$ term in the continuum amplitude is obtained. In this manner the ELC group obtains[33]

NERSC

(13) (20) (17) (22) (40)

■ $N_s=4$

● $N_s=10$




x WILSON

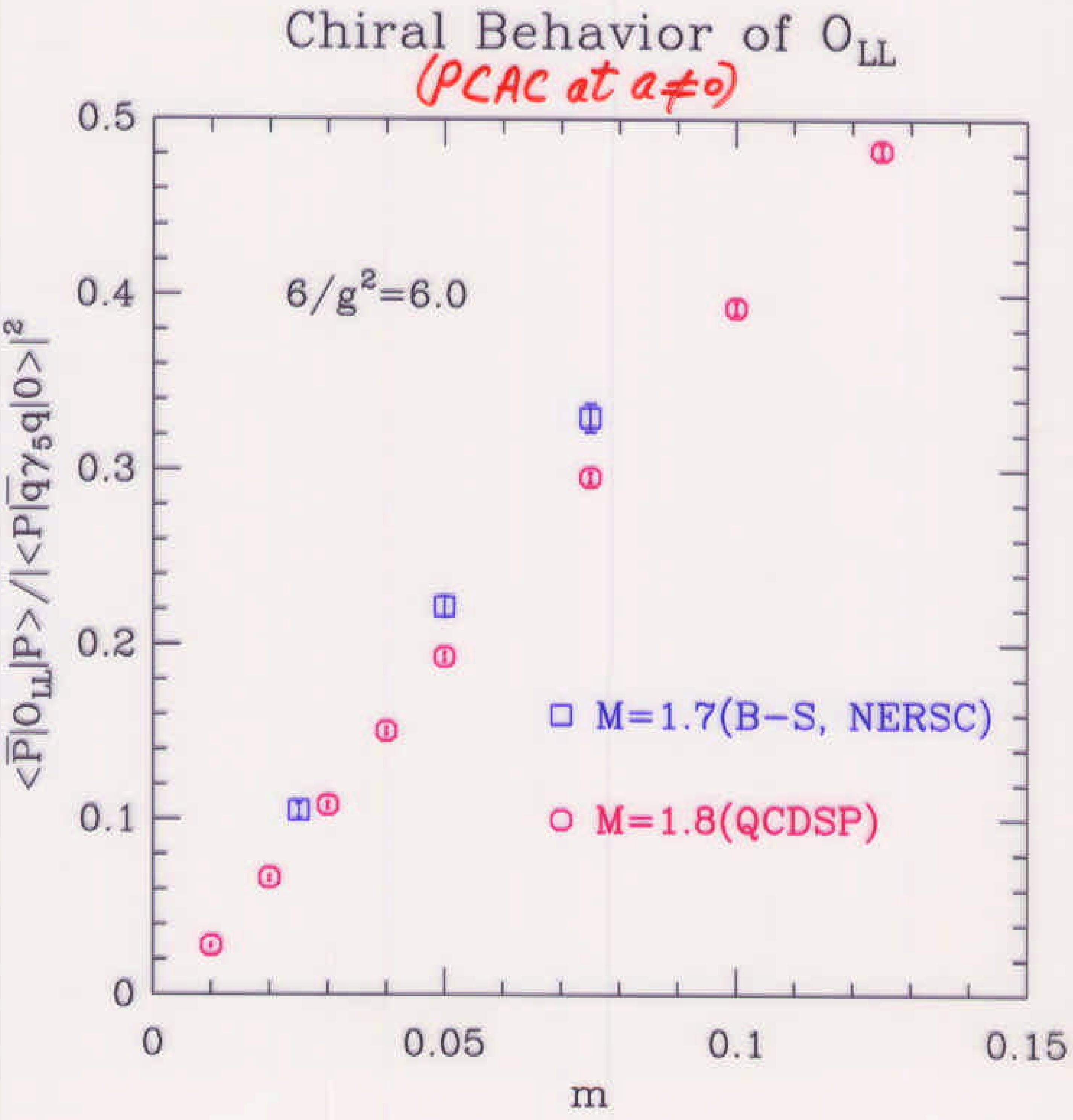
FIG. 2. The ratio of the four quark matrix element for $K_0 - \bar{K}_0$ mixing to the square of the pseudo-scalar density matrix element, calculated with domain wall fermions (octagons($N_s = 10$) and squares($N_s = 4$)). The $N_s = 10$ curve exhibits the correct behavior in the chiral limit. Also shown is the result using the same gauge field configurations for Wilson quarks (crosses) which extrapolates to zero far from $m = 0$ (note that for Wilson quarks the quark mass is defined as the difference of the inverse quark hopping parameter with the inverse critical hopping parameter, $m \equiv \frac{1}{2}(\kappa^{-1} - \kappa_c^{-1})$).

BLUM + A.S. PRD 97

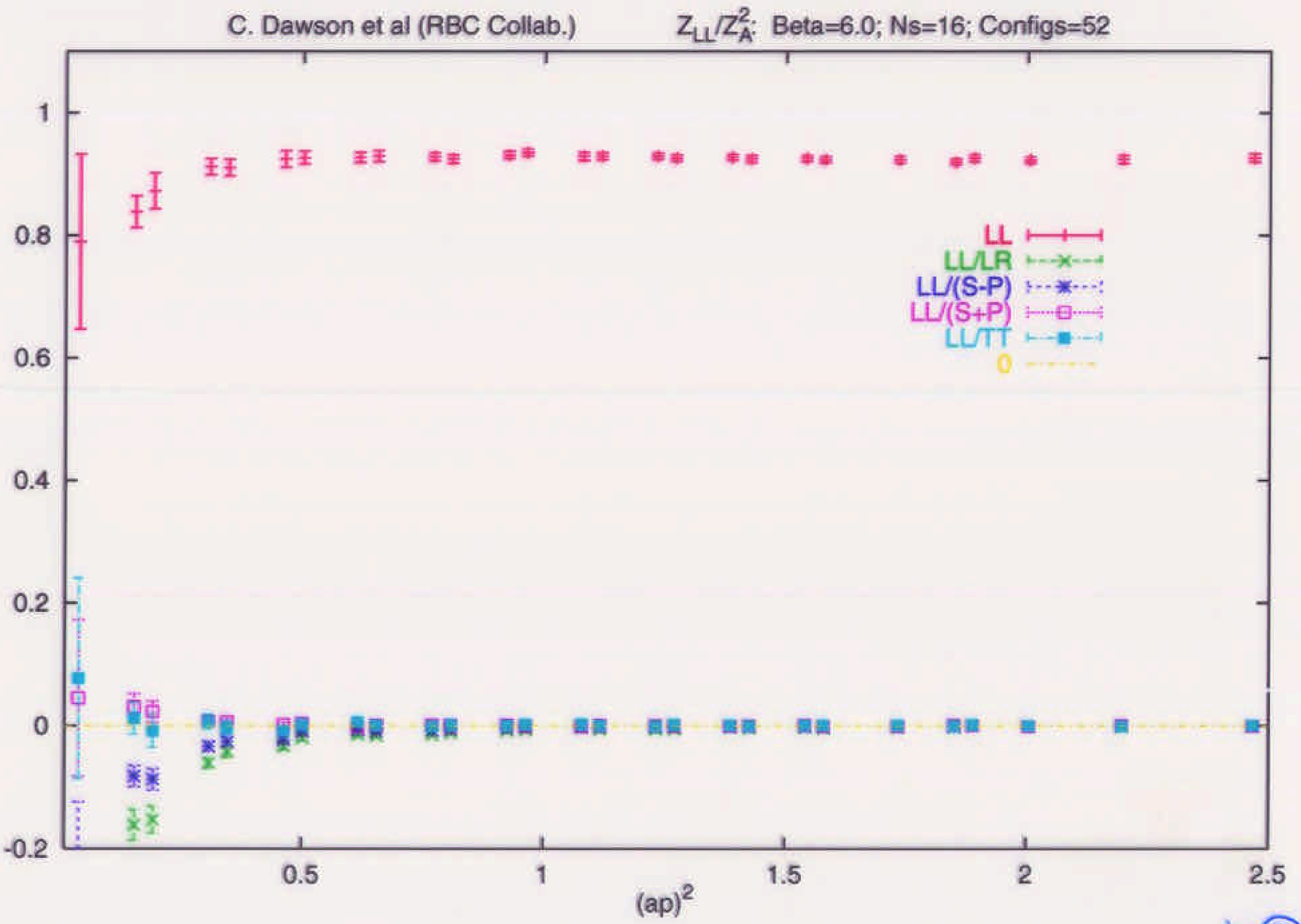
Recall $M_{LL}$ is extremely sensitive to presence of $\gamma_5$

PAPER I

(25) (14)



Chiral Behavior of $O_{LL}$
(PCAC at $a \neq 0$)
$6/g^2=6.0$
$\langle \bar{P}|O_{LL}|P\rangle / |\langle P|\bar{q}\gamma_5 q|0\rangle|^2$
m
M=1.7(B-S, NERSC)
M=1.8(QCDSP)
0
0.05
0.1
0.15
0.1
0.2
0.3
0.4
0.5



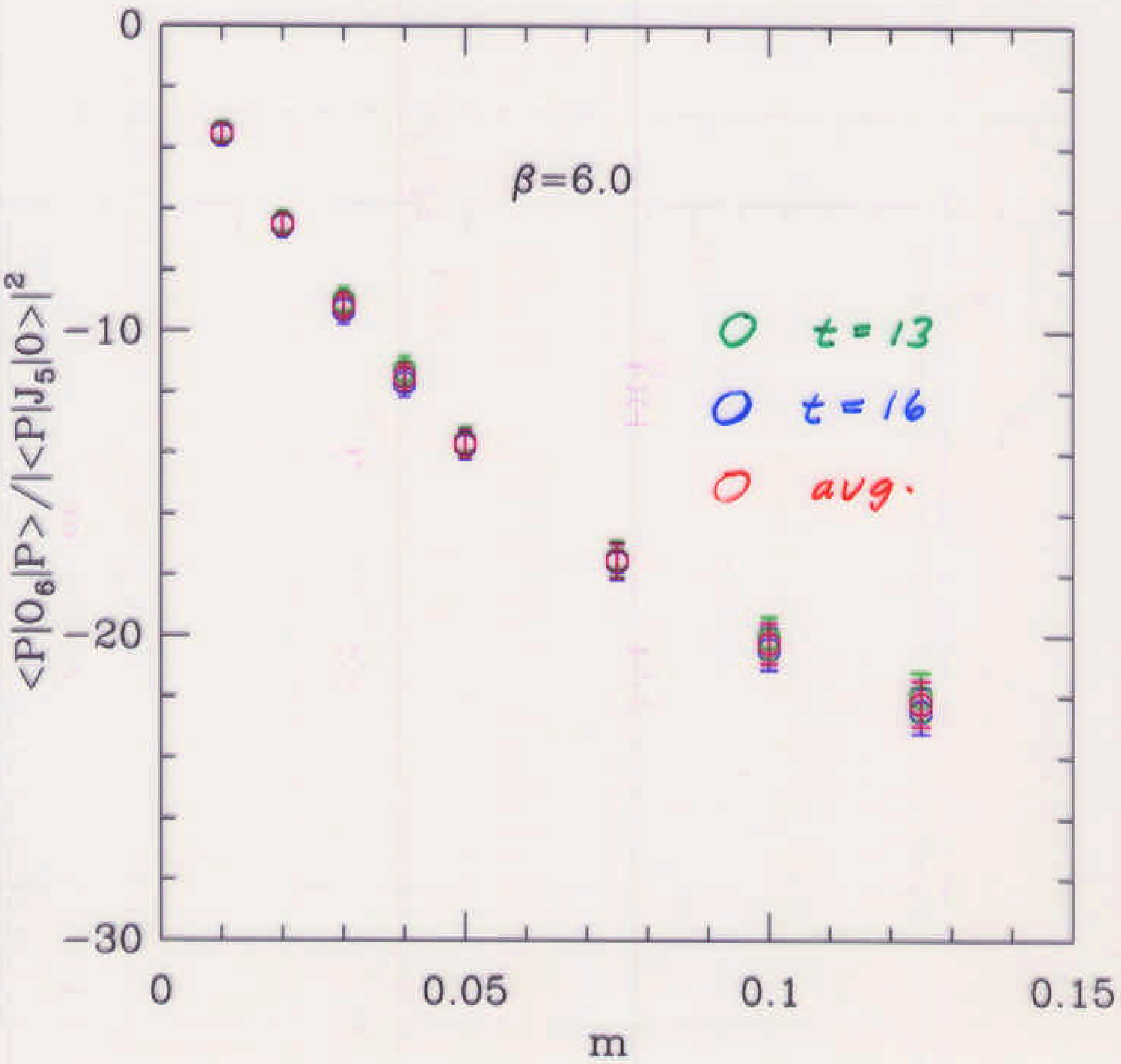
C. Dawson et al (RBC Collab.)
$Z_{LL}/Z_A^2$: Beta=6.0; Ns=16; Configs=52
LL
LL/LR
LL/(S-P)
LL/(S+P)
LL/TT
0
1
0.8
0.6
0.4
0.2
0
-0.2
0.5
1
1.5
2
2.5
$(ap)^2$

16 ~~43~~ ~~39~~

# QCD Penguin

total = ∞ + O
vanishes as $m \to 0$

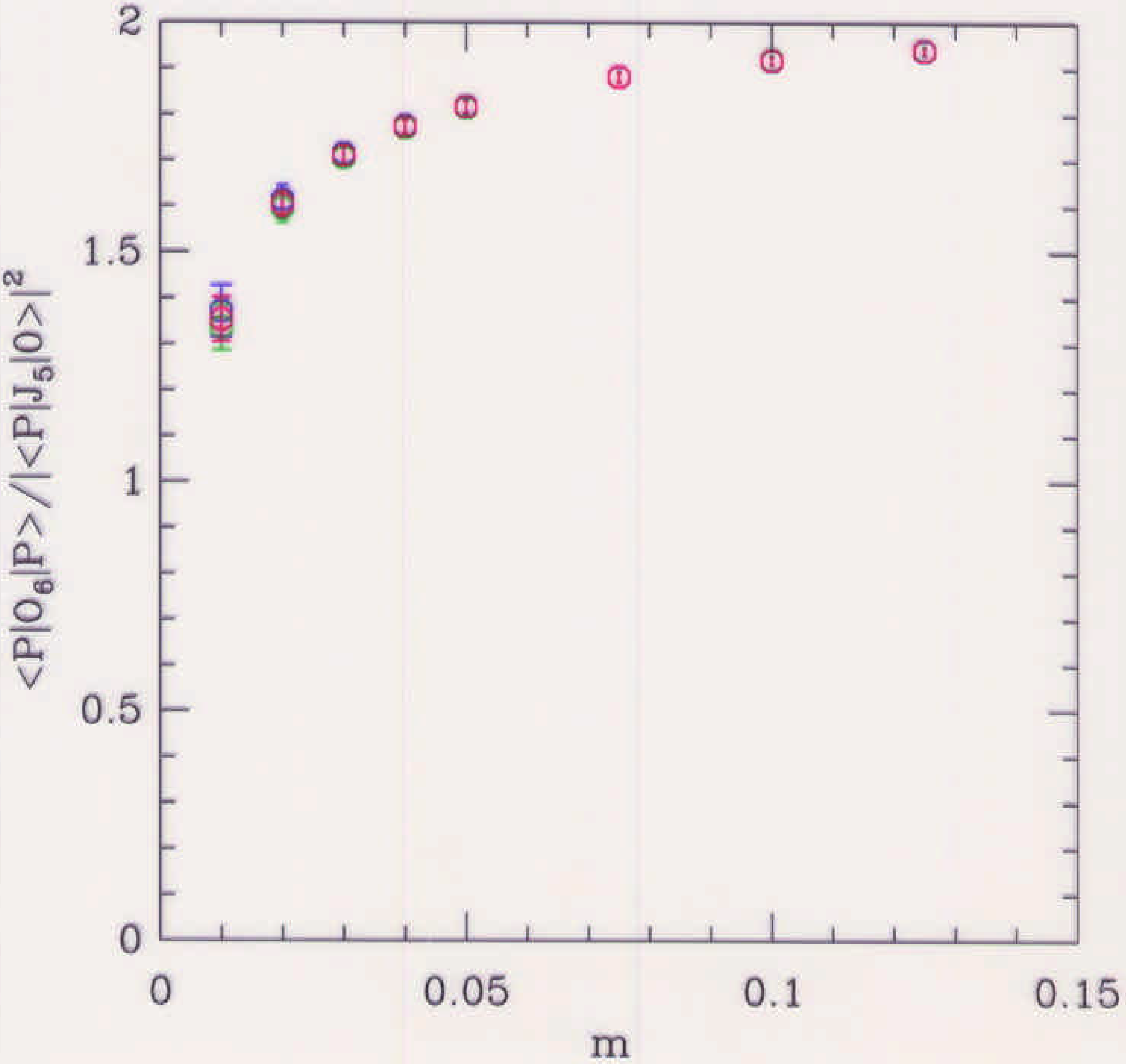
## Chiral Behavior of Lattice $\langle O_6 \rangle$



17 ~~44~~ ~~40~~

$\infty$ only,
does not vanish as $m \to 0$

Fig. Eight part of $<P|O_6|P>(\beta=6.0)$

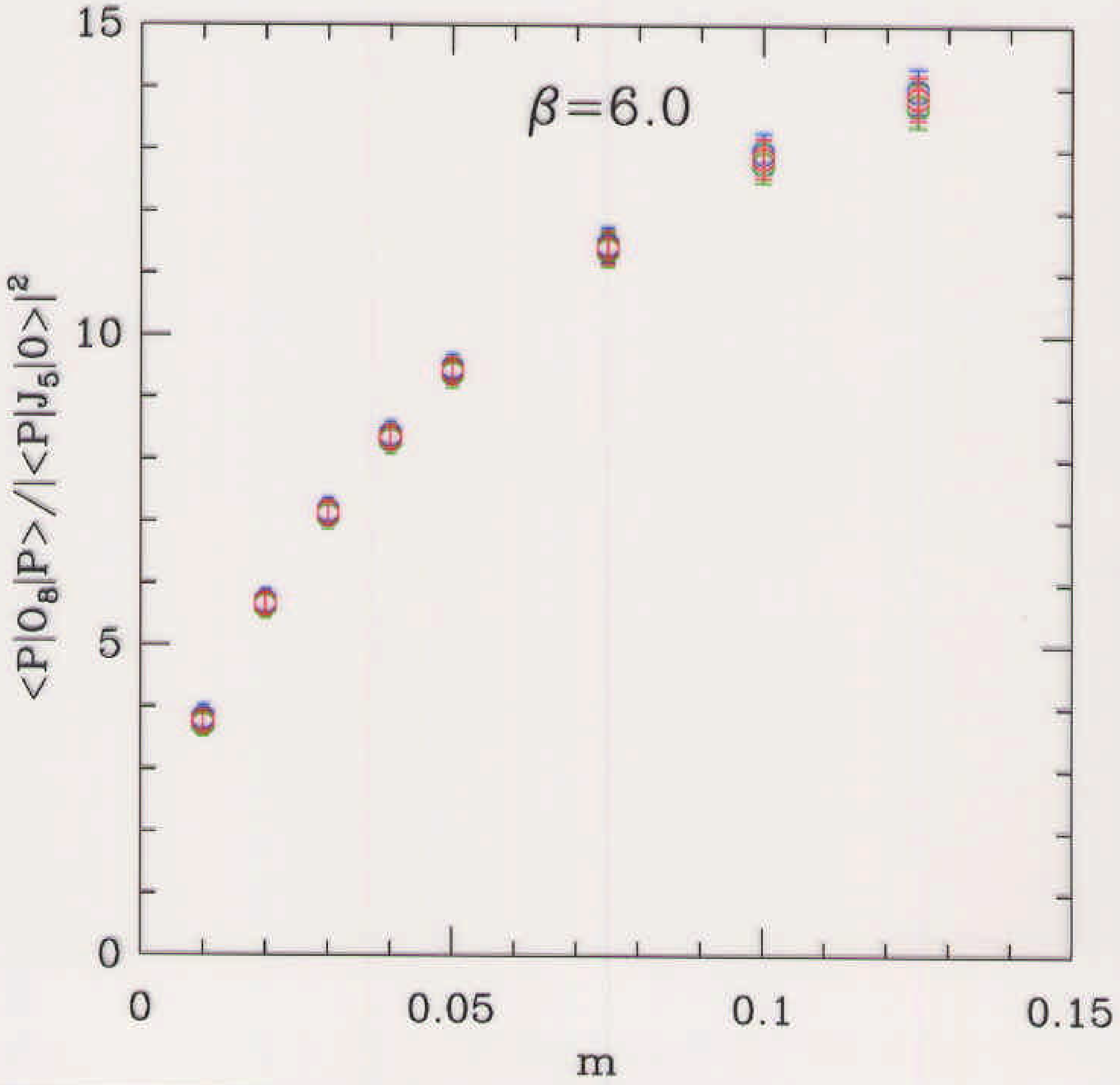


18 45 41

EW Penguin

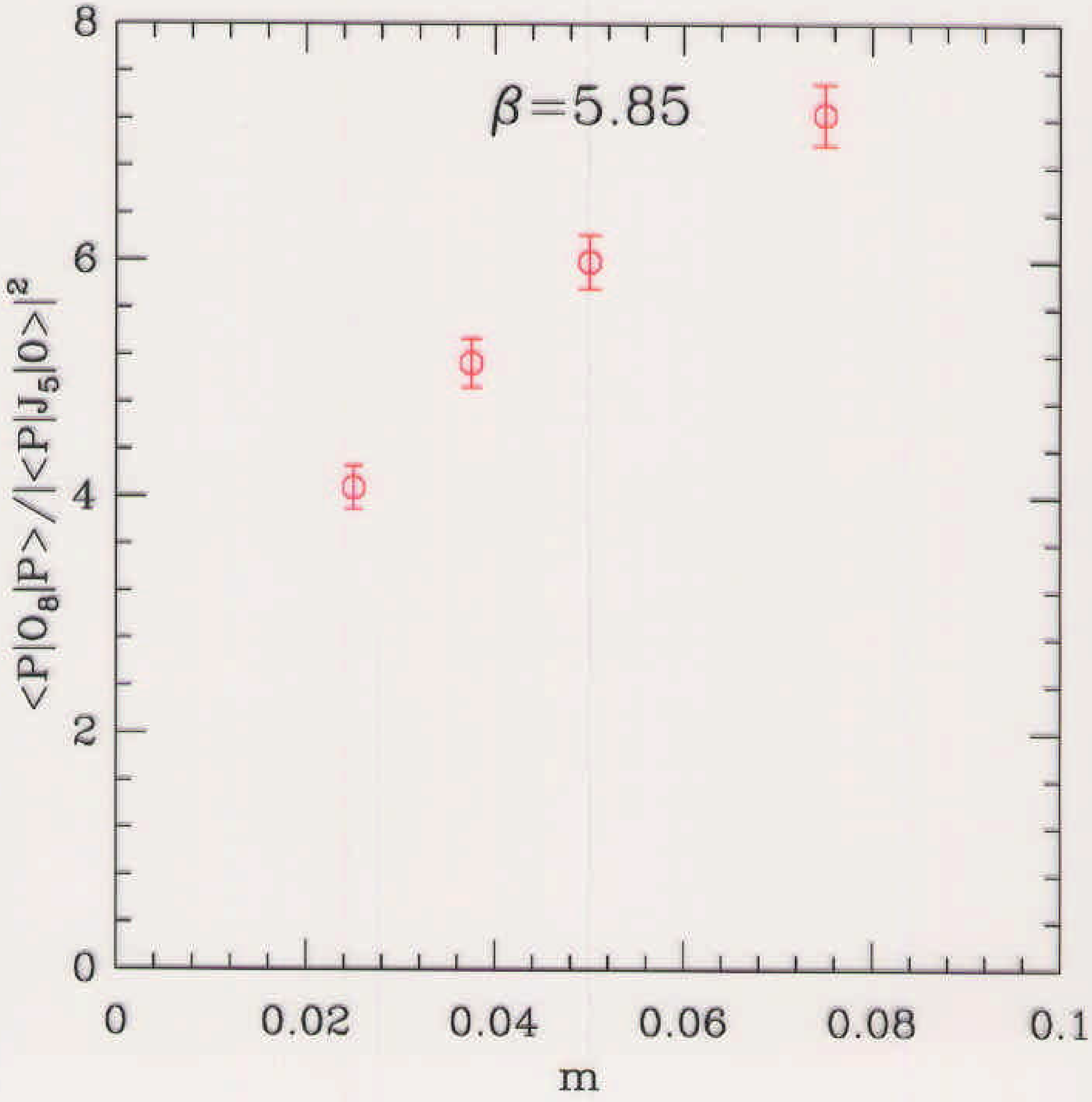
Should not vanish as $m \to 0$

does not " " "



48 19 42

EW Penguin



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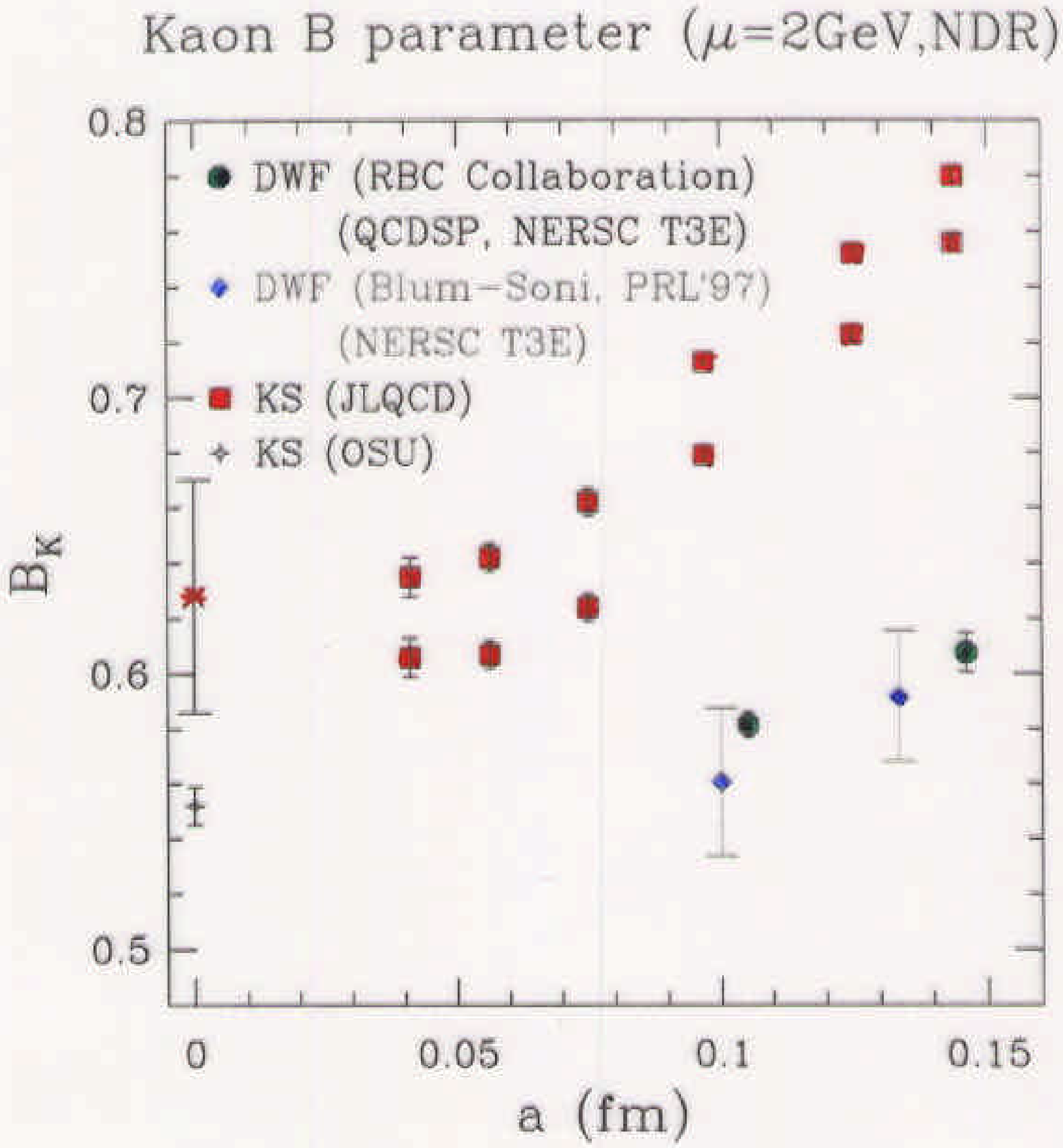



Figure 11: Scaling of the kaon $B$ parameter, $B_K$, renormalized at 2 GeV in the NDR scheme. Errors on the DWF points are statistical only.

SIMULATION COST ~ $a^{-6}$

IMPROVED SCALING WITH DWQ

# Illustrative Sample of Results for $B_K$ (2 GeV) from the Lattice and the Continuum

| Method | Result | Ref. |
|---|---|---|
| Lattice | $B_K^{\mathrm{NDR}}$ (2 GeV) | |
| Clover, $\beta = 6.0$, NPR | $.66 \pm .11$ | L. Conti *et al.* PLB **421**, 273 (98). |
| " " , BPT | $.65 \pm .11$ | " |
| Wilson, $\beta = 6.0$, BPT | $.74 \pm .04 \pm .05$ | T. Bhattacharya *et al.* PRD**55**, 4036 (98) |
| Wilson, NPR($\chi$WI) $\beta = 5.9 - 6.5$ | $0.69 \pm 0.07$ | S. Aoki *et al.*(JLQCD) hep-lat/9901018 |
| Staggered, $\beta = 5.7$–$6.65$ | $.628 \pm \cancel{0.42}$ .042 | S. Aoki *et al.*(JLQCD), PRL **80**, 5271 (98) |
| " $\beta = 6.0$, 6.2,6.4 | $.62 \pm .02 \pm .02$ | G. Kilcup *et al.* PRD**57**, 1654 (98) |
| Stagg, TI $\beta = 5.70$–$6.20$ | $.552 \pm .007$ | G. Kilcup *et al.* NP PS**63**, 293 (98) |
| MF-SW, $\beta = 6.0$, 6.2 | $.72 \pm .08$ | Lellouch and Lin (UKQCD), hep-lat/9809142 |
| DWQ, NPR $\beta = 5.85$–$6.3$ | $\approx 0.6 \pm 0.1$ | T. Blum *et al.* (RBRC) Preliminary |
| Continuum | $\hat{B}_K$ | |
| Lattice Summary $\left(\substack{\text{Quenched} \\ m_d = m_s}\right)$ | $.85 \pm .13$ | including statistics and systematics |
| VSA | 1.0 | Jane and John Doe* |
| LO$\chi$PT | $\approx .33$ | G. Donoghue *et al.* PLB **119**,412 (82) |
| Duality | $\sim .39 \pm .10$ | J. Prades et al ZPC 51, 287(91) |
| Large $N_c$ | $.70 \pm .07$ | W. Bardeen *et al.* PLB **211**, 343(88). |
| $\chi$PT + $N_c$ | $.69 \pm .10$ | J. BIJNENS hep-ph/9910263. |
| $\chi$QM(NLO$\chi$PT) | $1.1 \pm .2$ | S. Bertolini *et al.* hep-ph/9802405 |

EWP B - Parameters

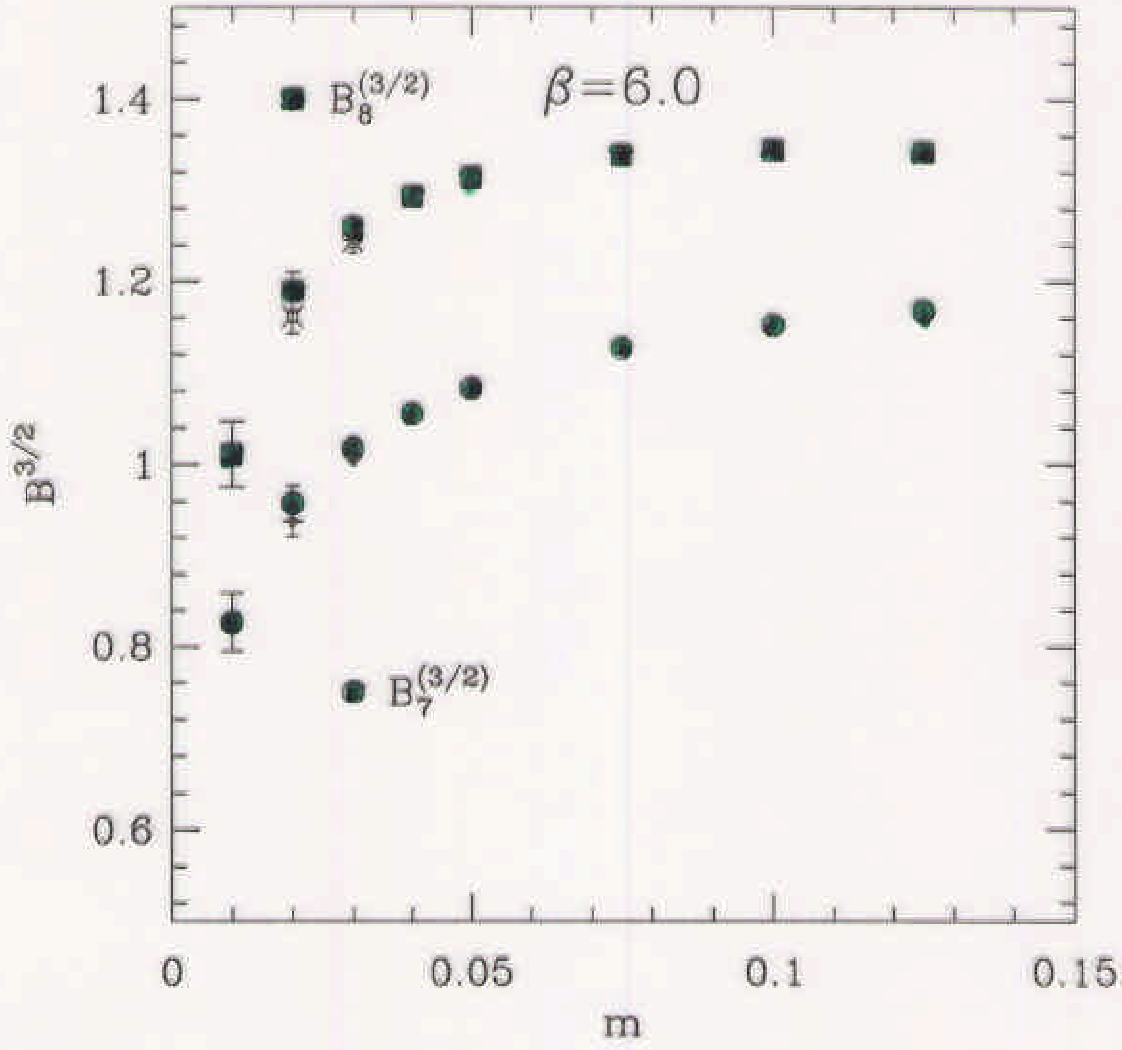



Figure 12: The $\Delta I = 3/2$ electro-weak $B$ parameters, $B_7^{(3/2)}$ (octagons) and $B_8^{(3/2)}$ (squares). The operators have been renormalized at a scale of 2.0 GeV in the NDR scheme. Fancy symbols refer to the 182 configuration ensemble.

$Q_7, Q_8$ (8,8) under $SU(3)_L \times SU(3)_R$



TEND TO CAUSE CANCELLATION ENHANCED BY $\Delta I = 1/2$ Rule.

OTHER Lattice ~ 0.7 — 1.2

Large N ~ 0.4 — 0.6

$\chi$QM ~ .75 — .79

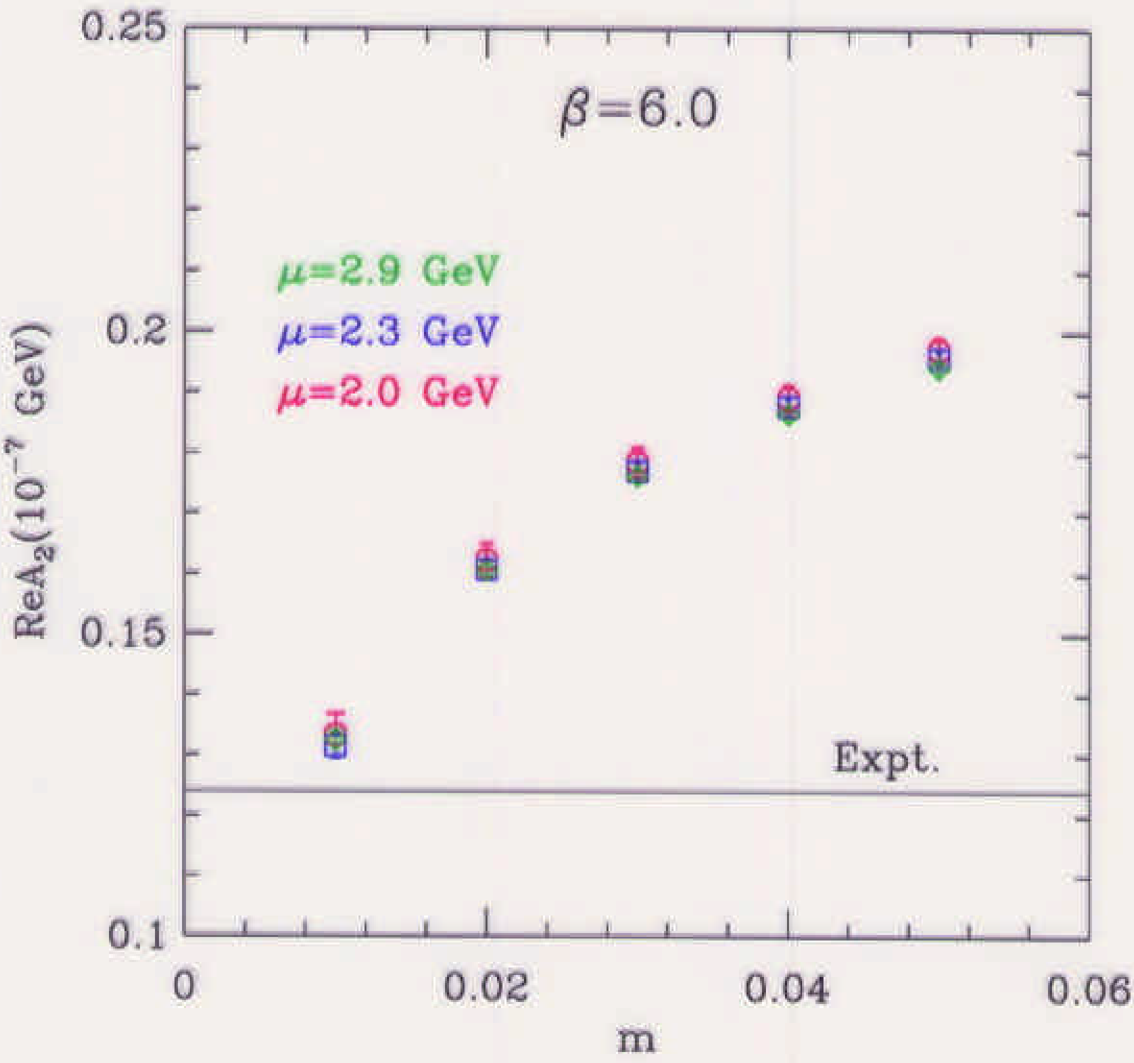
## $\Delta I = 3/2$ $B$ parameters

| Method | $B_7^{3/2}$ | $B_8^{3/2}$ | |
|---|---|---|---|
| Stagg, BPT, $\beta = 6.0, 6.2, 6.4$ | $.62 \pm .03 \pm .06$ | $.77 \pm .04 \pm .04$ | G. Kilcup *et al.*, PRD**57**, 1654 (98) |
| Wilson, BPT, $\beta = 6.0$ | $.58 \pm .02 \pm .07$ | $.81 \pm .03 \pm .03$ | R. Gupta *et al.* PRD(**55**), 4036(97) |
| MF-SW, $\beta = 6.0, 6.2$ | $.58 \pm .05 \pm$ | $.80 \pm .08 \pm$ | Lellouch and Din(UKQCD), hep-ph/9809142 |
| Clover, 6.0, NPR<br>" BPT | $.72 \pm .05$<br>$.58 \pm .02$ | 1.03(3)<br>.83(3) | L. Conti *et al.* PLB (98)<br>" |
| DWQ, NPR $\beta = 5.85, 6.0$ | .75±.15 | .92±.15 | T. Blum *et al.* (RBRC) |

PRELIMINARY

# Isospin 2 Amplitude ($Q_1^{ren}$ and $Q_2^{ren}$, fig 8's only)

## *Significant* mass dependence




$Re\, A_2^{expt} \sim .124 \times 10^{-7}$ GeV

$Re\, A_2^{\chi L} \sim 0.115 \times 10^{-7}$

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# SUMMARY + OUTLOOK.

1. DWQ extremely attractive for arenas (such as K decays) where chiral sym is crucial.

   SIGNIFICANTLY IMPROVED SCALING OVER CONV. DISCRETION
   TENDS TO offset THE XTRA COST of 5th DIM.

2. $\Delta S=1$, $\Delta I=1/2$ Computations for $K\to 2\pi$ & $\epsilon'/\epsilon$ not yet complete

3. $\hat{B}_K \sim .8 \pm .15$ — central value a bit smaller

   $B_7^{3/2} \sim .75 \pm .15$

   $B_8^{3/2} \sim .92 \pm .15$ — Numbers Preliminary ERROR ANALYSIS INCOMPLETE

   $Re A_2^{\chi L} \sim .115 \times 10^{-7}$ GeV

4. OUTLOOK : VERY PROMISING

- After ~ 15 years of standstill, $K\to 2\pi$ becoming amenable to lattice due new formulation of lattice fermions with improved $\chi L$ behavior

  NOTE RBCC, CP-PACS } using DWQ for $K\to 2\pi$

  MARTINELLI et al IMP WILSON Recomputing

  LELLOUCH + LUSCHER ... $K\to 2\pi$ FSI accessible via finite vol correlation studies.