The Weak OPE
and
Dimension-eight Operators

OUTLINE

Modifying The Weak OPE
Calculating Dimension – eight Operators
Revised Estimate of $B_{7,8}^{(3/2)}$

Gene Golowich
Univ. of Massachusetts
Amherst MA 01003 USA
(Report on work by Cirigliano, Donoghue, Golowich)

Parallel Session PA6 (ICHEP2000)

Calculating with Kaons

- Weak OPE

Amplitude — Wilson Coeff. — Matrix Element

$$\mathcal{M} = \sum_d \sum_i \mathcal{C}_i^{(d)}(\mu) \; \langle \mathcal{O}_i^{(d)} \rangle_\mu$$

- Hybrid Methodology

$\mathcal{C}_i^{(d)}(\mu)$: $\overline{\mathrm{MS}}$ renormalization

$\langle \mathcal{O}_i^{(d)} \rangle_\mu$: Cutoff renormalization → Quark Model / 1/N Expansion / Lattice

- Typical Scales (μ)

The Weak OPE (without QCD)

- **Full and Effective Hamiltonians**[1]

$$\mathcal{H}_{\text{full}} \equiv \mathcal{H}_{\text{SM}} = \frac{g_2}{2\sqrt{2}} W_\mu^+ J_{\text{ch}}^\mu$$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\mathcal{C}_2^{(6)} \mathcal{Q}_2^{(6)} + \ldots \right] \qquad \mathcal{Q}_2^{(6)} = J_{\text{ch}}^\mu J_{\text{ch},\mu}^\dagger$$

- **An Example**

Full:  $\rightarrow \frac{g_2^2}{8M_W^2} Q_2^{(6)} + \mathcal{O}\left(\frac{q^2}{M_W^2}\right)$ **Eff:**  $\rightarrow \frac{G_F}{\sqrt{2}} C_2^{(6)} Q_2^{(6)} + \ldots$

- **Preliminary Findings**

1. $\mathcal{C}_2^{(6)} = 1 \qquad (G_F/\sqrt{2} = g_2^2/(8M_W^2))$

2. $d > 6$ operators suppressed by $1/M_W^2$.

[1]Notation: (a) Omit CKM dependence, (b) Denote dimension via superscripts.

The Weak OPE (with QCD)

- **Effect of QCD**

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \mathcal{C}_i^{(6)} \, \mathcal{Q}_i^{(6)}$$

- **Categorizing the $d = 6$ Operators**

Current-current ($\{\mathcal{Q}_{1,2}^{(6)}\}$)

$$\mathcal{Q}_2^{(6)} = \bar{d}_a \Gamma_{\rm L}^{\mu} u_a \; \bar{u}_b \Gamma_{\mu}^{\rm L} s_b \; , \ldots$$

QCD Penguin ($\{\mathcal{Q}_{3,4,5,6}^{(6)}\}$)

$$\mathcal{Q}_5^{(6)} = \bar{d}_a \Gamma_{\rm L}^{\mu} u_a \; \sum_q \; \bar{q}_b \Gamma_{\mu}^{\rm R} q_b \; , \ldots$$

Electroweak Penguin ($\{\mathcal{Q}_{7,8,9,10}^{(6)}\}$)

$$\mathcal{Q}_7^{(6)} = \frac{3}{2} \, \bar{d}_a \Gamma_{\rm L}^{\mu} u_a \; \sum_q \; e_q \; \bar{q}_b \Gamma_{\mu}^{\rm R} q_b \; , \ldots$$

Dimension-eight Operators in the Weak OPE

- This issue is somewhat subtle:

  1] In a pure cutoff scheme, dimension-eight operators occur in the weak hamiltonian at order $G_F \alpha_s / \mu^2$, μ being the separation scale.

  2] This is explicitly demonstrated in a calculation involving a LR weak hamiltonian.

  3] In dimensional regularization (DR), the $d = 8$ operators do *not* appear explicitly in the hamiltonian at order $G_F \alpha_s$. However, the use of a cutoff scheme (such as lattice or quark model methods) for the calculation of the matrix elements of dimension-six operators requires a careful matching onto DR for which dimension-eight operators *do* play an important role.

Dimension-eight Operators in the Weak OPE

Vincenzo Cirigliano, John F. Donoghue and Eugene Golowich
Department of Physics and Astronomy, University of Massachusetts
Amherst, MA 01003 USA
vincenzo@het2.physics.umass.edu donoghue@physics.umass.edu
golowich@physics.umass.edu

Abstract

We argue that there is a potential flaw in the standard treatment of weak decay amplitudes, including that of ϵ'/ϵ. We show that (contrary to conventional wisdom) dimension-eight operators *do* contribute to weak amplitudes, at order $G_F\alpha_s$ and without $1/M_W^2$ suppression. We demonstrate the existence of these operators through the use of a simple weak hamiltonian. Their contribution appears in different places depending on which scheme is adopted in performing the OPE. If one performs a complete separation of short and long distance physics within a cutoff scheme, dimension-eight operators occur in the weak hamiltonian at order $G_F\alpha_s/\mu^2$, μ being the separating scale. However, in an $\overline{\rm MS}$ renormalization scheme for the OPE the dimension-eight operators do not appear explicitly in the hamiltonian at order $G_F\alpha_s$. In this case, matrix elements must include physics above the scale μ, and it is here that dimension eight effects enter. The use of a cutoff scheme (especially quark model methods) for the calculation of the matrix elements of dimension-six operators is inconsistent with $\overline{\rm MS}$ unless there is careful matching including dimension-eight operators. The contribution of dimension-eight operators can be minimized by working at large enough values of the scale μ. We find from sum rule methods that the contribution of dimension-eight operators to the dimension-six operator $Q_7^{(6)}$ is at the 100% level for $\mu = 1.5$ GeV. This suggests that presently available values of μ are too low to justify the neglect of these effects. Finally, we display the dimension-eight operators which appear within the Standard Model at one loop.

Physics of a LR Hamiltonian

- **Calculating ϵ'/ϵ**

Need $\langle(\pi\pi)_0|\mathcal{Q}_6^{(6)}|K\rangle$ and $\langle(\pi\pi)_2|\mathcal{Q}_8^{(6)}|K\rangle$

Chiral limit: $\langle(\pi\pi)_2|\mathcal{Q}_{7,8}^{(6)}|K\rangle \to \langle 0|\mathcal{O}_{1,8}^{(6)}|0\rangle$

- **The Two DG Sum Rules**

DG1:

$$\frac{16\pi^2}{3}\langle\mathcal{O}_1^{(6)}\rangle_\mu^{(\mathrm{c.o.})} = \int_0^\infty ds\ s^2 \ln\left(\frac{s+\mu^2}{s}\right)\Delta\rho(s)$$

DG2:

$$2\pi\langle\alpha_s\mathcal{O}_8^{(6)}\rangle_\mu^{(\mathrm{c.o.})} = \int_0^\infty ds\ s^2\frac{\mu^2}{s+\mu^2}\Delta\rho(s)$$

A Chiral Matrix Element

$$\mathcal{M}(p) = \langle \pi^-(p)|\mathcal{H}_{\mathrm{LR}}|K^-(p)\rangle$$

$$\mathcal{M} \equiv \mathcal{M}(0) = \lim_{\mathrm{p}=0} \mathcal{M}(p)$$

$$= \frac{3G_F M_W^2}{32\sqrt{2}\pi^2 F_\pi^2}\int_0^\infty dQ^2 \, \frac{Q^4}{Q^2+M_W^2}\,\Delta\Pi_3(Q^2)$$

$$= \frac{G_F}{2\sqrt{2}F_\pi^2}\Big[\underline{\langle \mathcal{O}_1^{(6)}\rangle_\mu^{(\mathrm{c.o.})}} + \underline{\frac{3}{8\pi}\ln\left(\frac{M_W^2}{\mu^2}\right)\langle \alpha_s \mathcal{O}_8^{(6)}\rangle_\mu}$$

$$+ \underline{\frac{3}{16\pi^2}\frac{\mathcal{E}_\mu^{(8)}}{\mu^2}} + \ldots\Big]$$

$$\simeq \begin{cases} -0.12 - 3.84 + 0.64 + \ldots & (\mu = 1\ \mathrm{GeV}) \\ -0.28 - 3.49 + 0.30 + \ldots & (\mu = 1.5\ \mathrm{GeV}) \\ -0.44 - 3.24 + 0.17 + \ldots & (\mu = 2\ \mathrm{GeV}) \\ -0.89 - 2.63 + 0.04 + \ldots & (\mu = 4\ \mathrm{GeV}) \end{cases}$$

$B_7^{(3/2)}$ and $B_8^{(3/2)}$

- **Determine** $\langle \mathcal{O}_{1,8} \rangle_{2\ \mathrm{GeV}}^{(\overline{\mathrm{MS}})}$

  (i) Sum rules DG1, DG2 at $\mu = 4$ GeV.

  (ii) RGE running down to $\mu = 2$ GeV.

  $$\langle \mathcal{O}_8 \rangle_{2\ \mathrm{GeV}}^{(\overline{\mathrm{MS}}[\mathrm{NDR}])} = -(1.29 \pm 0.25) \cdot 10^{-3}\ \mathrm{GeV}^6\ ,$$

  $$\langle \mathcal{O}_1 \rangle_{2\ \mathrm{GeV}}^{(\overline{\mathrm{MS}}[\mathrm{NDR}])} = -(1.02 \pm 0.17) \cdot 10^{-4}\ \mathrm{GeV}^6\ .$$

- **Determine** $B_7^{(3/2)}$ **and** $B_8^{(3/2)}$

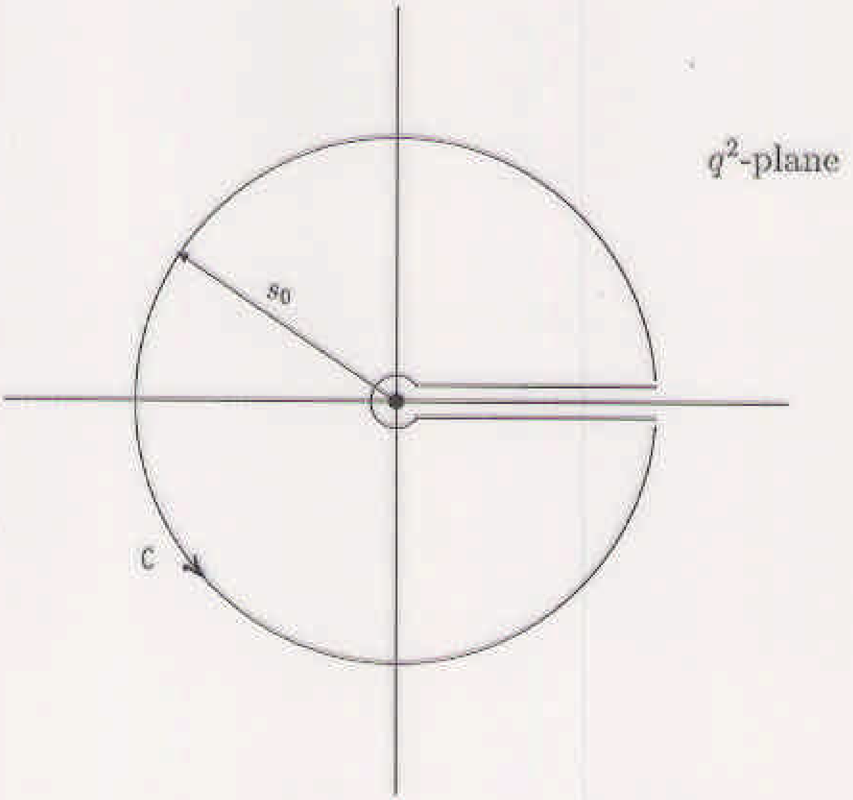
$$B_7^{(3/2)}\Big|_{2\ \mathrm{GeV}}^{(\overline{\mathrm{MS}}[\mathrm{NDR}])} =$$

$$B_8^{(3/2)}\Big|_{2\ \mathrm{GeV}}^{(\overline{\mathrm{MS}}[\mathrm{NDR}])} =$$

PRELIMINARY

FESR Analysis

(**Maltman** and **Golowich**)



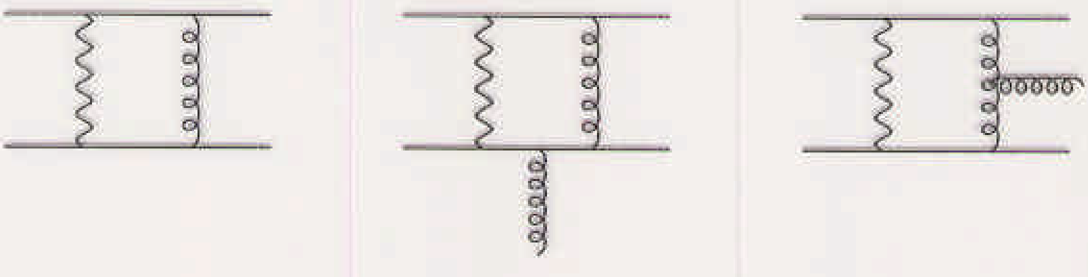
$$-\frac{1}{2\pi i}\oint_C dq^2\ w(q^2)\ \Delta\Pi(q^2) = \int_0^{s_0} ds\ w(s)\ \Delta\rho(s)$$

$d = 8$ Operators in the Standard Model

$\to Q_2^{(6)}$

QCD PENGUIN

CURRENT-CURRENT



Current-current

• Dimension-eight Operators[1]

$$
\begin{aligned}
\mathcal{Q}_1^{(8)} &= \bar{u}\,\overleftarrow{\mathcal{D}}_\mu \overleftarrow{\mathcal{D}}^\mu\,\Gamma_\nu^a s\;\bar{d}\Gamma_a^\nu u + \bar{u}\Gamma_\nu^a \mathcal{D}_\mu \mathcal{D}^\mu s\;\bar{d}\Gamma_a^\nu u + \bar{u}\Gamma_\nu^a s\;\bar{d}\,\overleftarrow{\mathcal{D}}_\mu \overleftarrow{\mathcal{D}}^\mu\,\Gamma_a^\nu u + \bar{u}\Gamma_\nu^a s\;\bar{d}\Gamma_a^\nu \mathcal{D}_\mu \mathcal{D}^\mu u \\
\mathcal{Q}_2^{(8)} &= \bar{u}\Gamma_\nu^a \mathcal{D}_\mu s\;\bar{d}\Gamma_a^\nu \mathcal{D}^\mu u + \bar{u}\,\overleftarrow{\mathcal{D}}^\mu\,\Gamma_\nu^a s\;\bar{d}\,\overleftarrow{\mathcal{D}}_\mu\,\Gamma_a^\nu u \\
\mathcal{Q}_3^{(8)} &= \bar{u}\,\overleftarrow{\mathcal{D}}_\mu\,\Gamma_\nu^a s\;\bar{d}\Gamma_a^\nu \mathcal{D}^\mu u + \bar{u}\Gamma_\nu^a \mathcal{D}_\mu s\;\bar{d}\,\overleftarrow{\mathcal{D}}^\mu\,\Gamma_a^\nu u \\
\mathcal{Q}_4^{(8)} &= \bar{u}\,\overleftarrow{\mathcal{D}}_\mu \overleftarrow{\mathcal{D}}_\nu\,\Gamma_a^\nu s\;\bar{d}\Gamma_a^\mu u + \bar{u}\Gamma_a^\nu \mathcal{D}_\nu \mathcal{D}_\mu s\;\bar{d}\Gamma_a^\mu u + \bar{u}\Gamma_a^\nu s\;\bar{d}\,\overleftarrow{\mathcal{D}}_\nu \overleftarrow{\mathcal{D}}_\mu\,\Gamma_a^\mu u + \bar{u}\Gamma_a^\nu s\;\bar{d}\Gamma_a^\mu \mathcal{D}_\mu \mathcal{D}_\nu u \\
\mathcal{Q}_5^{(8)} &= \bar{u}\,\overleftarrow{\mathcal{D}}_\mu\,\Gamma_a^\nu s\;\bar{d}\,\overleftarrow{\mathcal{D}}_\nu\,\Gamma_a^\mu u + \bar{u}\Gamma_\nu^a \mathcal{D}_\mu s\;\bar{d}\Gamma_a^\mu \mathcal{D}^\nu u \\
\mathcal{Q}_6^{(8)} &= \bar{u}\,\overleftarrow{\mathcal{D}}_\mu\,\Gamma_a^\nu s\;\bar{d}\Gamma_a^\mu \mathcal{D}_\nu u + \bar{u}\Gamma_a^\nu \mathcal{D}_\mu s\;\bar{d}\,\overleftarrow{\mathcal{D}}_\nu\,\Gamma_a^\mu u \\
\mathcal{Q}_7^{(8)} &= g_3 \delta^{ab} \tilde{F}^{\mu\nu,b} \left[\bar{u}\Gamma_\mu^a s\;\bar{d}\Gamma_\nu u - \bar{u}\Gamma_\mu s\;\bar{d}\Gamma_\nu^a u\right] \;.
\end{aligned}
$$

• Dimension-eight Wilson Coefficients

$$
\mathcal{C}_i^{(8)} = \frac{\alpha_s}{4\pi} \cdot \frac{1}{\mu^2} \cdot \eta_i^{(8)} \quad .
$$

$\eta_1^{(8)}$	$\eta_2^{(8)}$	$\eta_3^{(8)}$	$\eta_4^{(8)}$	$\eta_5^{(8)}$	$\eta_6^{(8)}$	$\eta_7^{(8)}$
5/3	22/3	8/3	−1/3	16/3	14/3	1/3

---

[1] (a) $\Gamma_\mu^a \equiv \frac{\lambda^a}{2}\Gamma_\mu^L \equiv \frac{\lambda^a}{2}\gamma_\mu(1+\gamma_5)$, (b) Cutoff scheme adopted, (c) Chiral limit!

QCD Penguin[1]

• Gauge Invariant Basis ($d = 6$)

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left[\mathcal{C}_2^{(6)}\mathcal{Q}_2^{(6)} + \mathcal{C}_{\text{P}}^{(6)}\mathcal{Q}_{\text{P}}^{(6)}\right]$$

$$\mathcal{Q}_{\text{P}}^{(6)} = \bar{d}\frac{\lambda^A}{2}\Gamma^{\text{L}}_{\mu}s\; q_\mu F_A^{\mu\nu}$$

• Matching (Two Generations)

Defining $g_c = \dfrac{m_c^2}{\mu^2 + m_c^2}$, find

$$\mathcal{C}_{\text{P}}^{(6)}(\mu) = \frac{g_3}{4\pi^2}\left[\frac{4}{3}\ln\frac{\mu^2 + m_c^2}{\mu^2} - 2g_c + 2g_c^2 - \frac{2}{9}g_c^3\right]$$

[1] REMINDER – Working in the chiral limit!

QCD Penguin

• Dimension-eight Operators

$$\mathcal{O}^{(8)}_{\rm P1} = \bar{d}\Gamma^a_\nu s\ \mathcal{D}^2\mathcal{D}_\mu F^{\mu\nu}_a \ ,$$

$$\mathcal{O}^{(8)}_{\rm P2} = i\bar{d}\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]\Gamma^\mu_L s\ F^{\alpha\beta}_b\mathcal{D}_\mu F^a_{\alpha\beta} \ ,$$

$$\mathcal{O}^{(8)}_{\rm P3} = \bar{d}\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]_+\Gamma^\mu_L s\ F^{\alpha\beta}_a\mathcal{D}_\alpha\tilde{F}^b_{\beta\mu} \ .$$

• Dimension-eight Wilson Coefficients

$$\mathcal{C}^{(8)}_{\rm P1}(\mu) = \frac{g_3}{(4\pi)^2}\frac{1}{\mu^2+m_c^2}\left[\frac{1}{3}\frac{m_c^2}{\mu^2}-\frac{2}{3}g_c+\frac{4}{3}g_c^2-\frac{2}{3}g_c^3+\frac{1}{15}g_c^4\right] \ ,$$

$$\mathcal{C}^{(8)}_{\rm P2}(\mu) = \frac{\alpha_s}{3\pi}\frac{1}{\mu^2+m_c^2}\left[-\frac{4}{3}\frac{m_c^2}{\mu^2}-3g_c+\frac{10}{3}g_c^2-\frac{13}{6}g_c^3+\frac{2}{5}g_c^4\right] \ ,$$

$$\mathcal{C}^{(8)}_{\rm P3}(\mu) = \frac{\alpha_s}{3\pi}\frac{1}{\mu^2+m_c^2}\left[\frac{4}{3}\frac{m_c^2}{\mu^2}+2g_c-\frac{4}{3}g_c^2-\frac{1}{3}g_c^3\right] \ .$$

Dimensional Regularization – I

- **Comments**

  Most used approach

  Calculate in d dimensions

  Introduce scale $\mu_{\text{d.r.}}$

  Observe: $\mu_{\text{d.r.}}$ is *not* a separation scale.

- **Chiral Matrix Element**

$$\langle \mathcal{O}_1^{(6)} \rangle_\mu^{(\text{d.r.})} = \langle \mathcal{O}_1^{(6)} \rangle_\mu^{(\text{c.o.})}$$

$$+ \frac{d-1}{(4\pi)^{d/2}} \cdot \frac{\mu_{\text{d.r.}}^{4-d}}{\Gamma(d/2)} \int_{\mu^2}^{\infty} dQ^2 \, Q^d \, \Delta\Pi_3(Q^2)$$

Dimensional Regularization – II

- **$\overline{\text{MS}}$-NDR Matrix Element**

  $\overline{\text{MS}}$: Remove $2/\epsilon - \gamma + \ln(4\pi)$

  NDR: Chirality in d-dimensions

- **Relation between C.O. and D.R.**

$$\langle \mathcal{O}_1^{(6)} \rangle_\mu^{(\overline{\text{MS}}-\text{NDR})} = \langle \mathcal{O}_1^{(6)} \rangle_\mu^{(\text{c.o.})}$$

$$+\frac{3}{8\pi}\left[\ln\frac{\mu_{\text{d.r.}}^2}{\mu^2} - \frac{1}{6}\right]\langle \alpha_s \mathcal{O}_8^{(6)} \rangle_\mu + \underline{\frac{3}{16\pi^2} \cdot \frac{\mathcal{E}_\mu^{(8)}}{\mu^2}} + \ldots$$

- **Comments:**

  The effect of the $d = 8$ contribution to the weak OPE now appears in the $d = 6$ $\overline{\text{MS}}$-NDR operator *matrix element.*

Conclusions

- A new insight to the weak OPE ...
- Inferring $\langle \mathcal{O}^{(6)} \rangle_\mu^{(\overline{\rm MS})}$ from $\langle \mathcal{O}^{(6)} \rangle_\mu^{(\rm c.o.)}$ requires careful matching.
- Dimension-eight effects play a role!
- Omitted in the 'usual' approach.
- Existing work on ϵ'/ϵ affected.
- Numerically important for $\mu \leq 2$ GeV.
- Updated values for $B_{7,8}^{(3/2)}$ presented.