Estimating ε'/ε

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More Definitions

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S \rangle} \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S \rangle},$$

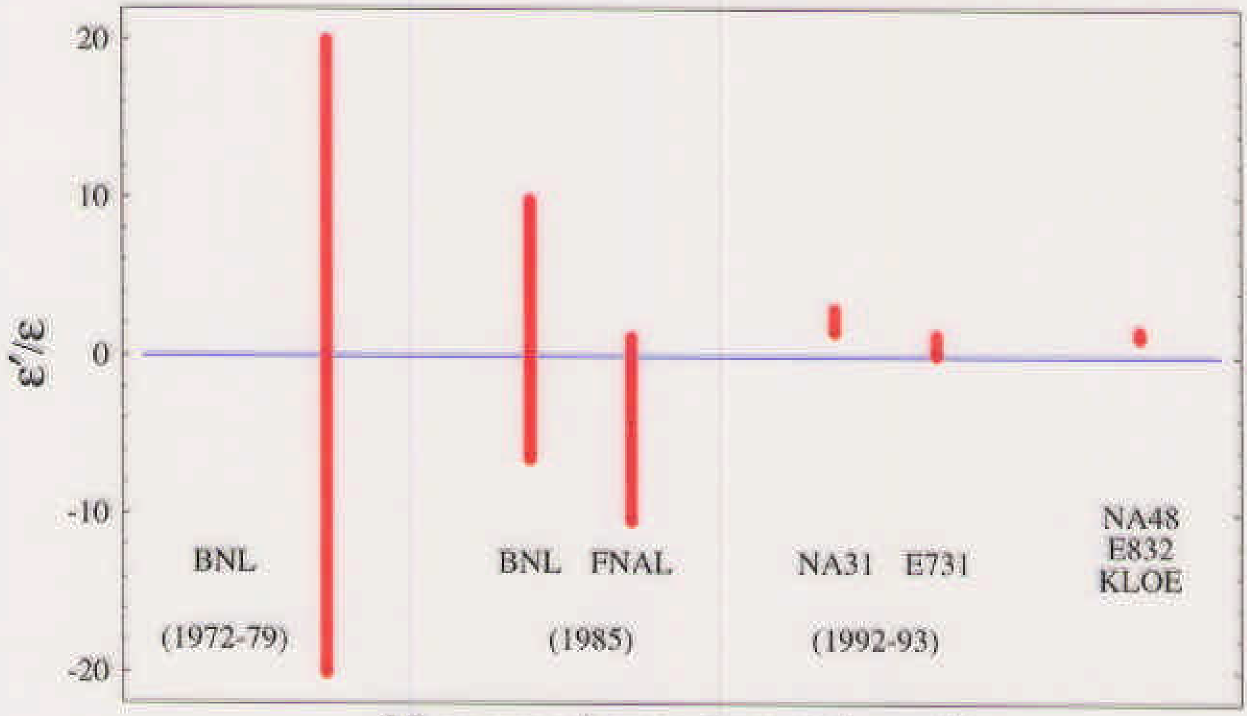
$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = 1 + 6 \,\mathrm{Re}\, \frac{\varepsilon'}{\varepsilon}$$

$$\langle (\pi\pi)_{(I)} | \mathcal{H}_W | K^0 \rangle = i A_I \exp(i\,\delta_I) \quad \langle (\pi\pi)_{(I)} | \mathcal{H}_W | \bar{K}^0 \rangle = i A_I^* \exp(i\,\delta_I)$$

$$\varepsilon' = e^{i(\pi/2 + \delta_2 - \delta_0)} \frac{\omega}{\sqrt{2}} \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right)$$

$$\omega \simeq \frac{\mathrm{Re} A_2}{\mathrm{Re} A_0}$$

The Experiments (circa 1998)



25 years of experimental results

CERN (NA31): $\text{Re}\,(\varepsilon'/\varepsilon) = (23 \pm 3.6 \pm 5.4) \times 10^{-4}$

FNAL (E731): $\text{Re}\,(\varepsilon'/\varepsilon) = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4}$

The Effective Hamiltonian

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu)$$

$$\tau = -V_{td} V_{ts}^* / V_{ud} V_{us}^*$$

$$\begin{aligned}
Q_1 &= \left(\bar{s}_\alpha u_\beta\right)_{V-A} \left(\bar{u}_\beta d_\alpha\right)_{V-A} \\
Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \\
Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} \\
Q_{4,6} &= \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V\mp A} \\
Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_q \hat{e}_q (\bar{q}q)_{V\pm A} \\
Q_{8,10} &= \frac{3}{2} \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q \hat{e}_q (\bar{q}_\beta q_\alpha)_{V\pm A}
\end{aligned}$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F \omega}{2|\varepsilon| \operatorname{Re} A_0} \operatorname{Im} \lambda_t \left[\Pi_0 - \frac{1}{\omega} \Pi_2 \right]$$

$$\Pi_0 = \frac{1}{\cos \delta_0} \sum_i y_i \langle Q_i \rangle_0$$

$$\Pi_2 = \frac{1}{\cos \delta_2} \sum_i y_i \langle Q_i \rangle_2 + \omega \sum_i y_i \langle Q_i \rangle_0 \, \Omega_{\eta+\eta'}$$

$$\operatorname{Im} \lambda_t \equiv \operatorname{Im} V_{td} V_{ts}^*$$

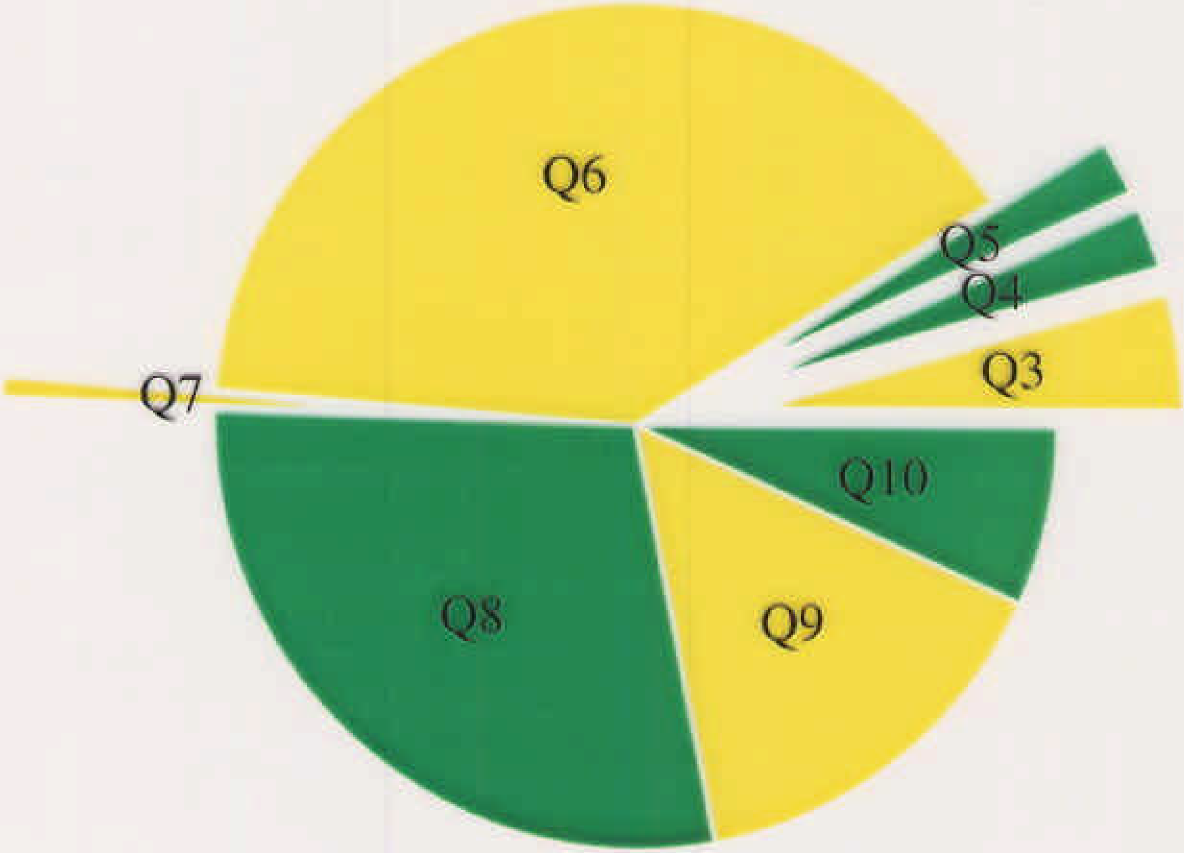
Back-of-the-Envelope Estimate

$$\frac{G_F \omega}{2|\varepsilon| \operatorname{Re} A_0} \simeq 10^3 \text{ GeV}^{-3} \qquad \operatorname{Im} \lambda_t \simeq 10^{-4}$$

$$\Pi_{0,2} \simeq \frac{\alpha_s}{\pi} [m_K]^3 \simeq 10^{-2} \text{ GeV}^3$$

$$\varepsilon'/\varepsilon \simeq 10^{-3}$$

The Problem

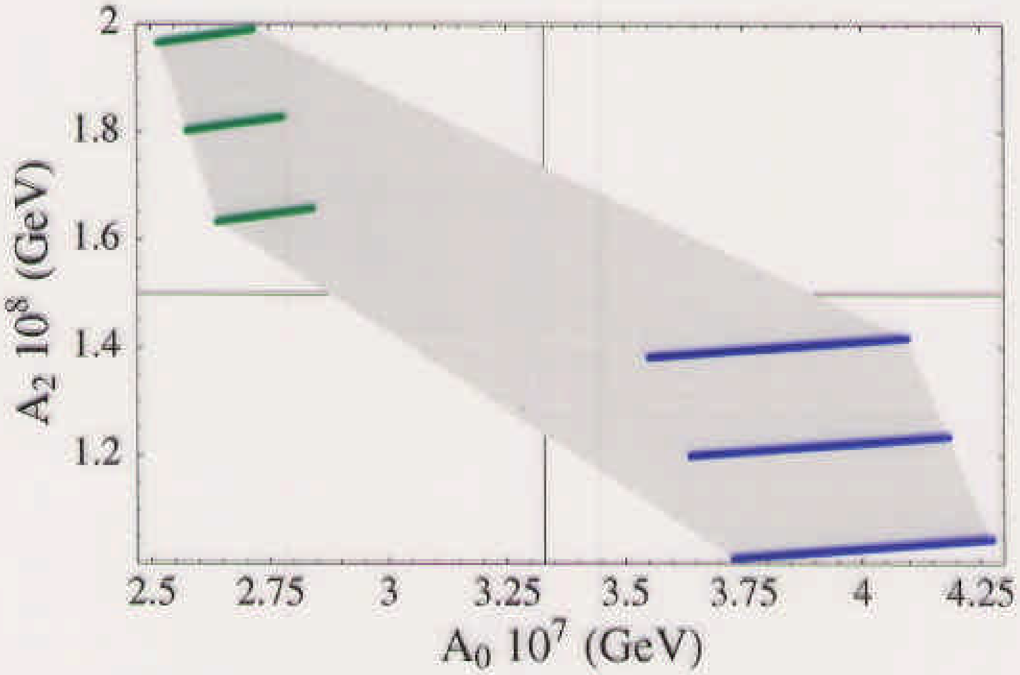


yellow = positive   green = negative

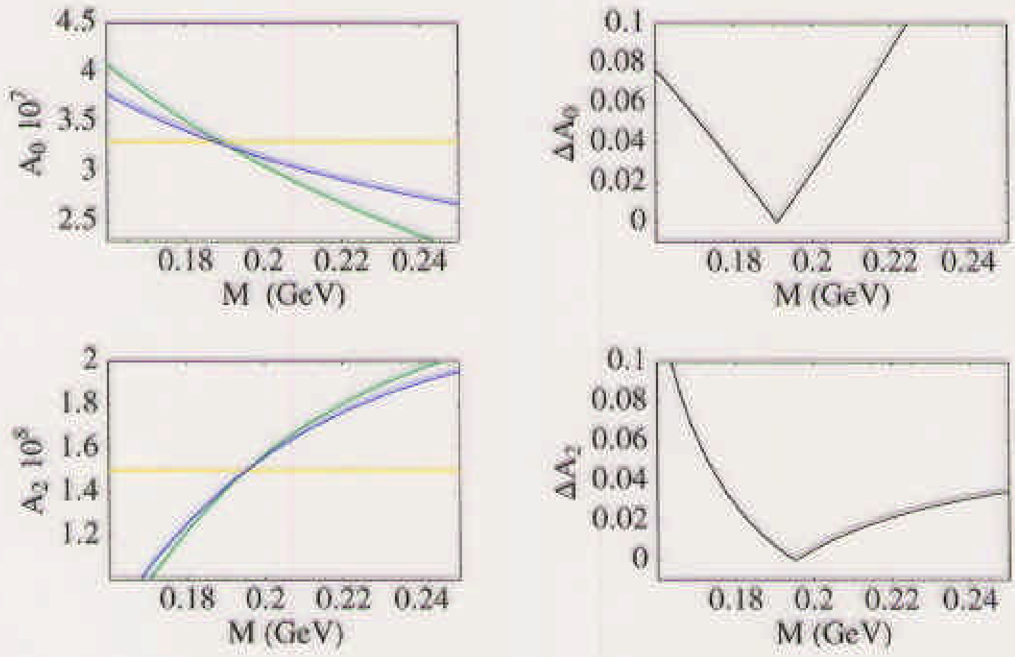
Hadronic Matrix Elements in the χQM

- very simple
  - to be able to understand it analytically ($\Longrightarrow$ in terms of few parameters)
- but not too simple
  - to be able to model the relevant physics ($\Longrightarrow \Delta I = 1/2$ rule and non-factorization)

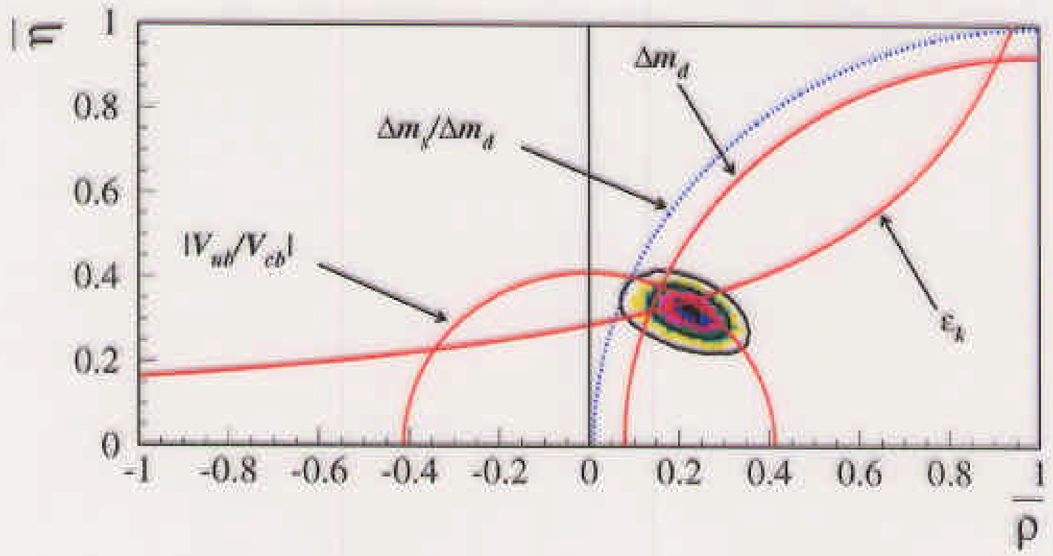
Fitting the $\Delta I = 1/2$ Rule



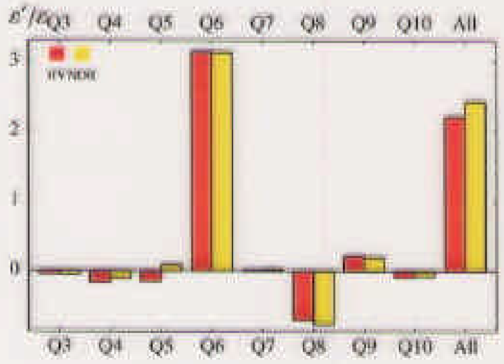
γ_5-Scheme Independence

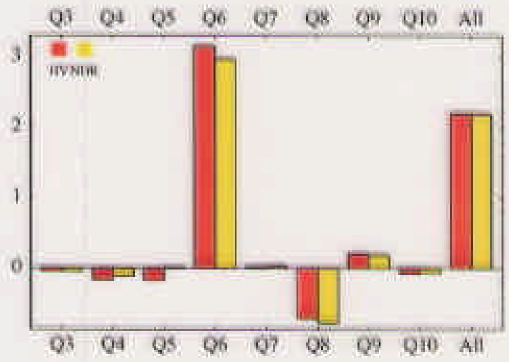


CKM Matrix Elements

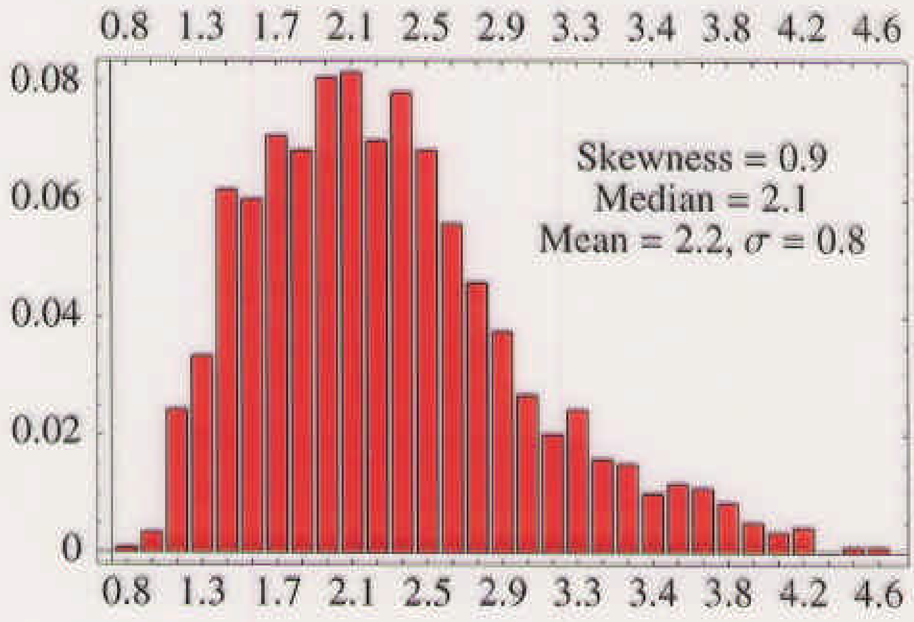


Scale Independence



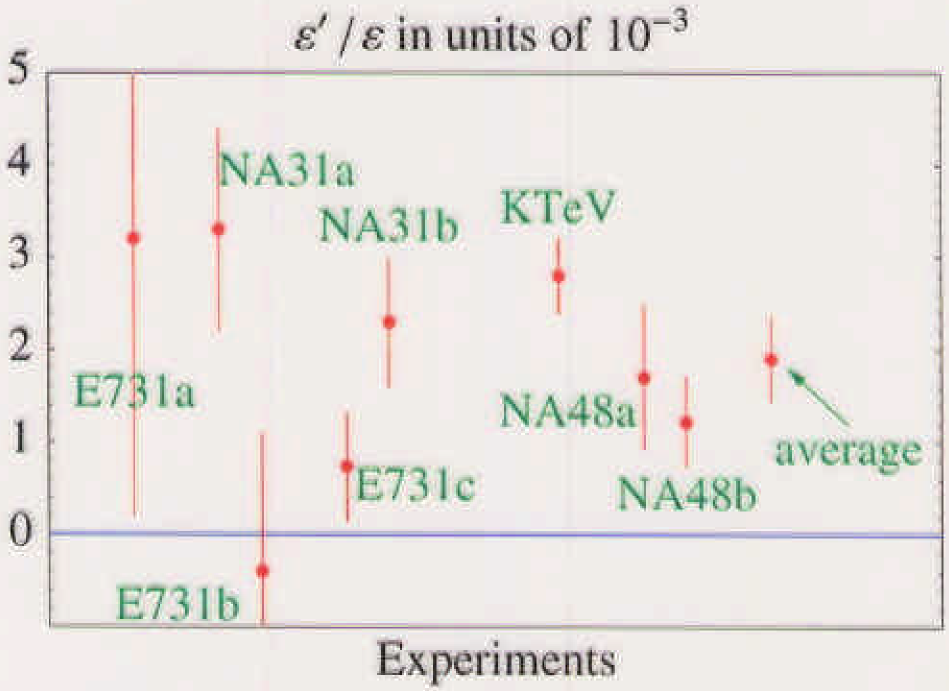


The Estimate



χQM: $\mathrm{Re}\ (\varepsilon'/\varepsilon) = (2.2 \pm 0.8) \times 10^{-3}$

The Experiments (circa 2000)

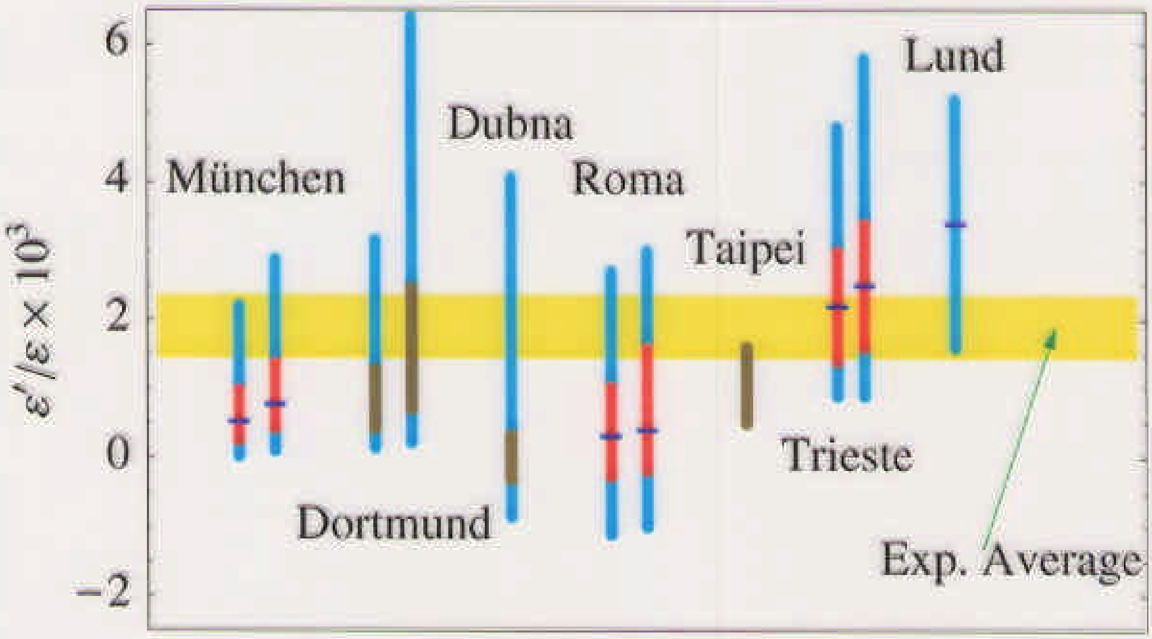


Grand Average: $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (1.9 \pm 0.46) \times 10^{-3}$

Hadron Matrix Elements

- Lattice
- Phenomenlogical $1/N_c$
- Chiral Quark Model
- $1/N_c$
- NJL Model + Chiral Loops Corrections
- Generalized Factorization
- ENJL Model in the Chiral Limit

Experiments vs. Theoretical Predictions



Future Improvments

- Reduction of the theoretical error:
  - uncertainty on Im λ_t: 20% on the total error
  - uncertainty on Im λ_t dominated by the determination of $\bar{B}_K$
  - a precise determination of Im λ_t from $K_L \to \pi^0 \nu \bar{\nu}$ or B-physics alone will reduce the impact of non-perturbative QCD
- Progress in the lattice estimate of hadronic elements (unquenching) is to be expected
- Premature to speak of new physics

Comments on the Theoretical Estimates, I

- The entire range between zero and 3×10^{-3} is taken by the standard model predictions
- Should the new experimental results converge to a common error range of the order of few 10^{-4},
  - focus attention on central values of various approaches to better understand most relevant effects at work

    The cancellation between the gluon and electroweak penguin operators is less effective once the FSI phase effects, chiral loops, a (possible) smaller value for m_s and a complete inclusion of the electroweak $O(p^2)$ corrections are taken into account