The $\Delta I = 1/2$ rule and ε'/ε

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Outlines

1. Difficulties with chiral approach

2. Generalized factorization

3. $\Delta I = 1/2$ rule in kaon decay

4. Direct *CP* violation ε'/ε

5. Conclusions

Difficulties with chiral approach

$$
\begin{aligned}
A(K \to 2\pi) &= \sum c_i(\mu) \langle \pi\pi | Q_i(\mu) | K \rangle \\
&= \sum c_i(\mu) B_i(\mu) \langle Q_i \rangle_{\rm VIA}
\end{aligned}
$$

where $c_i(\mu)B_i(\mu)$ are scheme and scale independent. Since μ dependence of $\langle Q(\mu) \rangle$ is lost under VIA, how to obtain scheme and scale dependence of hadronic matrix elements or $B_i(\mu)$?

Chiral approach:

$$\langle Q(\mu) \rangle = \langle Q \rangle_{\rm VIA} + \text{ chiral loop corrections,}$$

and cutoff is identified with the scale of $c(\mu)$.

Difficulties:

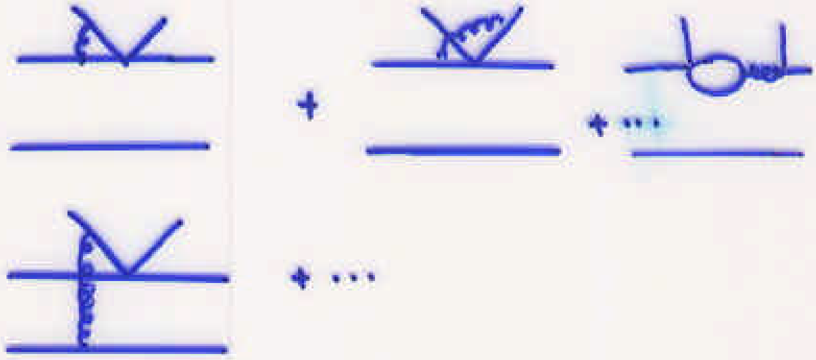
- $c(\mu)$ is reliable down to 1 GeV, whereas ChPT is applicable only up to 600 MeV $\Rightarrow$ matching problem.

- How to match quadratic scale dependence of chiral corrections with logarithmic μ dependence of $c(\mu)$? $\Rightarrow$ dimensional regularization is needed to regularize chiral loops. (Missimer et al.)
- How to furnish scheme dependence for chiral corrections ?
- While A_0 is largely enhanced, the predicted A_2 is still too large compared to experiment $\Rightarrow$ need nonfactorized effects (e.g. gluon condensate) other than chiral loops to explain A_2.

  (Antonelli, Bertolini, Eeg, Fabbrichesi)
- It is not applicable to heavy meson decays.

Generalized factorization

$$\langle Q(\mu)\rangle = \langle Q\rangle_{\rm VIA} +$$



$$\sum c_i(\mu)\langle Q_i(\mu)\rangle = \sum a_i \langle Q_i\rangle_{\rm VIA}$$

For B decays

$$a_1 = \underbrace{c_1(\mu) + c_2(\mu)\left(\frac{1}{N_c} + \chi_1\right)}_{\text{naive factorization}} + \frac{\alpha_s}{4\pi}\Big(\underbrace{\gamma_V \ln\frac{m_b^2}{\mu^2}}_{\text{scale}} + \underbrace{r_V}_{\text{scheme}}\Big)c$$

- Vertex corrections to 4-quark operators ensure that the effective parameters $a_{1,2}$ be scheme and scale independent.
- Nonfactorized terms $\chi_i = \chi_i(\alpha_s, \Lambda_{\rm QCD}/m_b)$ are complex. In $m_b \to \infty$ limit, χ_i are short-distance dominated and hence calculable. (Beneke et al.)

For K decays

- $\ln m_b^2/\mu^2 \to \ln \mu_f^2/\mu^2$ with factorization scale $\mu_f \gtrsim 1$ GeV.
- χ_i are large and arise mainly from soft gluon exchange $\Rightarrow$ large nonfactorizable corrections to naive factorization.

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[V_{ud}V_{us}^*\sum_{i=1}^{10} z_i(\mu)Q_i(\mu) + V_{td}V_{ts}^*\sum_{i=1}^{10} y_i(\mu)Q_i(\mu)\right]$$

$$A(K \to \pi\pi) = \sum a_i\langle Q_i\rangle_{\rm VIA} + \sum b_i\langle Q_i\rangle_{\rm VIA}$$

$$a_{2i} = z_{2i}^{\rm eff} + z_{2i-1}^{\rm eff}(\frac{1}{N_c} + \chi_{2i}),$$
$$b_{2i} = y_{2i}^{\rm eff} + y_{2i-1}^{\rm eff}(\frac{1}{N_c} + \chi_{2i}).$$

Further assumptions on nonfactorized effects are needed:

$$\chi_{LL} \equiv \chi_1 = \chi_2 = \chi_3 = \chi_4 = \chi_9 = \chi_{10},$$
$$\chi_{LR} \equiv \chi_5 = \chi_6 = \chi_7 = \chi_8,$$

$\chi_{1,2}$ can be extracted from $K^+ \to \pi^+\pi^0$:

$$\begin{aligned} {\rm Re}A_2 &= \frac{G_F}{\sqrt{2}}\frac{{\rm Re}(V_{ud}V_{us}^*)}{\cos\delta_2}\Big\{[a_1 + a_2 + \frac{3}{2}(-a_7 + a_9 + a_{10})]\frac{\sqrt{2}}{3}X \\ &+ \sqrt{3}f_\pi v^2 a_8\Big\}\frac{1}{1-\Omega_{\rm IB}}, \end{aligned}$$

data $\Rightarrow \chi_{LL} = -0.73$ (assuming to be real) at μ_f=1 GeV

A large negative χ_{LL} necessary for suppressing A_2 will enhance A_0 by a factor of 2.

No constraints on χ_{LR} can be extracted from $K^0 \to \pi\pi$.

Bag parameters

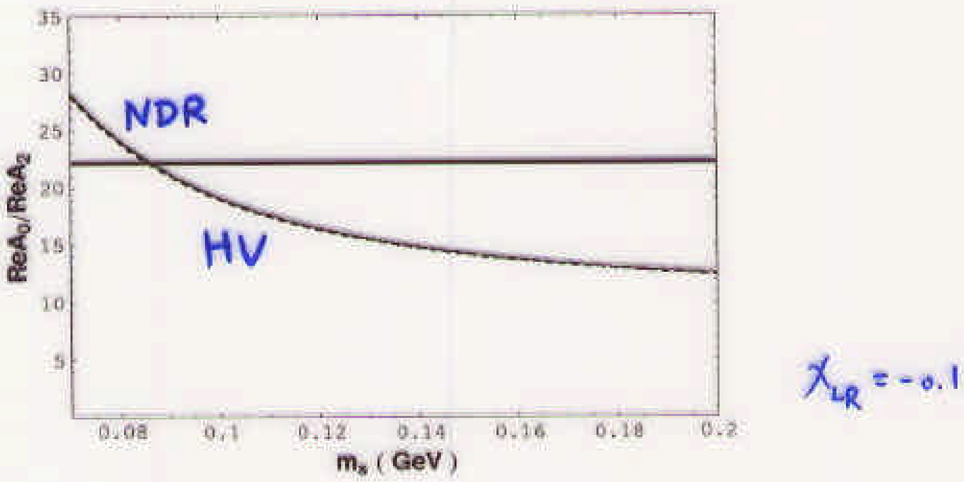
$$B_i^{(0)}(\mu) \equiv \frac{\langle Q_i(\mu)\rangle_0}{\langle Q_i\rangle_0^{\rm VIA}}, \qquad B_i^{(2)}(\mu) \equiv \frac{\langle Q_i(\mu)\rangle_2}{\langle Q_i\rangle_2^{\rm VIA}},$$

	$B_1^{(0)}$	$B_2^{(0)}$	$B_3^{(0)}$	$B_4^{(0)}$	$B_5^{(0)}$	$B_6^{(0)}$	$B_7^{(0)}$	$B_8^{(0)}$	$B_9^{(0)}$	$B_{10}^{(0)}$
NDR	2.5	8.7	-0.1	2.7	1.0	1.5	1.0	1.6	3.1	3.1
HV	2.6	8.0	0.9	2.6	1.7	1.5	1.7	1.6	2.9	2.9

	$B_1^{(2)}$	$B_2^{(2)}$	$B_7^{(2)}$	$B_8^{(2)}$	$B_9^{(2)}$	$B_{10}^{(2)}$
NDR	0.34	0.34	1.0	1.6	0.35	0.35
HV	0.37	0.36	1.8	1.6	0.37	0.37

for $\chi_{LL} = -0.73$ and $\chi_{LR} = -0.1$ in naive dimensional regularization (NDR) and 't Hooft-Veltman (HV) schemes.

- $B_{1,2}^{(2)}$ and $B_{9,10}^{(2)}$ are small (~ 0.34) in order to suppress A_2.
- B_5, $B_7^{(0,2)}$ are quite sensitive to χ_{LR}, while B_6, $B_8^{(0,2)}$ stay stable.



$\mathrm{Re}A_0/\mathrm{Re}A_2 = 13-15$ if $m_s(1\,\mathrm{GeV}) = (125-175)\,\mathrm{MeV}$.

$\frac{A_0}{A_2} =$	0.9	in absence of QCD corrections
	↓	
	2.0	QCD corrections to z_1 and z_2
	↓	
	2.3	QCD penguins + electroweak penguins
	↓	
	1.7	isospin breaking
	↓	
	2.0	final state interactions
	↓	
	4.2	NLO corrections to $c(\mu)$ and radiative corrections to $\langle Q(\mu)\rangle$ for $m_s(1\,\mathrm{GeV}) = 150$ MeV
	↓	
	13.8	nonfactorized effects

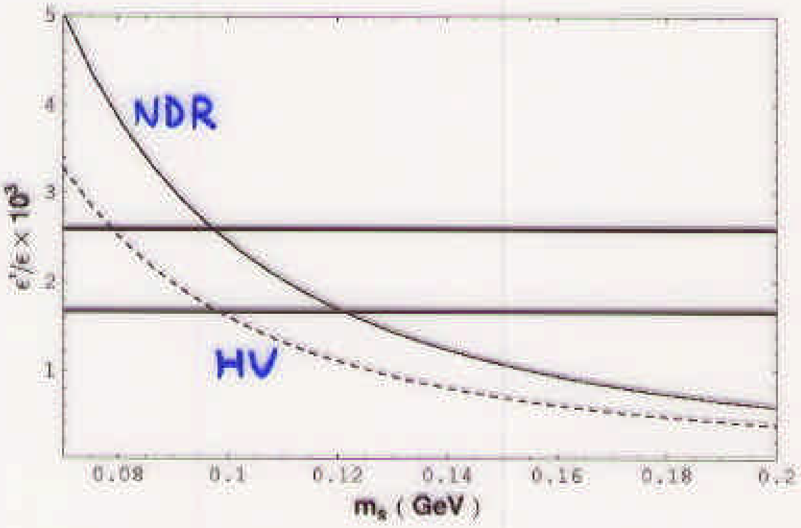
Experimentally $A_0/A_2 = 22.2 \pm 0.1$

Direct CP violation ε'/ε

$$\frac{\varepsilon'}{\varepsilon} = \begin{cases} 1.56 \pm 0.39\ (1.02 \pm 0.26) \times 10^{-3} & \text{at } m_s(1\,\text{GeV}) = 125\ \text{MeV}, \\ 1.07 \pm 0.27\ (0.70 \pm 0.18) \times 10^{-3} & \text{at } m_s(1\,\text{GeV}) = 150\ \text{MeV}, \\ 0.78 \pm 0.20\ (0.51 \pm 0.13) \times 10^{-3} & \text{at } m_s(1\,\text{GeV}) = 175\ \text{MeV}, \end{cases}$$

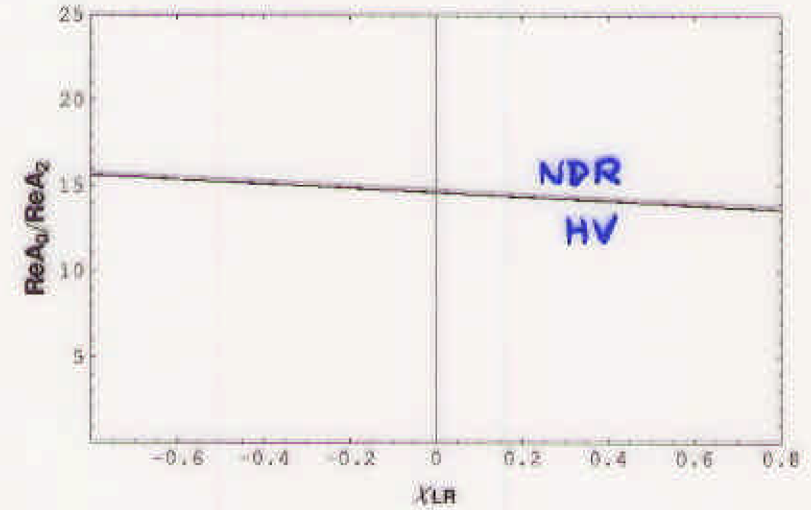
in NDR (HV) scheme, while experimentally

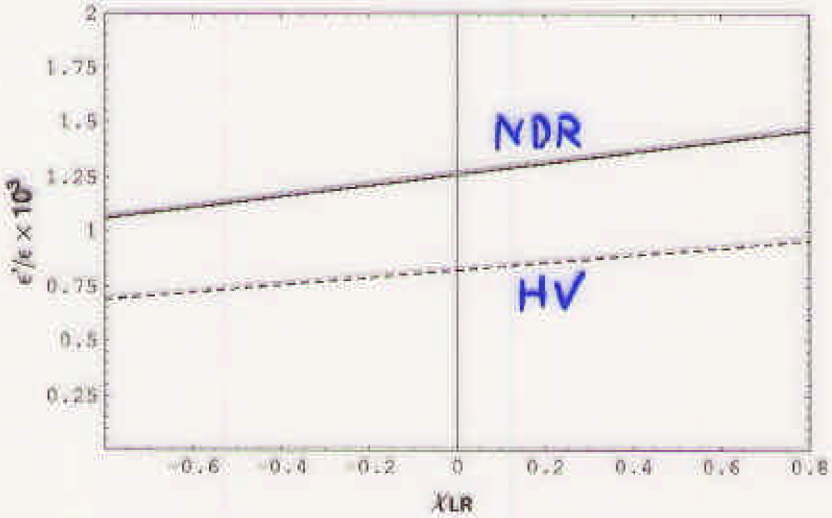
$$\text{Re}(\varepsilon'/\varepsilon) = (1.93 \pm 0.24) \times 10^{-3}$$



Scheme dependence of ε'/ε is probably due to effects of α_s^2. Moreover, scheme dependence of y_6 is further amplified by the strong cancellation between QCD penguin and electroweak penguin contributions, making it difficult to predict ε'/ε accurately.

dependence with χ_{LR}





ε'/ε and in particular A_0/A_2 are not sensitive to the nonfactorized term χ_{LR}.

Conclusions

- We work in the framework in which vertex-type and penguin-type corrections to 4-quark operators account for scheme and scale dependence of hadronic matrix elements. Nonfactorized terms $\chi_{1,2}$ extracted from $K^+ \to \pi^+\pi^0$ will suppress A_2 and enhance A_0.

- Two principal sources responsible for $\text{Re}A_0/\text{Re}A_2$: vertex-type as well as penguin-type corrections to matrix elements of four-quark operators, and nonfactorized effect due to soft-gluon exchange, which is needed to suppress the $\Delta I = \frac{3}{2}$ $K \to \pi\pi$ amplitude.

- $\text{Re}A_0/\text{Re}A_2 = 13 - 15$ if $m_s(1\text{ GeV})$ lies in the range $(125 - 175)$ MeV.

- $\varepsilon'/\varepsilon = (0.5 - 1.3) \times 10^{-3}$ if $m_s(1\,\text{GeV}) = 150$ MeV and $\varepsilon'/\varepsilon = (0.8 - 2.0) \times 10^{-3}$ if m_s is as small as indicated by recent lattice results.