Recoil and threshold resummation for hard scattering cross sections

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1 – Higher orders in hard scattering

Many hadronic sections in QCD factorize into parton distributions and hard scattering functions. E.g. prompt photon production:

$$p_T^3 \frac{d\sigma_{AB\to\gamma X}(x_T^2)}{dp_T} \quad = \\ \sum_{ab} \phi_{a/A}(x_a,\mu) \otimes \phi_{b/B}(x_b,\mu) \otimes \omega_{ab\to\gamma X}\left(\hat{x}_T^2,\mu\right)$$

- $x_T^2 \equiv 4p_T^2/S$ and $\hat{x}_T^2 \equiv 4p_T^2/x_a x_b s$
- hard scattering function ω_{ab} computed in perturbation theory

Computation of higher order (α_s^k) corrections

- tests convergence of perturbation theory
- moves dependence on unphysical scales ($\mu_{F,R}$) to higher orders

Resummation of classes of log corrections

- may “rescue” perturbation theory if in distress
- moves dependence on unphysical scales to subleading terms
- may explain large data-theory discrepancies

(For good understanding, both should be done even if data don’t seem to need it).

Two popular classes of large (Sudakov) logarithmic corrections (for Drell-Yan e.g.) $\frac{d\sigma}{dQ^2\,dQ_T^2}$

1. Threshold enhancements
   - $\alpha_s^i \left[\frac{\ln^{2i-1}(1-z)}{1-z}\right]_+$, $z = \frac{Q^2}{x_a x_b S}$ integration variable
   - possibly large corrections after integrations against smooth functions (PDF's)
2. Q_T (recoil) enhancements
   - $\alpha_s^i \frac{\ln^{2i-1}(Q_T/Q)}{Q_T}$, Q_T measurable
   - large if $Q_T \ll Q$

Both result from simultaneously soft and collinear gluon radiation.

Can they be resummed together? (H-n.Li ('98))

2 – Refactorization and threshold resummation

Scheme (sch) dependence in Drell-Yan:

$$Q^2 \frac{d\sigma_{AB\to\gamma^*+X}}{dQ^2} = \sigma_0 \sum_{ab} \phi^{\rm sch}_{a/A}(x_a,\mu) \otimes \phi^{\rm sch}_{b/B}(x_b,\mu) \otimes \omega^{\rm sch}_{ab\to\gamma^*+x}\left(z = Q^2/(x_a x_b S), \frac{Q}{\mu}\right)$$

If we use $\overline{\rm MS}$-scheme PDF's:

$$\omega^{(1),\overline{\rm MS}}_{ab\to\gamma^*+x} \simeq \alpha_s\; 4C_F \left[\frac{\ln(1-z)}{1-z}\right]_+$$

For DIS-scheme PDF's

$$\omega^{(1),{\rm DIS}}_{ab\to\gamma^*+x} \simeq \alpha_s\; 2C_F \left[\frac{\ln(1-z)}{1-z}\right]_+$$

i.e. some logs $\subset$ finite terms in the redefinition of the $\phi^{\rm sch}_{a/A}$. Systematize Sterman ('87) to factor all large corrections from ω_{ab}.

Define Sterman('87) the a' matrix element of time-like separated quark a fields.

$$\psi_{a/a'}(x, 2p_0, \epsilon) = \frac{1}{4\sqrt{2}N_c} \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\lambda x p^0} \langle a'(p)|\bar{q}_a(\lambda, \vec{0})\gamma^+ q_a(0)|a'(p)\rangle\,,$$

Fourier transform fixes energy fraction to be x.

$$\psi_{a/a'}(x, 2p_0, \epsilon) = \delta(1-x) + \alpha_s\Big(\frac{1}{\epsilon}P_{aa'}(x) + 2C_F\left[\frac{\ln(1-z)}{1-z}\right]_+ + \ldots\Big)$$

Define also U_{ab}: eikonalized ω_{ab} (with soft parts of ψ's subtracted). Then refactorize hard scattering function

$$\omega_{ab\to\gamma^*+x} = \psi_{a/a} \otimes \psi_{b/b} \otimes U_{ab} \times H_{ab} + Y_{thr}$$

H_{ab} has no large logs left! Large logs now in universal ψ's. Also in U, but "easy". "$\otimes$" includes energy conservation to $\mathcal{O}(1-z)$. Matching term $Y_{thr} = \mathcal{O}(1-z)$.

The ψ's and U can be resummed to all orders, e.g.

$$\ln\psi(N) = \ln\int_0^1 dx\, x^{N-1}\psi(x) =$$

$$\int_0^1 \frac{x^{N-1}-1}{1-x}\left[\int_{(1-x)^2Q^2}^{Q^2}\frac{d\mu^2}{\mu^2}A(\alpha_s(\mu)) + B((\alpha_s((1-x)Q))\right]$$

Sterman('87); Catani,Trentadue('89)

- $\psi' s$ universal, soft function U process dependent
- procedure extends to pure QCD cross sections, where color-coherence effects show up, Contopanagos, EL, Sterman ('97); Kidonakis, Oderda, Sterman ('98); Bonciani, Catani, Mangano, Nason ('98) and single particle inclusive kinematics EL, Oderda, Sterman('98)
- Accuracy achieved : NLL
- for $(1-x) < \Lambda/Q$ the μ integral diverges: "IR renormalon"

What are threshold-resummed cross sections good for?

- estimates of sizes of all-order cross sections (E.g. top quark cross section EL, Smith, van Neerven ('92,'94); Berger, Contopanagos ('98); Catani, Mangano, Nason, Trentadue ('96))
- generation of approximate higher orders (E.g. DY Magnea ('91); Higgs EL, Krämer, Spira ('98); F_2 A. Vogt ('99); $F_2(c\bar{c})$ EL, Moch ('99); $g_1(c\bar{c})$ Eynck, Moch ('00); prompt γ Kidonakis, Owens ('99); W, Z+jet Kidonakis, Del Duca ('99).

All order cross sections need a prescription to deal with IR renormalon:

- cut-off Appel, Sterman, Mackenzie ('88): EL, Smith, van Neerven ('92)
- principal value Contopanagos, Sterman ('94)
- avoid in N contour integration Catani, Mangano, Nason, Trentadue ('96)

3 – Refactorization and double resummation

Write parton-level prompt photon p_T spectrum via distribution in (unobserved) Q_T of $a + b \to \gamma + c$ process Lai, Li ('98)):

$$p_T^3 \frac{d\sigma^{(\mathrm{resum})}_{ab\to\gamma c}(\bar{\mu})}{dp_T} \simeq \int d^2\mathbf{Q}_T \; p_T^3 \frac{d\sigma^{(\mathrm{resum})}_{ab\to\gamma c}}{d^2\mathbf{Q}_T \, dp_T} \, \Theta\left(\bar{\mu} - Q_T\right) \, .$$

Refactorize

$$p_T^3 \frac{d\sigma^{(\mathrm{resum})}_{ab\to\gamma c}}{d^2\mathbf{Q}_T \, dp_T} = R_{a/a} \otimes R_{b/b} \otimes S_{ab} \otimes J_c \times h_{cd} + Y_{both}$$

with

$$R_{a/a}(x, \mathbf{k}, \epsilon) = \frac{1}{4\sqrt{2}N_c} \int \frac{d\lambda}{2\pi} \, \frac{d^2 b}{(2\pi)^2} \mathrm{e}^{-i\lambda x p^0 + i\mathbf{b}\cdot\mathbf{k}} \langle a(p)|\bar{q}_a(\lambda, \mathbf{b}, 0)\gamma^+ q_a(0)|a(p)\rangle \, ,$$

Fourier transform fixes energy fraction *and* transverse momentum of quark a. "$\otimes$" includes both k_T and energy conservation. J_c is final state "jet" function (has only threshold logs).

J_c, S_{ab} and $R_{a/a}$ can be resummed (via eikonal approximation, and "webs" Gatherall ('83)):

$$\frac{p_T^3 d\sigma^{(\mathrm{resum})}_{AB\to\gamma}}{dp_T} =$$

$$\sum_{ij} \frac{p_T^4}{8\pi S^2} \int_{\mathcal{C}} \frac{dN}{2\pi i}\, \tilde{\phi}_{i/A}(N,\mu)\tilde{\phi}_{j/B}(N,\mu)$$

$$\int_0^1 d\tilde{x}_T^2 \left(\tilde{x}_T^2\right)^N \frac{|M_{ij}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}$$

$$\times \int \frac{d^2\mathbf{Q}_T}{(2\pi)^2}\, \Theta\left(\bar{\mu} - Q_T\right)$$

$$\left(\frac{S}{4\mathbf{p}_T'^{\,2}}\right)^{N+1} P_{ij}\left(N, \mathbf{Q}_T, \frac{4p_T^2}{\tilde{x}_T^2}, \mu\right) ,$$

where

- $\mathbf{p}_T' = \mathbf{p}_T - \mathbf{Q}_T/2$
- $x_T^2 \equiv 4p_T^2/S$ and $\tilde{x}_T^2 \equiv 4\mathbf{p}_T^2/\hat{s}$
- "normal" PDF's can be used!
- $P_{ij} \simeq \int d^2b \exp(E_{ij}(N, \mathbf{b}, ..)$ "profile function"

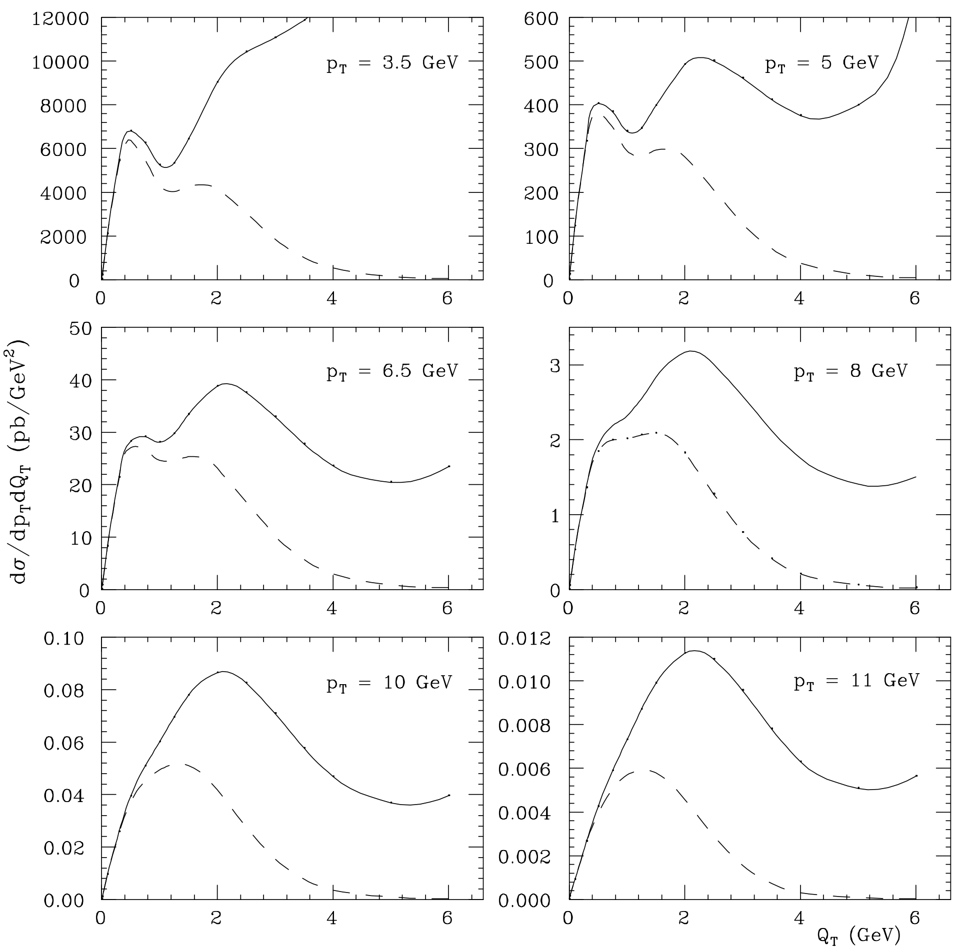
Here the "Drell-Yan $\ln(1-z)$" is $\ln(1-\tilde{x}_T^2)$.

What is double resummation good for?

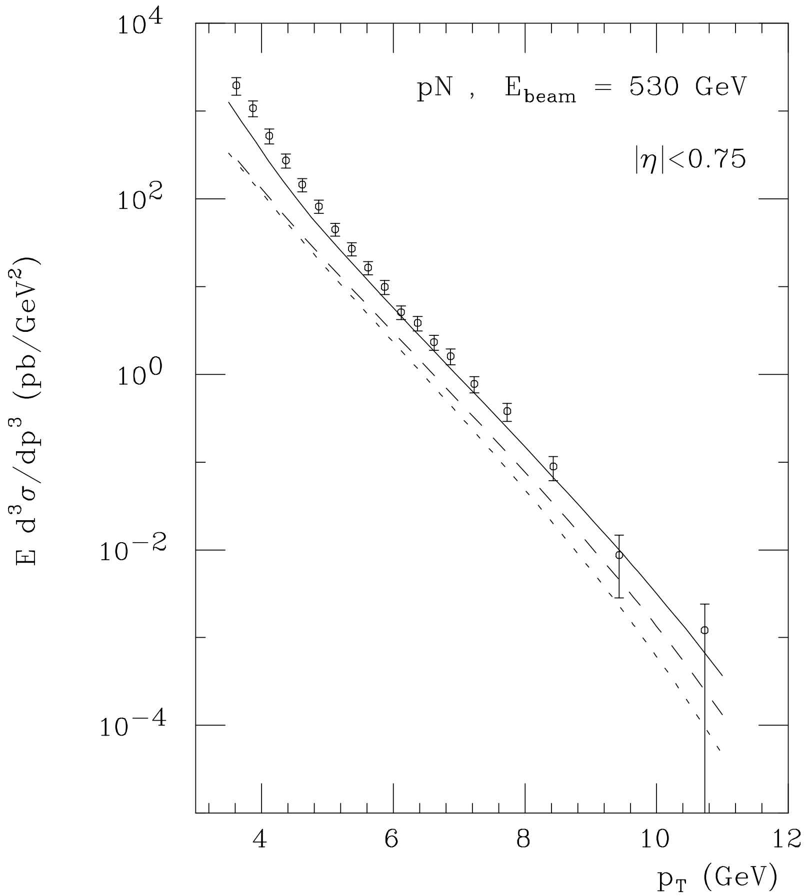
- estimates of <u>shapes + sizes</u> of all-order cross sections
- perturbative recoil effects included with energy conservation
- less need for phenomenological "intrinsic k_T"?

4 – Is double resummation important?

Effect of profile function ($\sqrt{S} = 31.5$ GeV (E706)), vs. Q_T for $d\sigma^{(\mathrm{resum})}_{pN\to\gamma X}/dQ_T\,dp_T$. Dashed lines: $(S/4p_T^2)^{N+1}$ (no recoil) instead of $(S/4\mathbf{p}'^{\,2}_T)^{N+1}$



Prompt photon cross section $Ed^3\sigma_{\mathrm{pN}\to\gamma X}/dp^3$ for pN collisions at $\sqrt{s} = 31.5$ GeV. Data from E706, GRV PDF's.



- dotted: full NLO Aurenche et al ('88); Baer, Ohnemus, Owens ('90); Gordon, Vogelsang ('90)
- dashed: threshold resummation Catani, Mangano, Nason, Oleari, Vogelsang ('99); EL, Oderda, Sterman ('99)
- solid: double resummation

- Substantial numerical effect: for a measured p_T, the $2 \to 2$ scattering has to contribute less, with an accompanying less heavy final state.
- no matching yet. We took cutoff $\bar{\mu} = 5\mathrm{GeV}$
- $\mu = p_T$, $F_{ij} = 0.5\mathrm{GeV}^2$
- both resummed curves have substantially reduced μ dependence
- recoil of partons in hadrons evidently relevant!

5 – Summary and outlook

Summary:

- Recoil and threshold resummation now in one consistent formalism, to NLL accuracy
- formalism ready for Drell-Yan (Higgs..) at measured Q_T, and for single particle inclusive $2 \longrightarrow 2$ scattering.
- (new principal value method for integration over impact parameter b)

Outlook:

- matching to finite order? Power corrections?
- other processes, numerical studies