

Multiplicities & Factorial Moments

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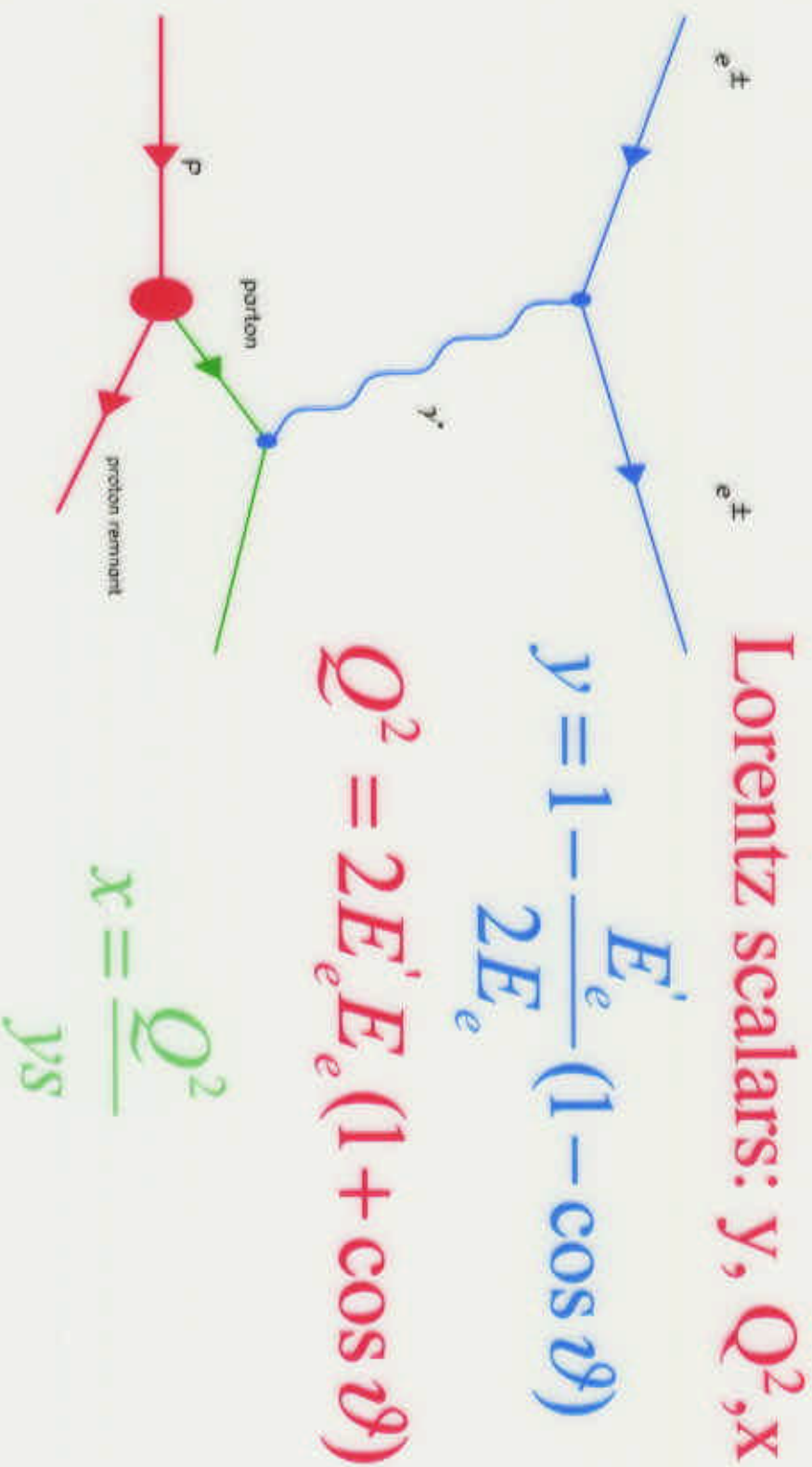
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(on behalf of the ZEUS Collab.)

- Introduction & Motivation
- Results
- Conclusions



Naïve Quark Parton Model (QPM)

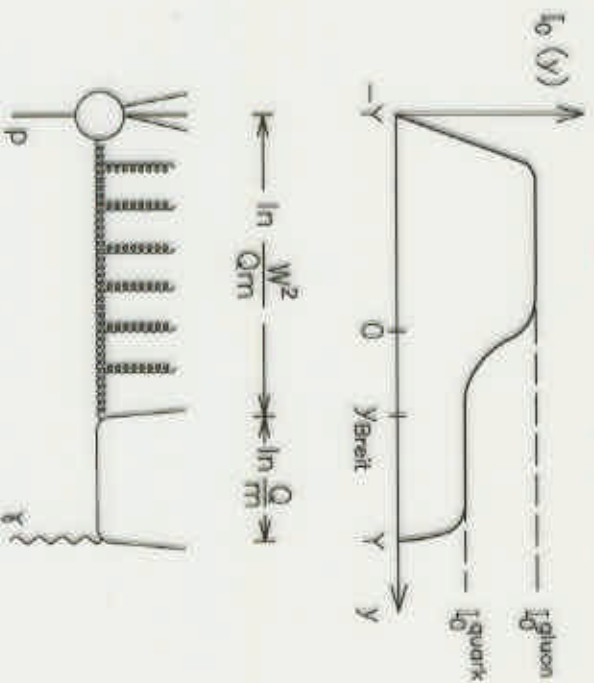


where \sqrt{s} is the eP centre of mass energy

Multiplicity Distributions

Investigation of dynamics of hadronisation process - study the $\langle n_{ch} \rangle$ as a function of invariant mass (M_{eff}) in a fixed rapidity range (in lab. frame)

M_{eff} is essentially measuring the rapidity “along” the gluon ladder



Gluon or quark initiated - measurement of colour charge

Factorial Moments

fractal dimension techniques (eg Rényi dimensions) to multiparticle production processes

investigation of cascading dynamics & “self-similarity” of branching processes

predictions exist within framework of pQCD

use of DLA and MLIA (in conjunction with LPHD)

[theoretical work: Dokshitzer, Ochs (MLIA)]

Dremin, Wosiek (Factorial moments)]

Mathematical Interlude

Fractal dimension, F_D

$$M \sim l^{F_D}$$

- M is mass & l is length. $F_D = 1$ for a line, $F_D = 2$ for a square...

Koch Curve



→ self-similar curve with dimension $F_D = \ln(4)/\ln(3)$

Generalise to multi-fractals $F_D \rightarrow D_q$ (Rényi dimension)

(cf replace our homogenous stick with an inhomogenous one)

Factorial Moments

$$F_q(\Delta\Omega) = \langle n(n-1)\dots(n-q+1) \rangle / \langle n \rangle^q, \quad q = 2, 3, \dots$$

where n is number of charged particles inside a phase-space region of size $\Delta\Omega$ and $\langle \dots \rangle$ denotes average.

For uncorrelated particle production within $\Delta\Omega$, $F_q = 1$ (Poisson stats) rise follows power law - "intermittency"

moments probe different dynamics depending on choice of $\Delta\Omega$

Kinematic Selections

Multiplicity: 1995 data (5.5 pb⁻¹)

$$8 < Q^2 < 1200 \text{ GeV}^2$$

$$70 < W < 260 \text{ GeV}$$

Particles with angular acceptance of $|\eta(\text{lab})| < 1.75$

Moments: 96+97 (38.4 pb⁻¹)

$$Q^2 > 1000 \text{ GeV}^2$$

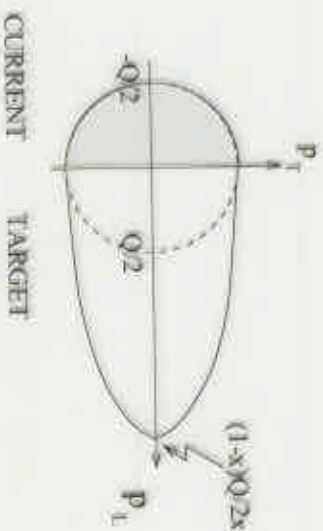
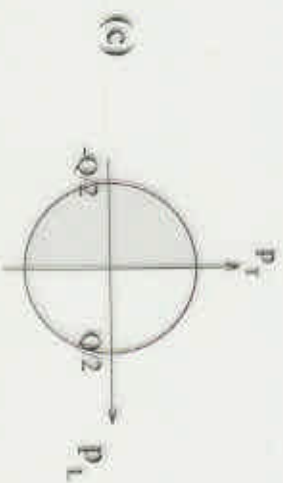
Moments measured in
current region of Breit frame

The Breit Frame 'Brickwall' frame

Electron-positron Annihilation



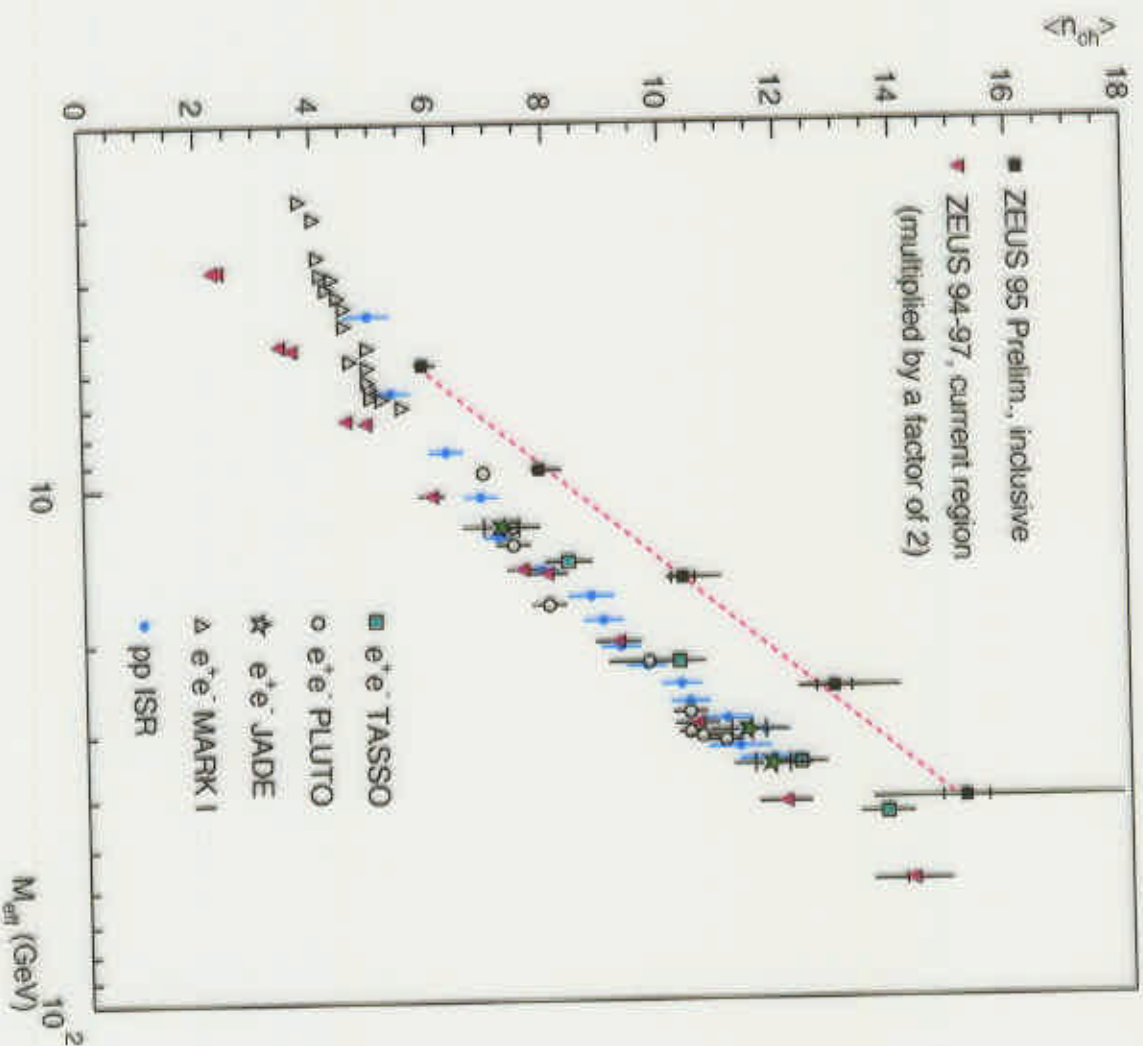
DIS in the Breit Frame



Phase space for e^+e^-
annihilation evolves
with $Q/2 = \sqrt{s}/2$

Current hemisphere of
Breit frame evolves as
 $Q/2$

Current region $\equiv e^+e^-$
annihilation



Data grows linearly with

$$\log(M_{eff})$$

$\langle n_{ch} \rangle$ higher than e⁺e⁻
 data ($M_{eff} \approx \sqrt{s_{ee}}$), low
 energy pp data and
 (Breit) current region in

$$DIS (M_{eff} \approx Q)$$

Data compatible with
 picture of additional
 coherent gluon radiation
 due to (octet) colour
 charge

Systematic Checks

Event Selection:

- checks on y_e (2%), y_{JB} (3%), E- p_z (1%), vertex cut (1%)

Trk selection:

- tightening of p_T cut (1%) and η cut (2%)

Monte Carlo dependence:

- LEPTO + POMPYT (up to $\sim 16\%$)

Analytic QCD Results

$$\Omega \equiv p_T^{\text{cut}} \text{ or } p^{\text{cut}} \text{ ie } p_T \lesssim p_T^{\text{cut}} \text{ (} |p| \lesssim p^{\text{cut}} \text{)}$$

$$F_q(p_T^{\text{cut}}) \approx 1 + \frac{q(q-1)}{6} \frac{\ln(p_T^{\text{cut}}/Q_0)}{\ln(E/Q_0)},$$

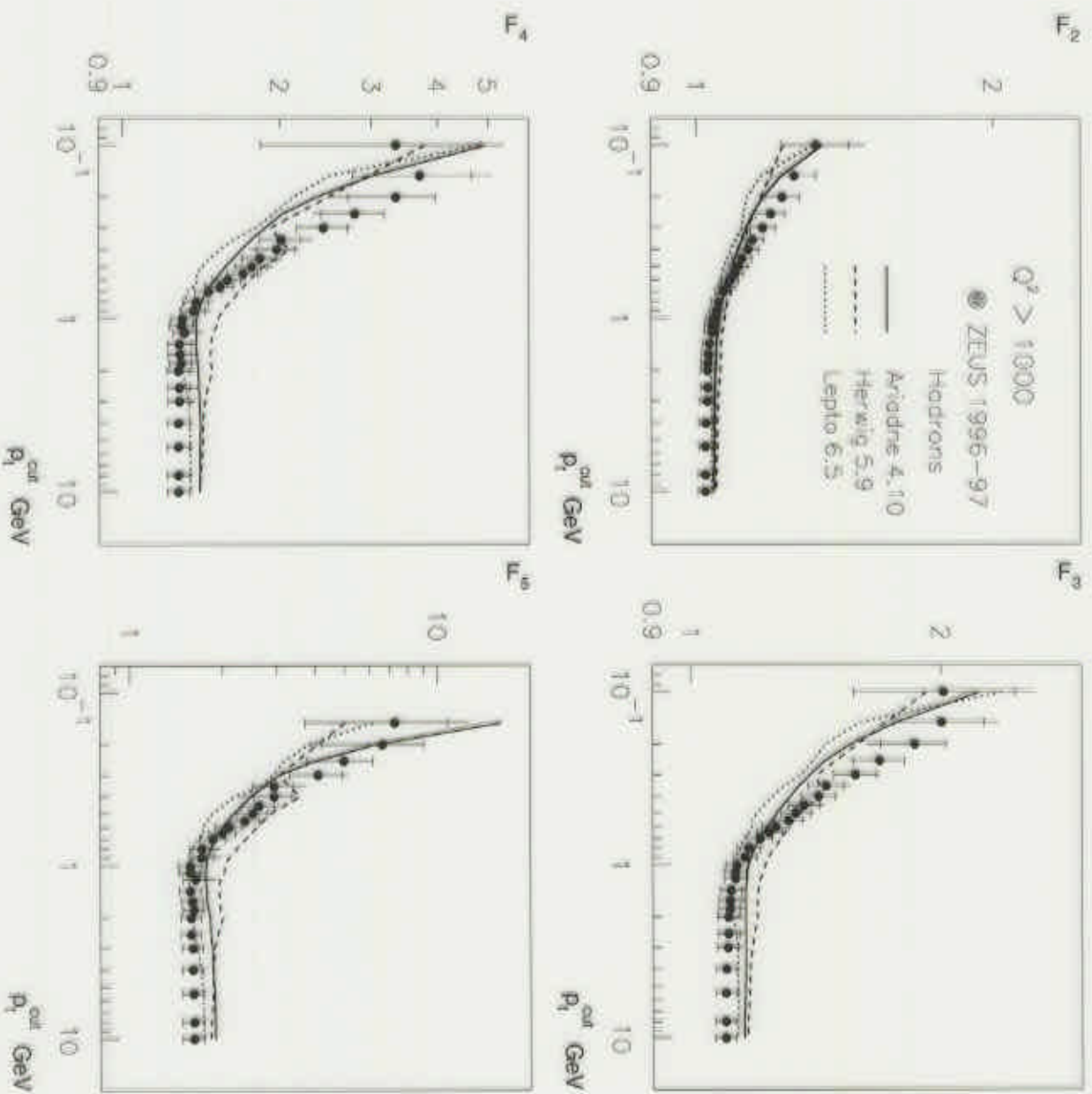
$$F_q(p^{\text{cut}}) \approx \text{const} > 1$$

E = jet energy, Q_0 = parton shower cut-off. DLIA approx
(Lupia, Ochs & Wosiek)

p_T : prediction of correlations (presence of gluon enhances probability of emission of another one)

As $p_T \rightarrow Q_0$ correlations vanish due to coherence effects

p : dist^{bn} of soft gluons remains non-Poisson



MC models
 qualitatively
 describe the data (in
 detail there are
 discrepancies)

data in disagreement
 with theoretical
 predictions

Result for
 momentum look
 similar to p_1

Systematic Checks

Event Selection:

- checks on Y_e , Y_{JB} , E-P_z, vertex cut

Trk selection:

- tightening of p_T cut and η cut

Typically a few % change - but can be ~20% at low p , p_T or high z

Analytic QCD Results

$\Omega \equiv$ polar rings of size Θ around axis centred at Θ_0 (see fig \blacktriangleright)

$$\ln \frac{F_q(z)}{F_q(0)} = z(1 - D_q)(q - 1) \ln \frac{E\Theta_0}{\Lambda}$$

$E =$ jet energy, $\Theta_0 =$ opening half angle of jet, $\Lambda =$ QCD scale

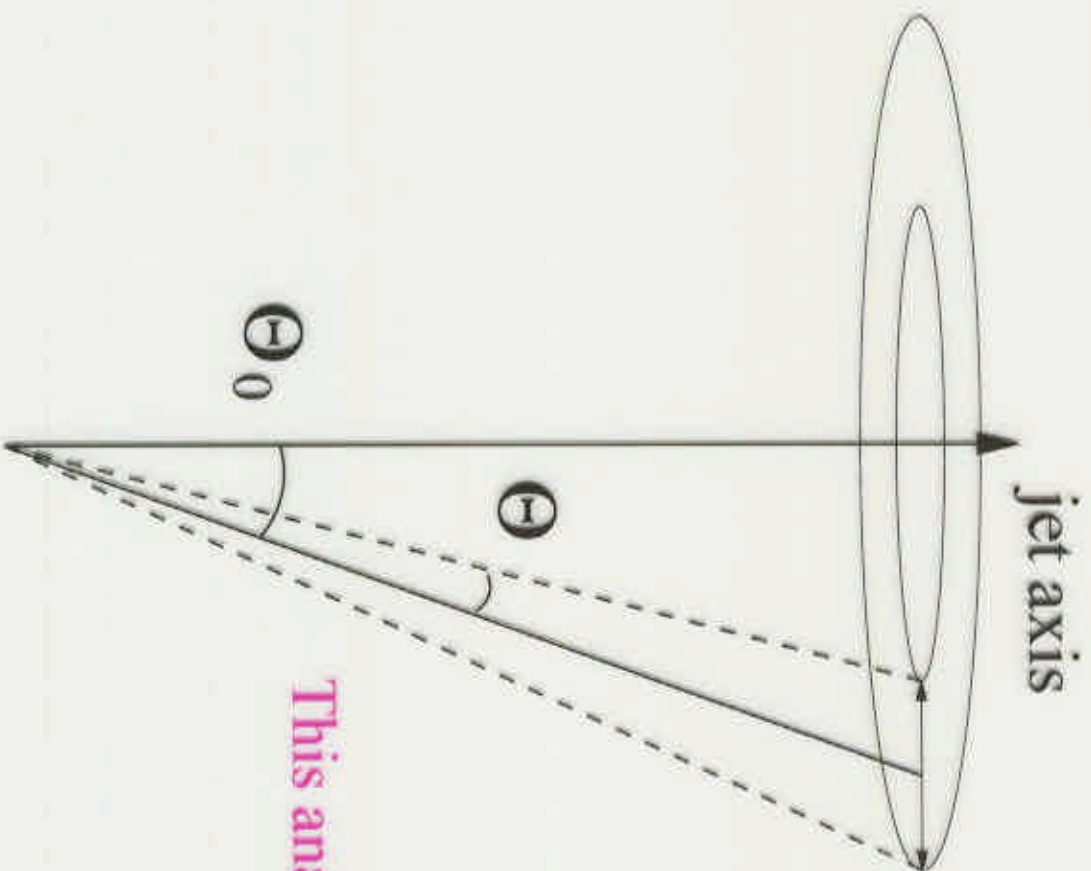
$$z = \frac{\ln(\Theta_0/\Theta)}{\ln(E\Theta_0/\Lambda)}$$

Number of predictions in DLA (Dokshitzer & Dremmin; Brax, Meunier & Peschanski; Ochs & Wosiek) and in MLLA (Dokshitzer & Dremmin) for the Rényi dimensions, D_q

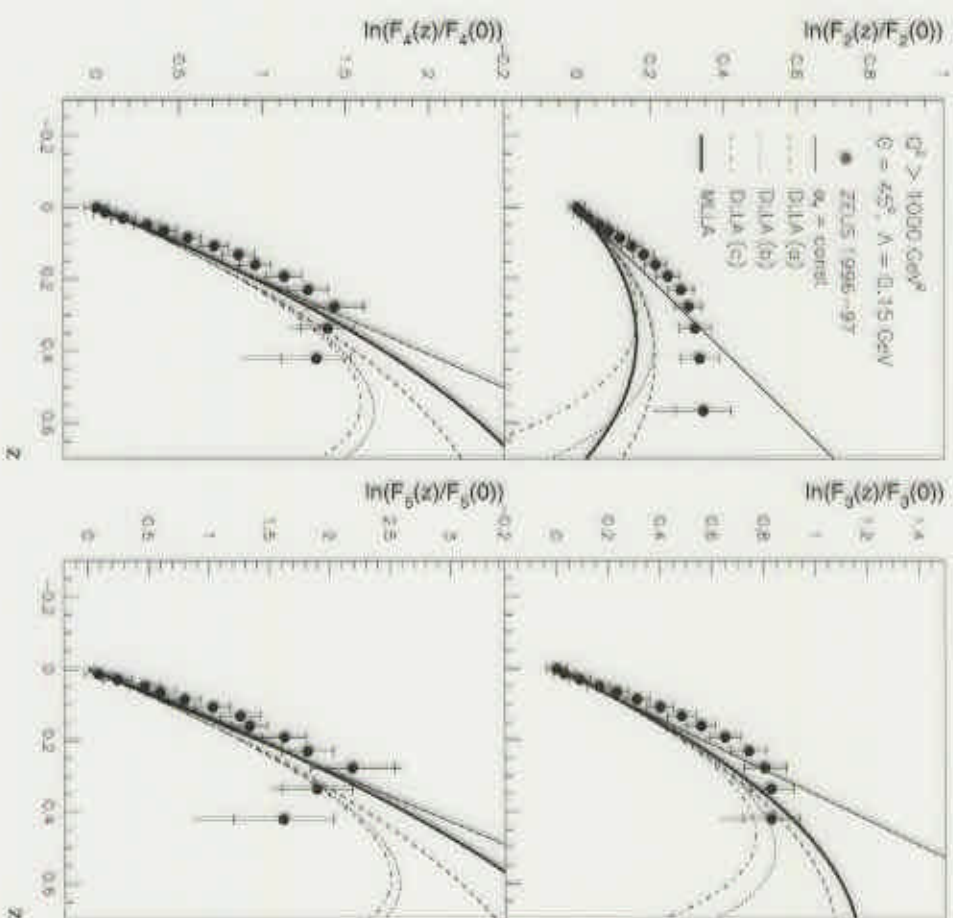
$$D_q \rightarrow 1 \quad \text{for Poisson dist}^{\text{bn}}$$

QCD predictions have $D_q = D_q(q, z)$

Defn of angular variable

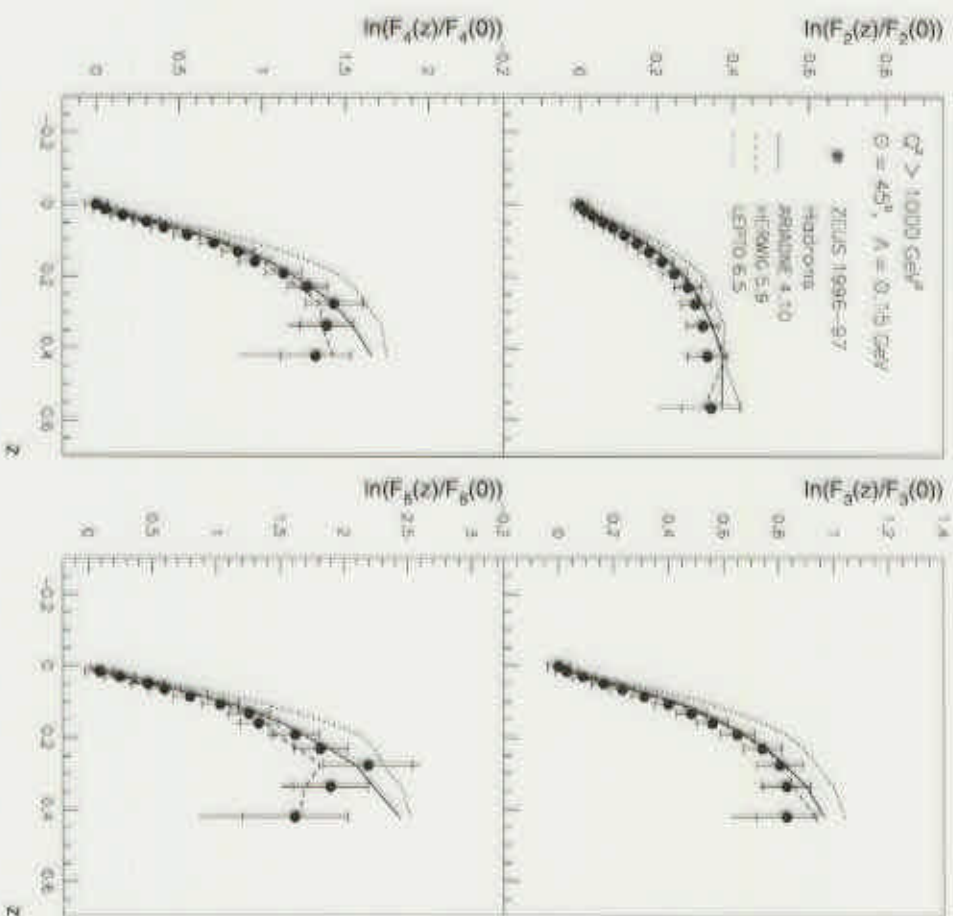


This analysis uses $\Theta_0 = 45^\circ$



For $\Theta_0 = 45^\circ$ & $\Lambda = 150 \text{ MeV}$,
 general trend of data & theory
 agree ie moments \uparrow as $z \uparrow$

Particular at lower order moments
 data & theory don't agree in
 detail.



**ARIADNE & HERWIG
reproduce the data**

LEP TO overshoots the data

**Non-negligible hadronisation
effects**

Conclusions

- $\langle n_{ch} \rangle$ higher than e^+e^- , low energy pp data & (Breit) DIS current region - colour charge effect ?
- Multiplicity factorial moments exhibit strong rise in restricted intervals as p_T , p & Θ decreases
- Analytic calculations (for partons) do not show the same increase for the factorial moments
- MC models (generally) reproduce the data
- Substantial contribution from hadronisation - LPHD hypothesis non-applicable