# UNITARISATION IN PROCESSES MEDIATED BY REAL AND VIRTUAL PHOTONS

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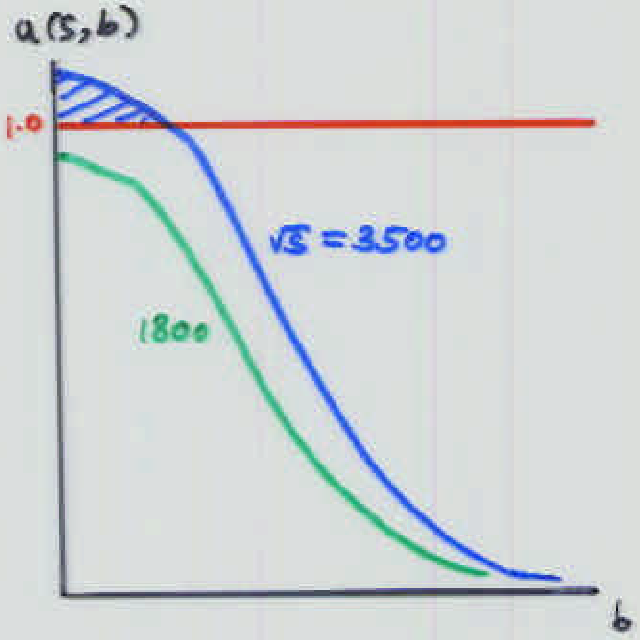
## A. A POOR MAN INTRODUCTION

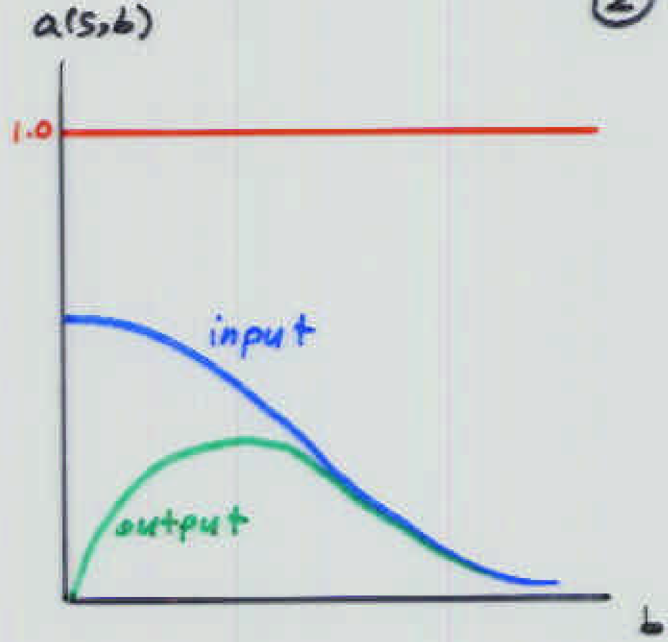
1. s-channel unitarity is easily presented in impact parameter b-space

$$|a_{el}(s,b)| \leq 1$$

* Implementing s-channel unitarity is a model dependent procedure.
* In the following we assume that unitarity is enforced through screening corrections (sc) which are calculated in an eikonal model with a b-space Gaussian input.
* This is a semi realistic toy model with which we can study, at a relative ease, the general features and signatures of unitarity below and at its saturation.

$$a_{out}(s,b) = i \int d^2b \left(1 - e^{-a_{in}(s,b)}\right)$$




In soft h-h reactions

CDF: $a_{el}(\sqrt{s}=1800, b=0) = 0.96 \pm 0.04$

* For $\sqrt{s}$ of a few TeV sc result in a small normalization re scale. The scale for severe sc is high $\sqrt{s} \simeq 10$ TeV.

* sc saturate in diffractive channels at a much lower scale $\sqrt{s} \simeq 100$ GeV This is dramatically seen at ISR SD data !!!

2. In hard pQCD s-channel unitarity is manifested by gluon saturation. In the eikonal model the input opacity is given by

$$a_{in}(x,Q^2,b) = \kappa_g(x,Q^2;b^2) = \frac{8\pi\alpha_s}{27Q^2} \times G^{DGLAP}(x,Q^2)\,\Gamma(b^2)$$

$$\Gamma(b^2) = \frac{1}{R^2}\, e^{-b^2/R^2}$$

The contour defined by $\kappa_g(x,Q^2;b^2) \equiv 1$ defines a natural scale for gluon saturation where we have the transition from pQCD with low gluon density to npQCD with high gluon density where collective phenomena play a dominant role.

* The study of gluon saturation is obviously connected with the problem of matching between hard pQCD and soft npQCD.

* With present HERA kinematics saturation is expected at $Q^2 \simeq 1\,GeV^2$, $x \simeq 10^{-3}$.

3. A practical (not ideological!) distinction should be made between the evolution equations and the dipole approximation:

* A Global DGLAP DIS analysis provides us with our p.d.f. Thus the study of $F_2$ and its $Q^2$ and $x$ derivatives is basically a DGLAP excersize.

* For DIS exclusive channels we utilize the dipole approximation in which the p.d.f. derived from DGLAP are serving as input.

## B. SEARCHING FOR UNITARITY EXPERIMENTAL SIGNATURES

We wish to examine if pQCD has to be modified in the small $Q^2$ and $x$ limits due to unitarity effects. The signature we are searching for is a significant deviation from DGLAP either in $F_2$ and/or exclusive dipole estimates.

* Global DGLAP analysis of $F_2$ provides a remarkable reproduction of the data. SC to $F_2$ in the small $Q^2 < 5$ GeV$^2$, $x < 5 \cdot 10^{-3}$ are too small to produce a realistic signature (Ayala, Gay-Ducati, Levin). Clearly we need a "magnifying glass" for a dedicated study at this kinematical corner.

* We anticipate the saturation scale in hard diffraction to be considerably lower than in $\sigma_{tot}(\gamma^* p)$. To this end a study of photo and DIS production of $J/\psi$ is promissing as its hardness scale is at relatively low $Q^2$.

In the following I wish to present a joint analysis of $\frac{\partial F_2}{\partial \ln Q^2}$ at small $Q^2$ and $x$ and photo and DIS production of $J/\psi$. Both channels are relatively rich in data.

$$\frac{\partial F_2}{\partial \ln Q^2} \propto x G(x, Q^2)$$

$$\sigma(\gamma p \to J/\psi\, p) \propto \left(x G(x, Q^2)\right)^2$$

Our initial observation is that none of the global p.d.f. (GRV, MRS, CTEQ) provides a simultaneous reproduction of both channels.

This may be a signature of unitarization since SC for $J/\psi$ are not the square of the SC for $\frac{\partial F_2}{\partial \ln Q^2}$.

Given a p.d.f. which describes well only one of these channel is not suffecient to support a "clean" pQCD interpretation!

This is a good opportunity to coment on the attempts to determine the saturation scale through turn over effects in $\frac{\partial F_2}{\partial \ln Q^2}$ (Caldwell plot, fixed W plots).

Actually, any semi reasonable model in the market does well on these turn over effects which can not serve as effective discriminators between different interpretations

GLMN: a hard pQCD with SC.

CKMT: a single semi hard Pomeron with SC.

G-BW: a sum of soft and saturated hard components.

DL: a two Pomeron model with $Q^2$ form factors.

## C. THE $Q^2$ LOGARITHMIC SLOPE OF $F_2$

DGLAP imply for small enough $x$

$$\frac{\partial F_2}{\partial \ln Q^2} = \frac{2\alpha_s}{9\pi} \times G^{DGLAP}(x, Q^2)$$

We have presented (GLM98, GLMN99) a SC calculation

$$\frac{\partial F_2^{SC}}{\partial \ln Q^2} = D_q(x, Q^2)\, D_g(x, Q^2)\, \frac{\partial F_2^{DGLAP}}{\partial \ln Q^2}$$

SC in the quark sector comes from the percolation of a $q\bar{q}$ through the target.

$$D_q(x, Q^2)\, \frac{\partial F_2^{DGLAP}}{\partial \ln Q^2} = \frac{Q^2}{3\pi^2} \int d^2b \left(1 - e^{-K_q(x, Q^2; b^2)}\right)$$

$$K_q = \frac{2\pi\alpha_s}{3Q^2} \times G^{DGLAP}(x, Q^2)\, \Gamma(b^2)$$

$$\Gamma(b^2) = \frac{1}{R^2}\, e^{-b^2/R^2}$$

SC in the gluon sector are given by

$$x\, G^{SC}(x,Q^2) = D_g(x,Q^2)\, x\, G^{DGLAP}(x,Q^2)$$

$$x\, G^{SC}(x,Q^2) = \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_0^{Q^2} dQ'^2 \left(1 - e^{-\kappa_g(x',Q'^2;\,b^2)}\right)$$

$$\kappa_g(x,Q^2;b^2) = \frac{4}{9}\,\kappa_q(x,Q^2;b^2)$$

$$x\, G(x, Q^2 < \mu^2) = \frac{Q^2}{\mu^2}\, x\, G(x,Q^2) \qquad \mu^2 \simeq Q_0^2$$

ZEUS data $\quad x\, G(x,Q^2) \propto Q^2$ when $Q^2 \to 0 \quad$ gauge invariance

For our calculation we need an input p.d.f. (GRV98 NLO) and $R^2 = 8\text{-}10\ \text{GeV}^{-2}$ which is determined directly from the $J/\psi$ forward differential slope.

Fig 1,2,3

H1 : for 21 data points we have $\frac{\chi^2}{ndf} = 0.75$
we took off 3 data points which are visibly off line.

ZEUS : reported errors are very small so $\chi^2$ is not as good. with estimated theoretical errors $\frac{\chi^2}{ndf} < 1$.
For $x > 10^{-3}$ there may be an additional soft contribution.

# D. PHOTO AND DIS PRODUCTION OF $J/\psi$

A LO dipole calculation (Ryskin, Brodsky...) for a non relativistic $c\bar{c}$ charmonium

$$\left.\frac{d\sigma(\gamma^* p \to J/\psi\, p)}{dt}\right|_{t=0} =$$

$$= \frac{\Gamma_{ee} M^3 \pi^3}{48\alpha} \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^8} \left(x\, G(x, \bar{Q}^2)\right)^2 \left(1 + \frac{Q^2}{M^2}\right)$$

$$x = \frac{M^2}{W^2} \qquad \bar{Q}^2 = \frac{1}{4}(Q^2 + M^2)$$

$$\sigma(\gamma^* p \to J/\psi\, p) = \frac{\left(\frac{d\sigma}{dt}\right)_0}{B}$$ Fig 4

The above pQCD calculation needs to be corrected for

1) Real part: $D_R^2 = (1 + \rho^2)$

$$\rho = \frac{Re\, F}{Im\, F} = tg\, \frac{\pi\lambda}{2}$$

$$\lambda = \frac{\partial \ln F}{\partial \ln \frac{1}{x}} = \frac{\partial \ln x G}{\partial \ln \frac{1}{x}}$$

2) Off diagonal (skewed) gluon distributions

$$R_g^2 = \left(\frac{2^{2\lambda+3}\, \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi}\, \Gamma(\lambda + 4)}\right)^2$$

3) Fermi motion : relativistic corrections in the $c\bar{c}$ system.

In as much as there is no dispute about the need for this correction, its numerics is debated.

FS et. al. estimate $R_F^2 \simeq 0.25$ with $m_c = 1.5$ GeV

Our estimate is $R_F^2 \simeq 0.65$ with $m_c = 1.52$ GeV

The above 3 corrections relate mostly to the normalization of $\sigma(\gamma^* p \to J/\psi\, p)$ but do not change significantly its W or x dependence.

$$\left(\frac{d\sigma}{dt}\right)_0 = D_q^2\, D_g^2 \left(\frac{d\sigma}{dt}\right)_0^{pQCD} R_R^2\, R_g^2\, R_F^2$$

$$D_q^T = \frac{1 + \left(1 - \frac{1}{K_q}\right) E_1\!\left(\frac{1}{K_q}\right) e^{\frac{1}{K_q}}}{2K_q}$$

$$D_q^L = \frac{E_1\!\left(\frac{1}{K_q}\right) e^{\frac{1}{K_q}}}{K_q}$$

different from $D_q\left(\frac{\partial F_2}{\partial \ln Q^2}\right)$

$$D_g(J/\psi) = D_g\left(\frac{\partial F_2}{\partial \ln Q^2}\right)$$

$$\left(\frac{\chi^2}{ndf}\right)_{all} = 0.80 \qquad \left(\frac{\chi^2}{ndf}\right)_{HERA} = 0.85$$

# D. TURN OVER EFFECTS IN $\frac{\partial F_2}{\partial \ln Q^2}$

1. Caldwell plot    Fig 7

   A choice of one x point for each $Q^2$ with a strong correlation does not provide a useful discrimination.

2. Fixed W plots suggested by G-B W
   ZEUS data    Fig 8

$$W^2 \cong \frac{Q^2}{x}$$

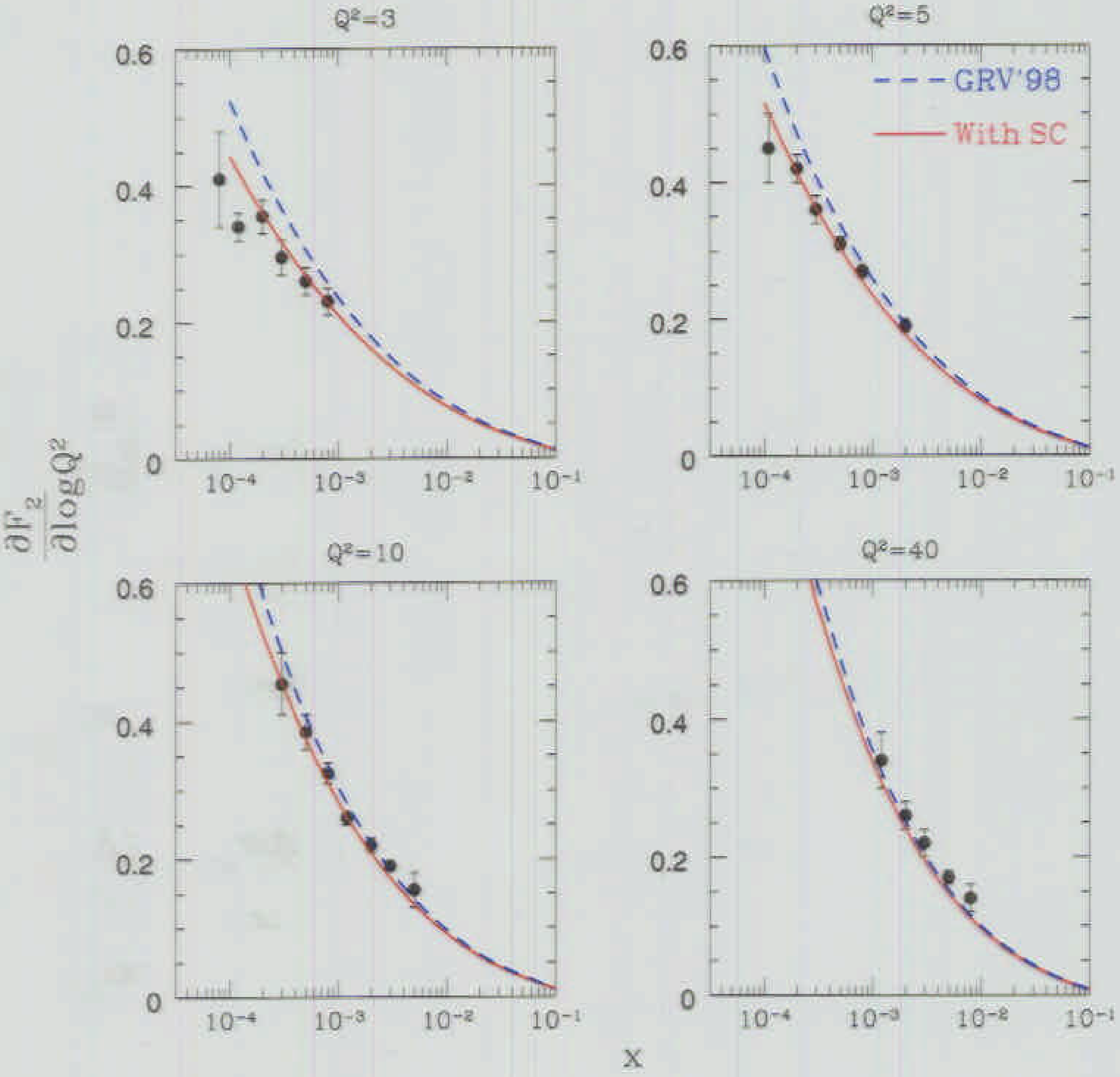
   Any model in which $\frac{\partial F_2}{\partial \ln Q^2}$ is monotonic in $\frac{1}{x}$ and $Q^2$ will produce a turn over. Its position depends on details.

   GLMN    Figs 9, 10

   DL    Figs 11, 12

Fig 1

H1




$R^2 = 8.5\ GeV^{-2}$

Fig 2

ZEUS preliminary

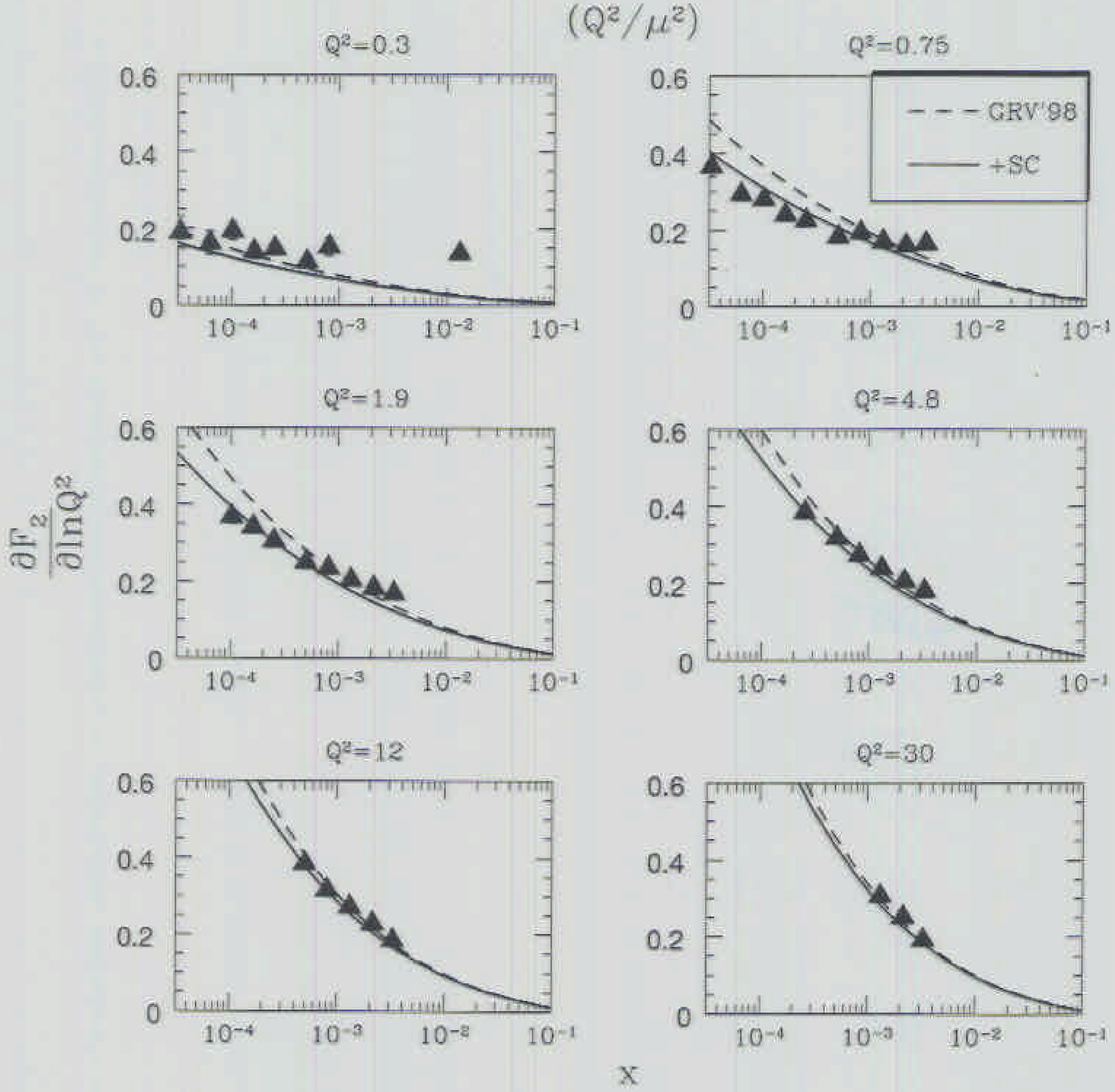
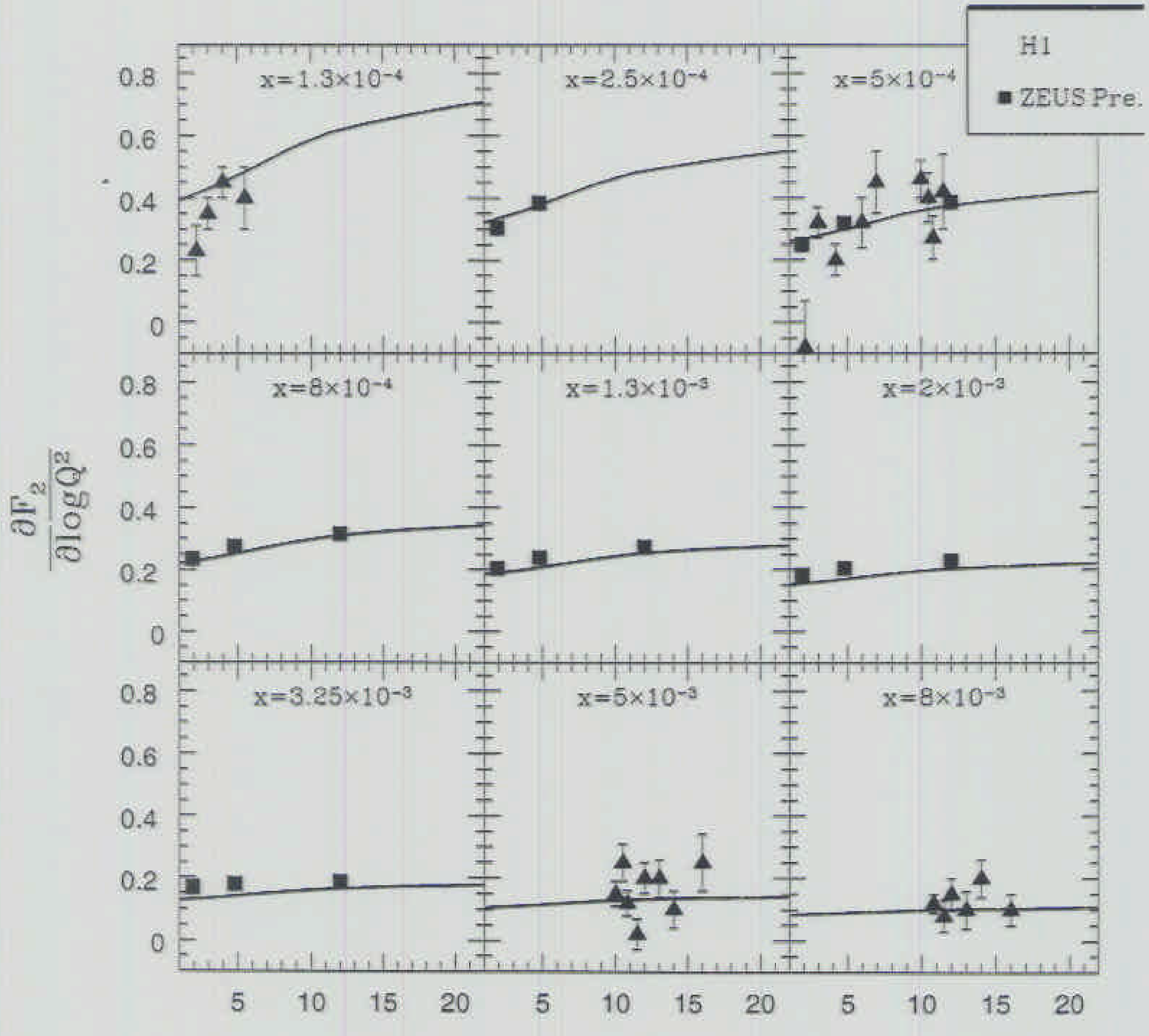


Fig 3



H1
ZEUS Pre.
x=1.3×10⁻⁴
x=2.5×10⁻⁴
x=5×10⁻⁴
x=8×10⁻⁴
x=1.3×10⁻³
x=2×10⁻³
x=3.25×10⁻³
x=5×10⁻³
x=8×10⁻³
$\frac{\partial F_2}{\partial \log Q^2}$
0.8
0.6
0.4
0.2
0
5
10
15
20

Fig 4

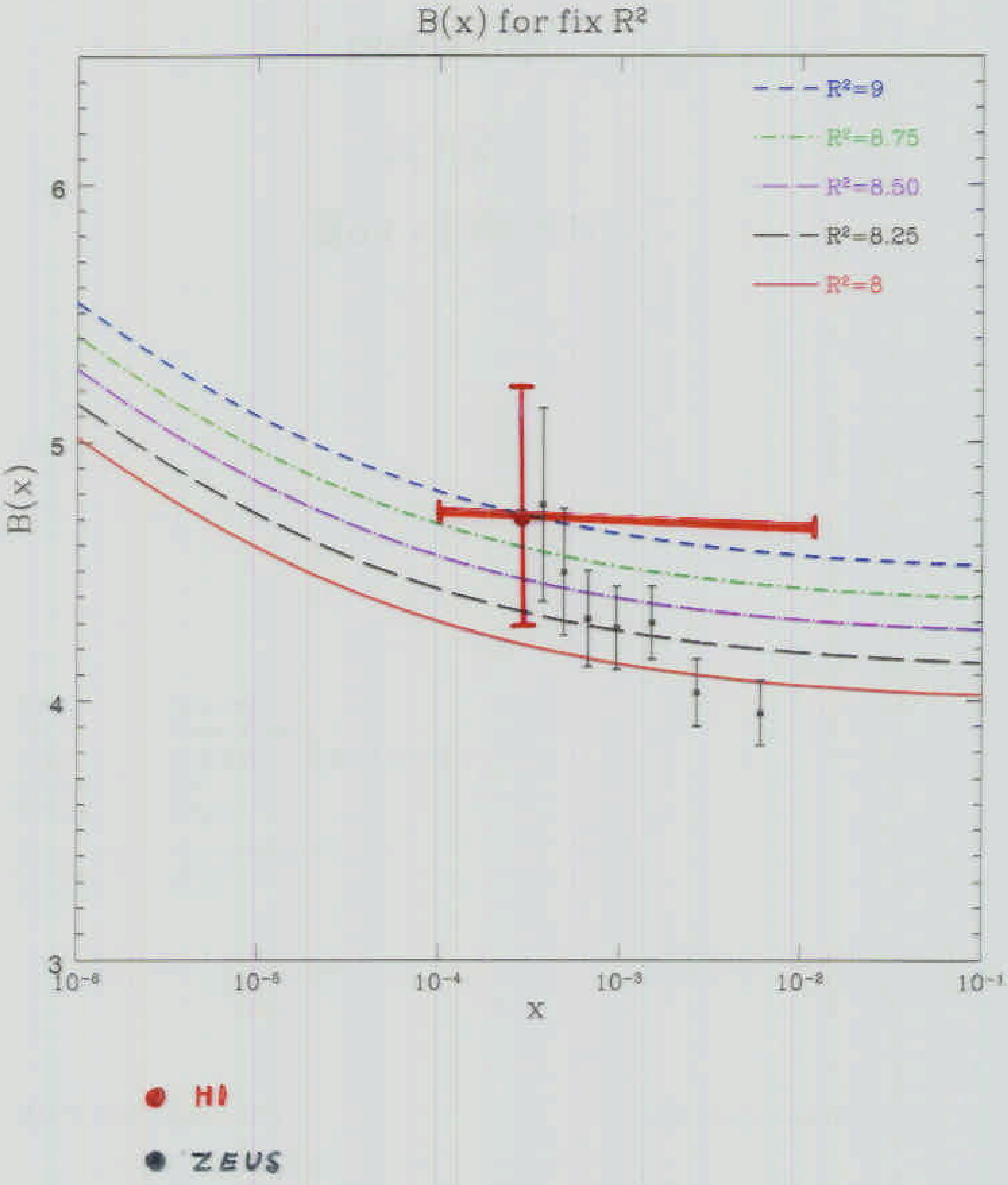


Fig 5

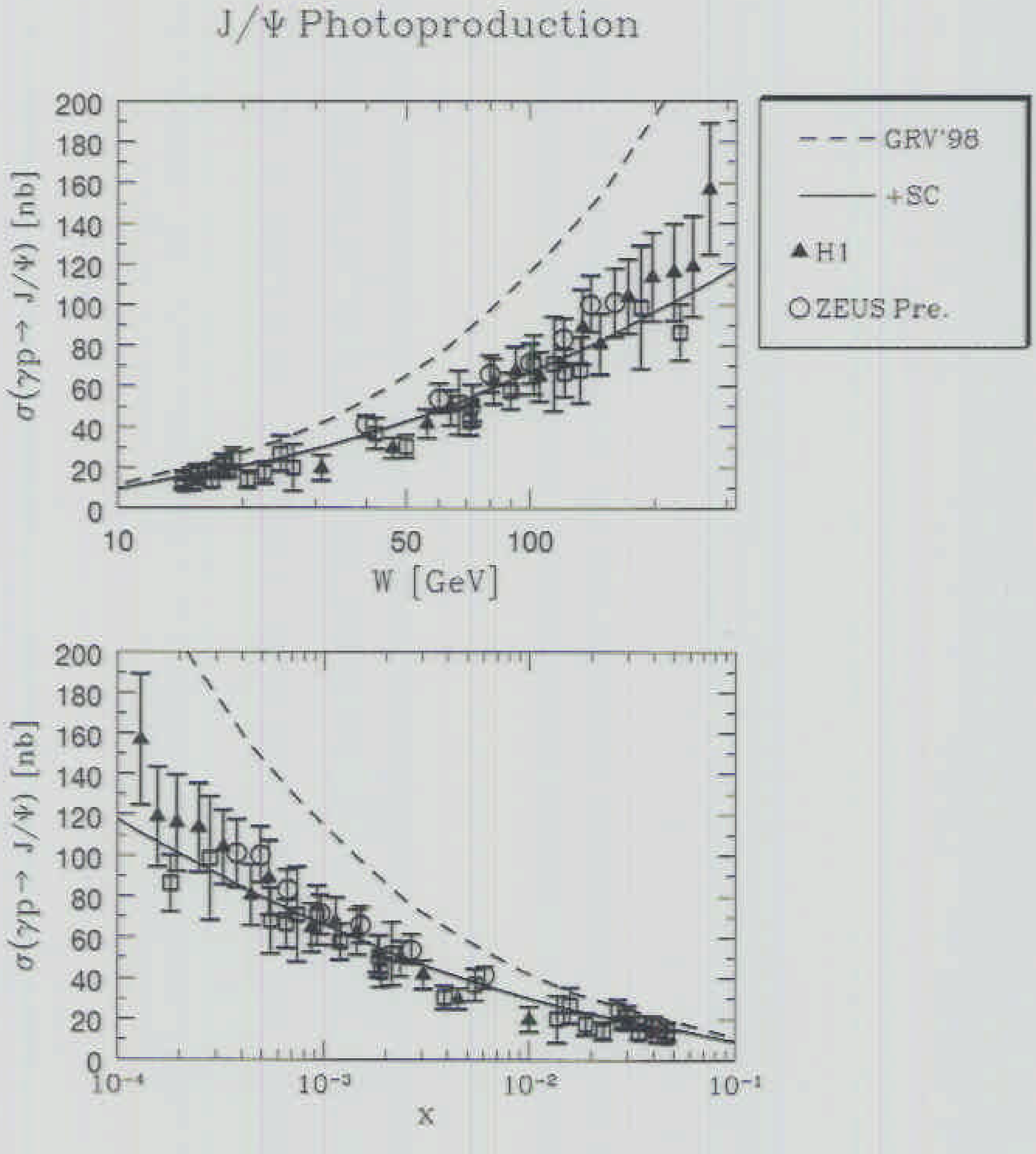
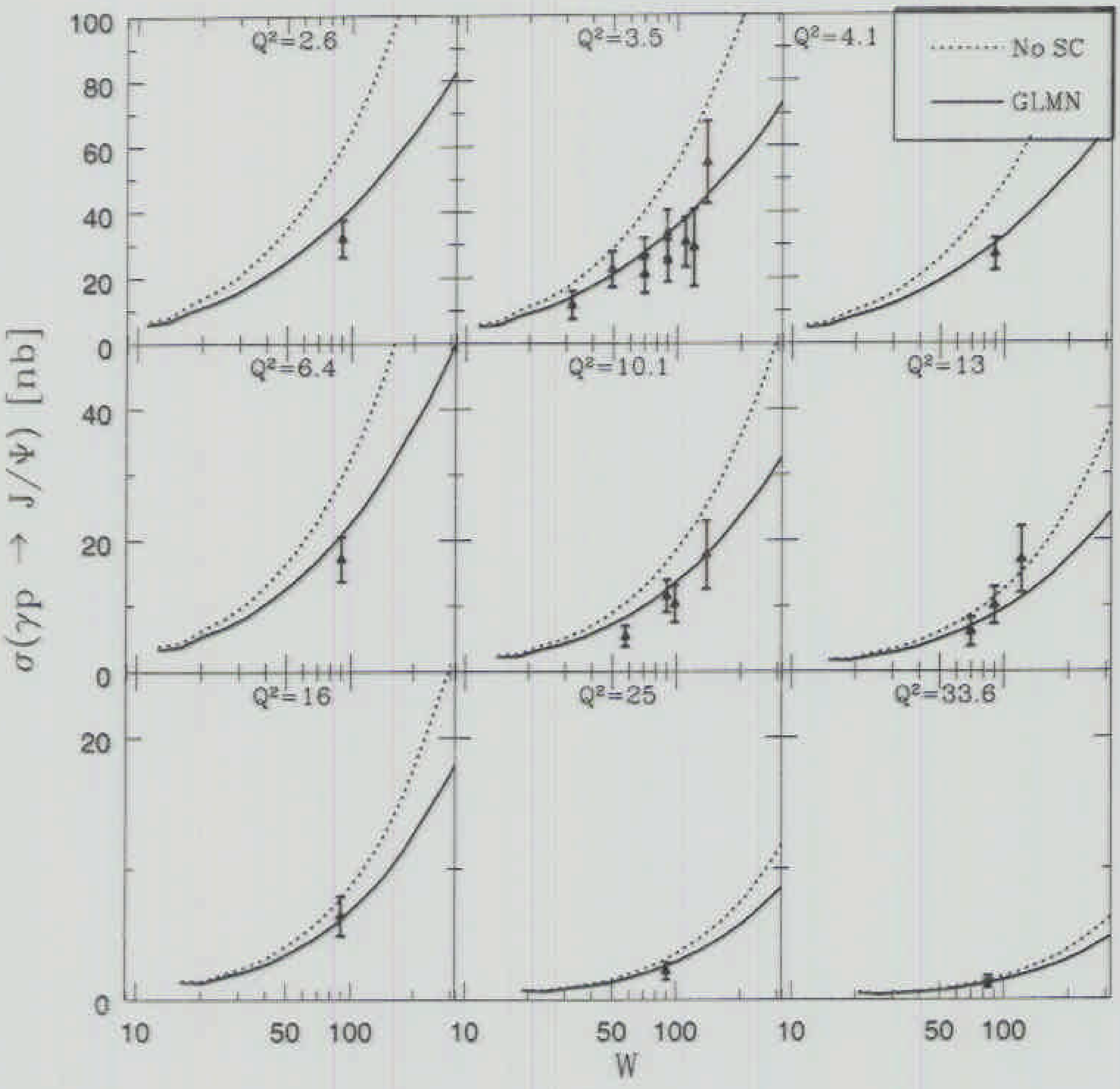
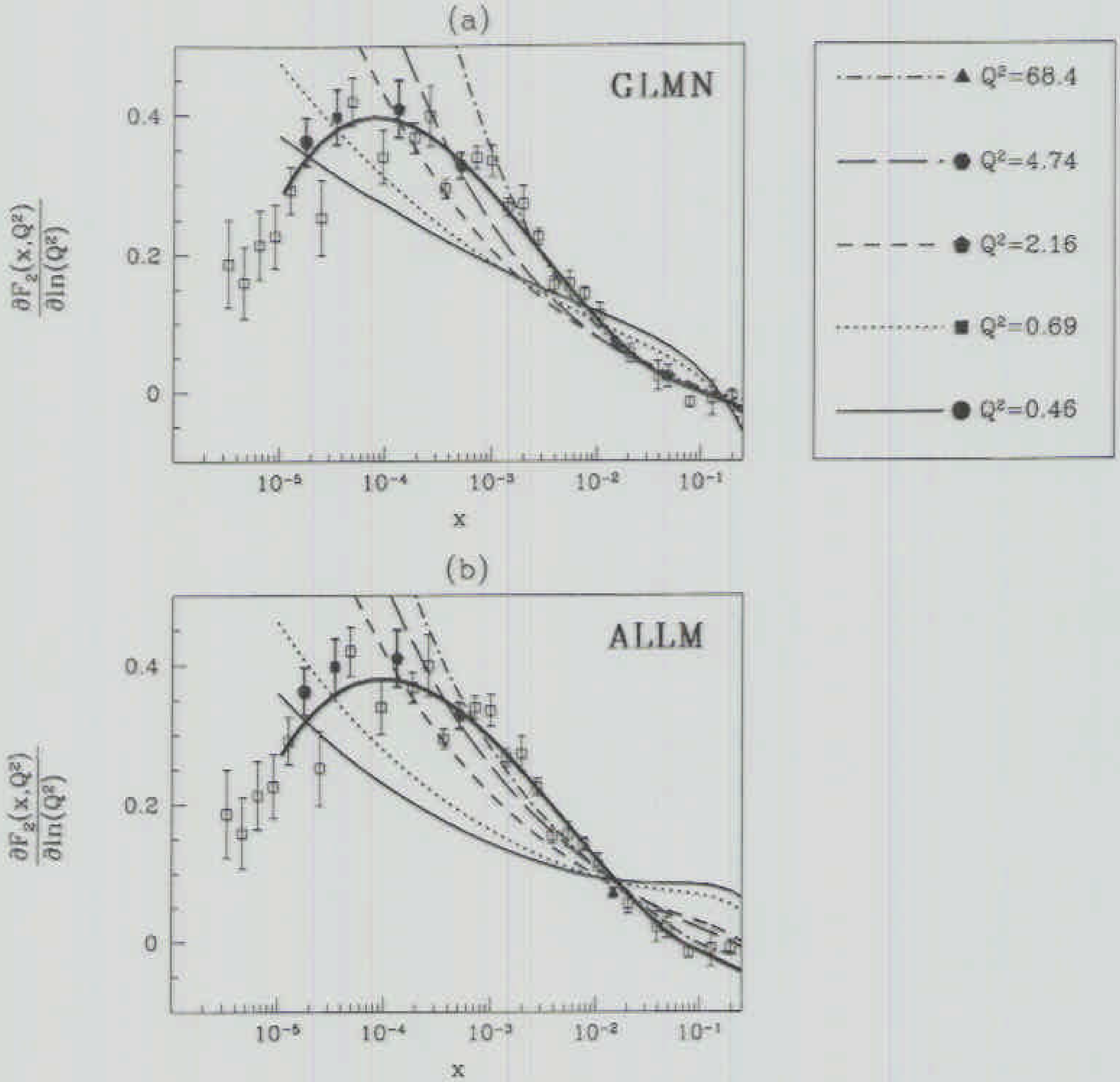


Fig 6



Q²=2.6
Q²=3.5
Q²=4.1
No SC
GLMN
Q²=6.4
Q²=10.1
Q²=13
Q²=16
Q²=25
Q²=33.6
σ(γp → J/Ψ) [nb]
W
100
80
60
40
20
0
10
50
100

Fig 7



(a)
GLMN
0.4
0.2
0
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
x
$\frac{\partial F_2(x,Q^2)}{\partial \ln(Q^2)}$
$Q^2$=68.4
$Q^2$=4.74
$Q^2$=2.16
$Q^2$=0.69
$Q^2$=0.46
(b)
ALLM
0.4
0.2
0
$10^{-5}$
$10^{-4}$
$10^{-3}$
$10^{-2}$
$10^{-1}$
x
$\frac{\partial F_2(x,Q^2)}{\partial \ln(Q^2)}$

Fig 8

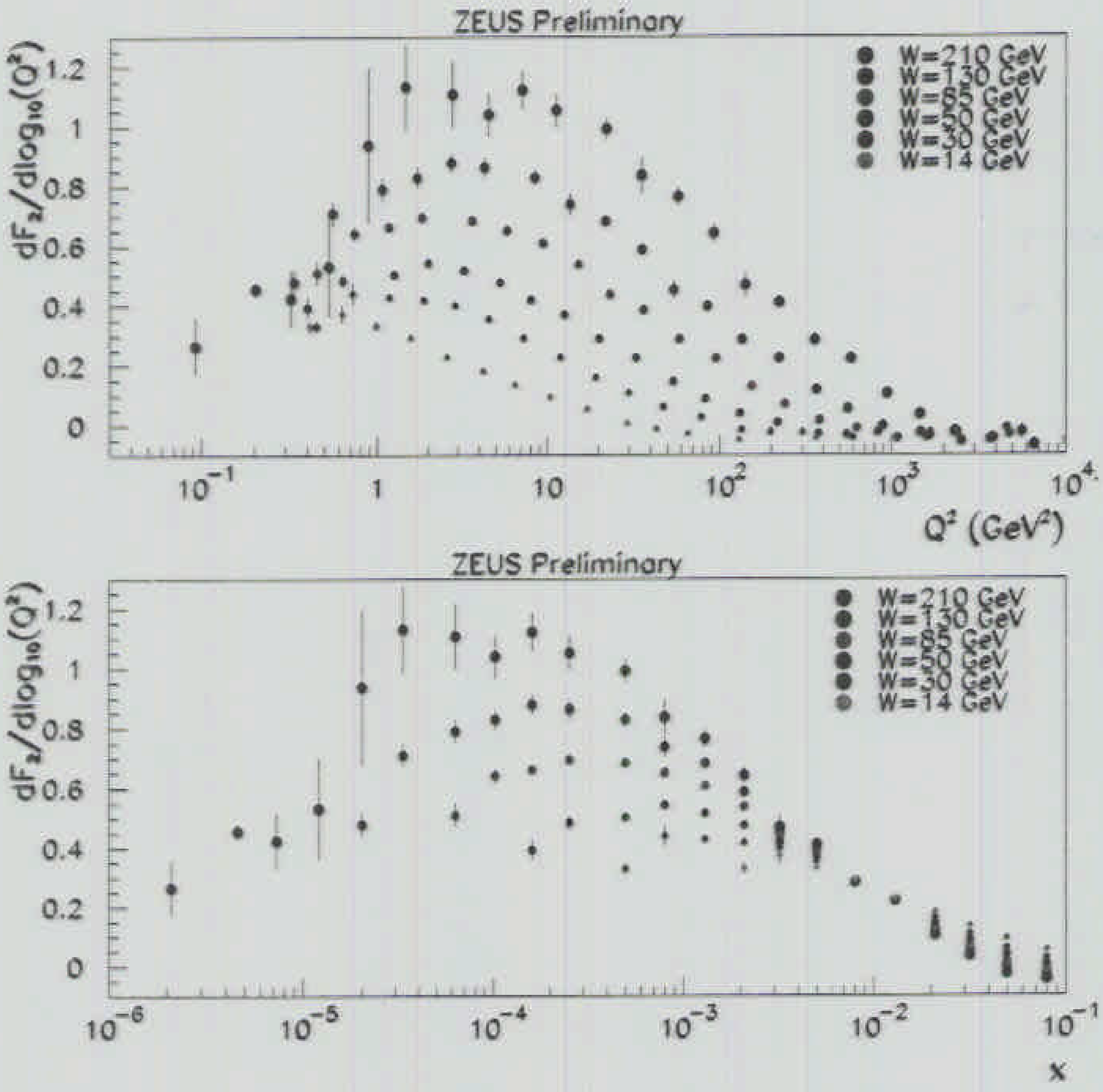

ZEUS Preliminary
W=210 GeV
W=130 GeV
W=85 GeV
W=50 GeV
W=30 GeV
W=14 GeV
$dF_2/d\log_{10}(Q^2)$
$Q^2$ (GeV$^2$)
ZEUS Preliminary
W=210 GeV
W=130 GeV
W=85 GeV
W=50 GeV
W=30 GeV
W=14 GeV
$dF_2/d\log_{10}(Q^2)$
x

Fig 9

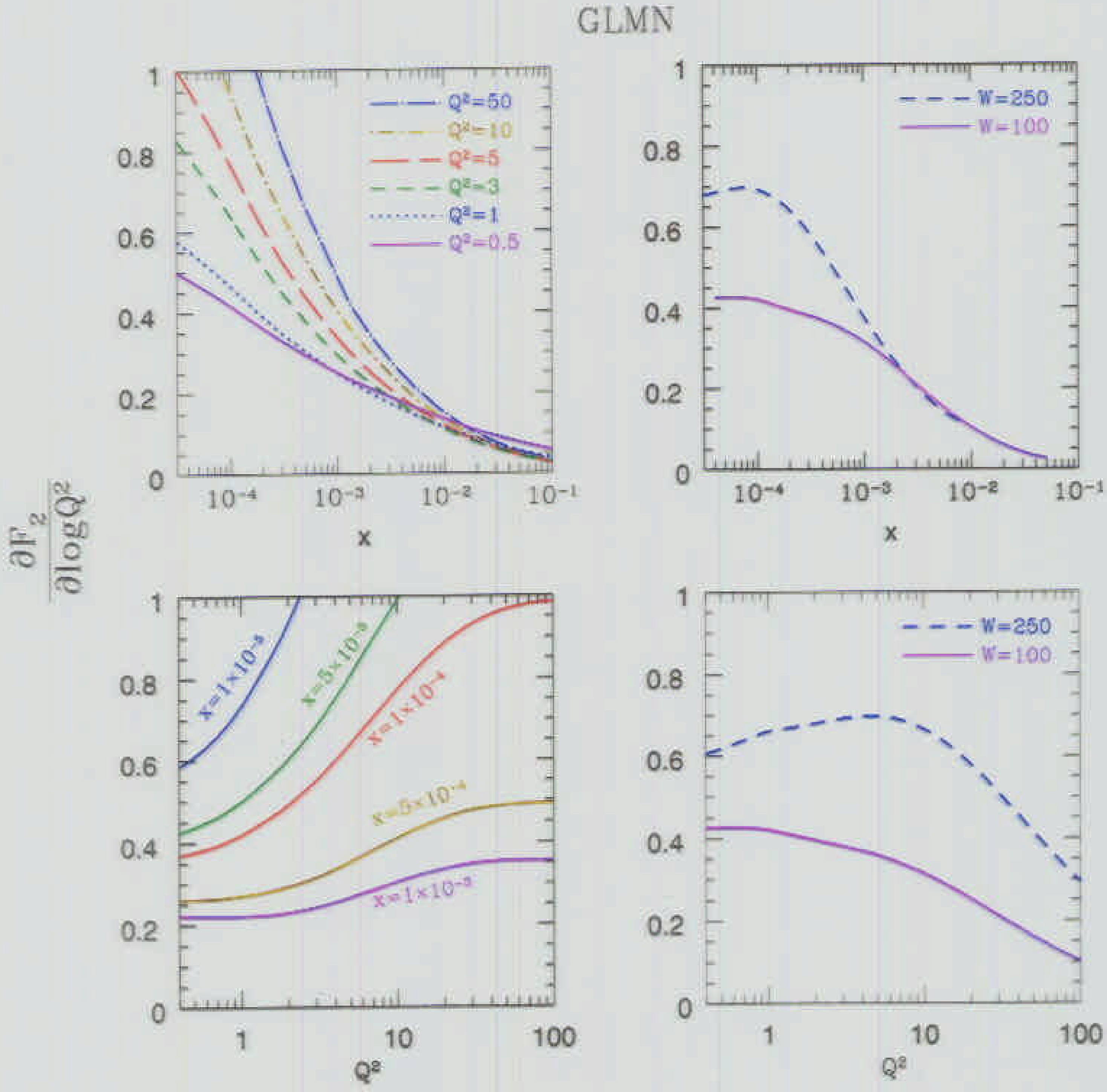

GLMN
Q²=50
Q²=10
Q²=5
Q²=3
Q²=1
Q²=0.5
W=250
W=100
x=1×10⁻⁵
x=5×10⁻⁵
x=1×10⁻⁴
x=5×10⁻⁴
x=1×10⁻³
∂F₂/∂logQ²
x
Q²

Fig 10

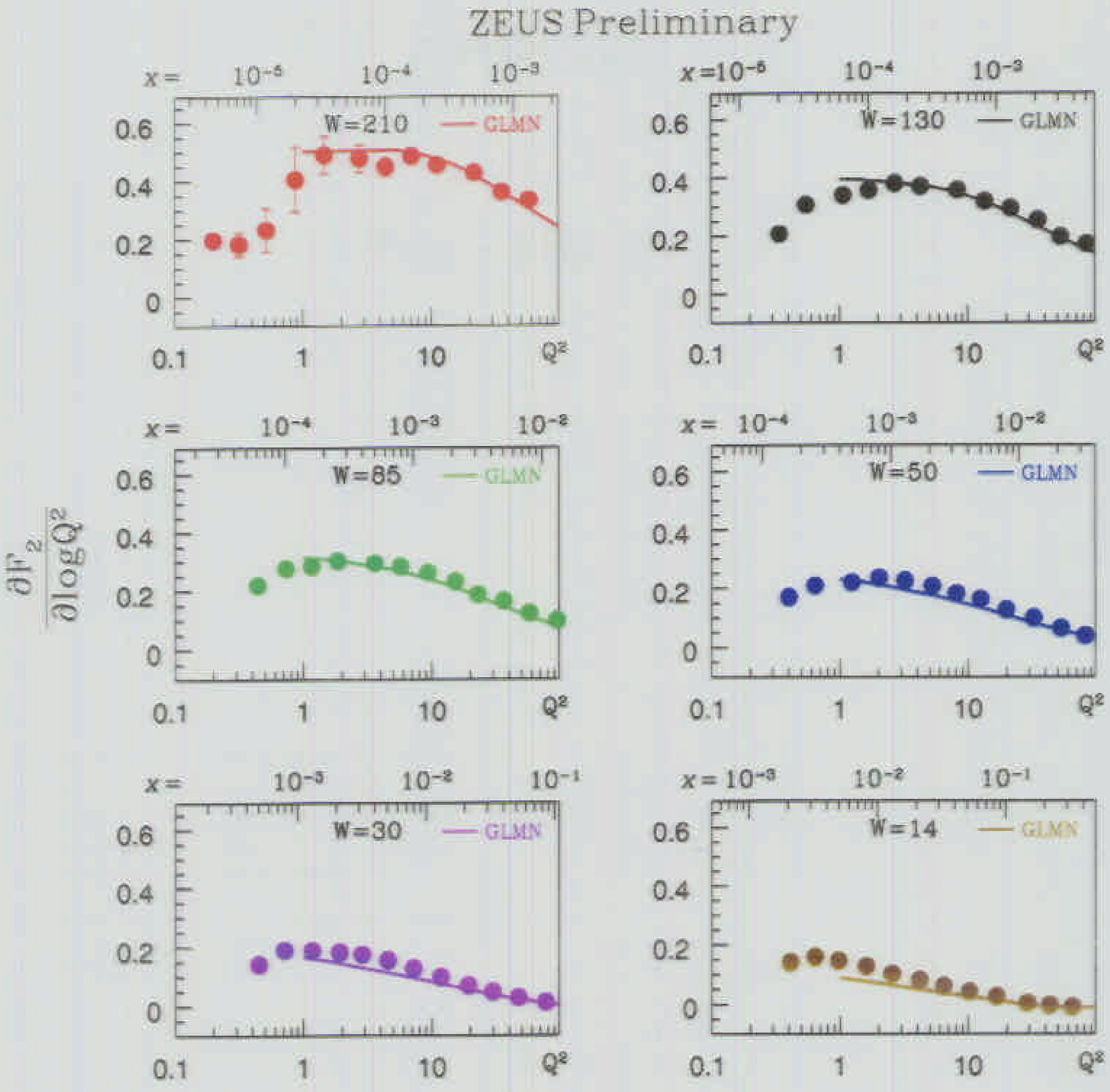

ZEUS Preliminary
x=
10^-5
10^-4
10^-3
W=210
GLMN
W=130
W=85
W=50
W=30
W=14
10^-2
10^-1
0.6
0.4
0.2
0
0.1
1
10
Q^2
∂F_2 / ∂logQ^2

Fig 11

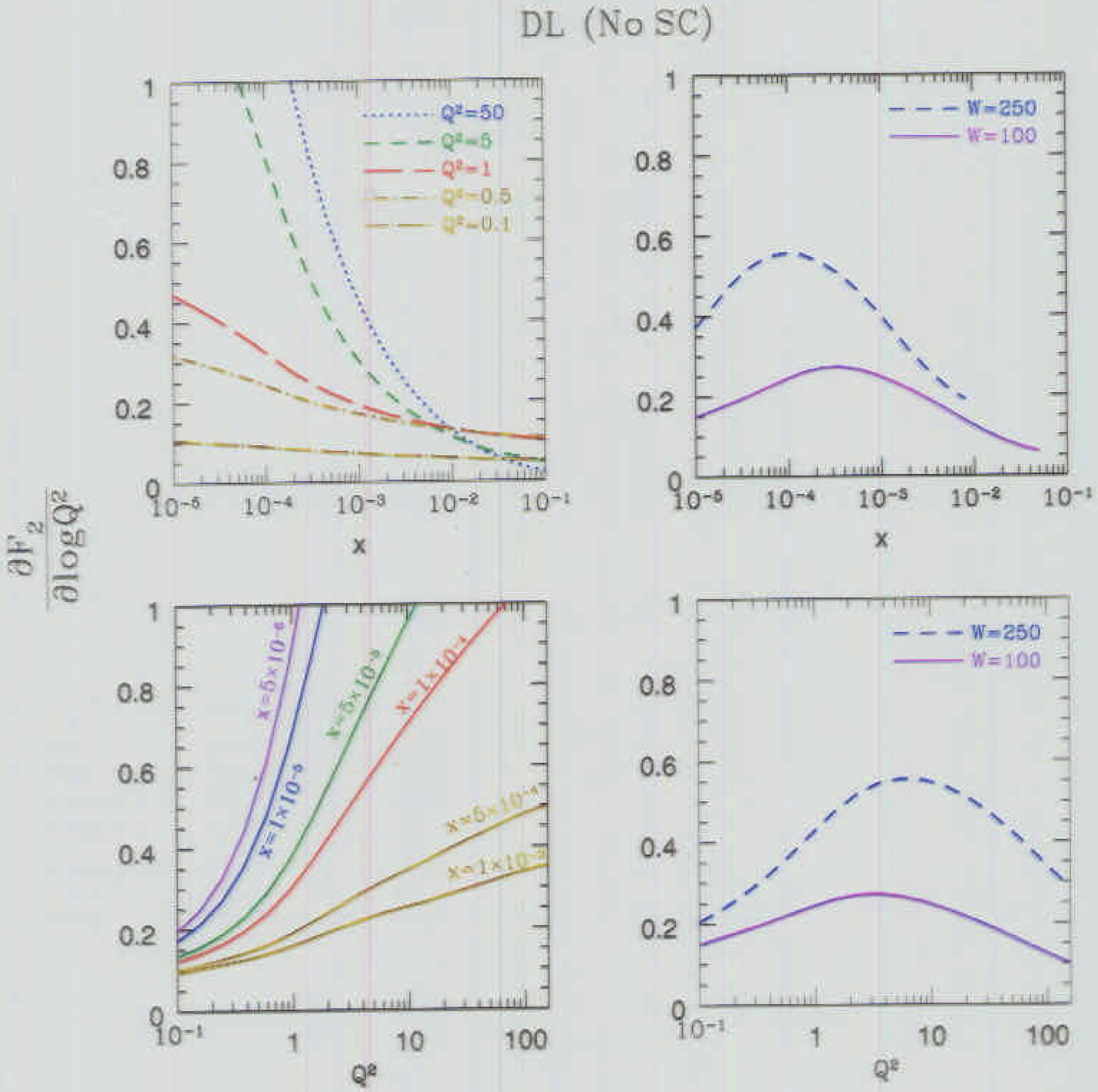

DL (No SC)
Q²=50
Q²=5
Q²=1
Q²=0.5
Q²=0.1
W=250
W=100
x=5×10⁻⁶
x=1×10⁻⁵
x=5×10⁻⁵
x=1×10⁻⁴
x=5×10⁻⁴
x=1×10⁻³
x
Q²
∂F₂/∂logQ²

Fig 12

ZEUS Preliminary