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LIGHT-FRONT QUANTIZED QCD IN LIGHT-CONE GAUGE REVISITED

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- LIGHT-FRONT-QUANTIZED QCD IN COVARIANT GAUGE, SLAC-PUB-8168, May 1999, hep-ph/9906423 (PPS and S.J. Brodsky, Phys. Rev. D61, 25013 (2000).

- Perspectives of Light-Front Quantized Field Theory, SLAC-PUB-8219, August 1999, hep-ph/9908492 (PPS)

- LIGHT-FRONT-QUANTIZED QCD IN LIGHT-CONE GAUGE REVISITED, SLAC-PUB-...., July 2000, (PPS and S.J. Brodsky).

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[2] We recall the discovery of Kruskal-Szekers coordinates which threw a new light on the problem of the Schwarzshild singularity.
The $\pm$ components of a tensor, for example, A^{μ} are defined by $A^{\pm} = A_{\mp} = (A^0 \pm A^3)/\sqrt{2}$ and the metric may be read from $A \cdot B = A^+B^- + A^-B^+ - A^1B^1 - A^2B^2$.

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Quantum action for QCD is based on the Lagrangian density (Covariant Gauge)

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^{a}{}_{\mu\nu} + \underline{B^a\partial_\mu A^{a\mu}} + \frac{\xi}{2}\underline{B^a B^a}$$
$$+ i\partial^\mu \chi_1{}^a \mathcal{D}^{ac}{}_\mu \chi_2{}^c + \bar{\psi}^i(i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j$$

- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a{}_\mu + g f^{abc} A^b{}_\mu A^c{}_\nu$
- $\mathcal{D}^{ac}{}_\mu = (\delta^{ac}\partial_\mu + g f^{abc} A^b{}_\mu)$
- $D^{ij}{}_\mu \psi^j = (\delta^{ij}\partial_\mu - igA^a{}_\mu(\lambda^a/2)^{ij})\psi^j$
- $\gamma^\pm = (\gamma^0 \pm \gamma^3)/\sqrt{2}, \gamma^1, \gamma^2$
- $(\gamma^+)^2 = (\gamma^-)^2 = 0,\ \gamma^+\gamma^- + \gamma^-\gamma^+ = 2I$
- $\Lambda^\pm = \frac{1}{2}\gamma^\mp\gamma^\pm, (\Lambda^\pm)^2 = \Lambda^\pm, \Lambda^+\Lambda^- = 0, ..$

The corresponding $\pm$ projections of the LF Dirac spinor are $\psi_\pm = \Lambda^\pm\psi$ **and** $\psi = \psi_+ + \psi_-, \bar{\psi} = \psi^\dagger\gamma^0 = \bar{\psi}_+ + \bar{\psi}_-$**,** $\gamma^\pm\psi_\mp = 0$ **etc.**

Spinor field propagator on the LF

The quark field term in QCD Lagrangian is

- $$\bar{\psi}^i(i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j = i\sqrt{2}\bar{\psi}^i_+\gamma^0 D^{ij}{}_+\psi_+{}^j + \bar{\psi}^i_+(i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_-{}^j + \bar{\psi}^i_-\left[i\sqrt{2}\gamma^0 D^{ij}{}_-\psi_-{}^j + (i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_+{}^j\right]$$

The minus components $\psi_-{}^j$ are nondynamical (Lagrange multiplier) fields. We find the gauge covariant constraint equation

- $$i\sqrt{2}D^{ij}{}_-\psi_-{}^j = -(i\gamma^0\gamma^\perp D^{ij}{}_\perp - m\gamma^0\delta^{ij})\psi_+{}^j \qquad (1)$$

The $\psi^j{}_-$ components may be eliminated in favor of the dynamical components ψ^j_+

- $$\psi_-{}^j(x) = \frac{i}{\sqrt{2}}\left[U^{-1}(x|A_-)\frac{1}{\partial_-}U(x|A_-)\right]^{jk}(i\gamma^0\gamma^\perp D^{kl}{}_\perp - m\gamma^0\delta^{kl})\psi_+{}^l(x) \qquad (2)$$

Here, for a fixed τ, $U \equiv U(x|A_-)$ is an $N_c \times N_c$ gauge matrix in the fundamental representation of $SU(N_c)$ and

- $$\partial_- U(x|A_-) = -ig\,U(x|A_-)\,A_-(x), \quad A_- = A^a{}_-\lambda^a/2 \qquad (3)$$

Formal solution:

- $$U(x^-, x^\perp|A_-) = U(x^-{}_0, x^\perp|A_-)\,\tilde{\mathcal{P}}\,exp\left\{-ig\int_{x^-{}_0}^{x^-} dy^- A_-(y^-, x^\perp)\right\} \qquad (4)$$

where $\tilde{\mathcal{P}}$ indicates the anti-path-ordering along the longitudinal direction x^-. U has a series expansion in the powers of the coupling constant.

The free field propagator for ψ_+ is determined from the following quadratic terms in the Lagrangian (suppressing the color index) density

$$i\sqrt{2}\psi_+{}^\dagger\partial_+\psi_+ + \psi_+{}^\dagger(i\gamma^0\gamma^\perp\partial_\perp - m\gamma^0)\psi_- \tag{5}$$

Here we have used the free field constraint equation $2i\partial_-\psi_- = (i\gamma^\perp\partial_\perp + m)\gamma^+\psi_+$ which determines the dependent field ψ_-. The equation of motion for the independent component ψ_+ is *nonlocal in the longitudinal direction*

$$\left[4\partial_+ + (m + i\gamma^\perp\partial_\perp)\gamma^-\frac{1}{\partial_-}(m + i\gamma^\perp\partial_\perp)\gamma^+\right]\psi_+ = 0 \tag{6}$$

The free field Hamiltonian formulation can be constructed by following the Dirac procedure. The constraint equation arises now as a second class constraint on the canonical phase space. The Dirac bracket which takes care of these constraints is easily constructed. The effective free LF Hamiltonian is found to be $\mathcal{H}^{lf} = -\bar{\psi}_+(i\gamma^\perp\partial_\perp - m)\psi_-$ and the canonical quantization performed by the correspondence of the Dirac brackets with the (anti-)commutators leads to the following nonvanishing *local* anticommutation relation

$$\{\psi_+(\tau, x^-, x^\perp), {\psi_+}^\dagger(\tau, y^-, y^\perp)\} = \frac{1}{\sqrt{2}}\Lambda^+\delta(x^- - y^-)\delta^2(x^\perp - y^\perp). \tag{7}$$

on the LF.

Covariant Phase Space Factor on the LF. Fourier transform of $\psi(x)$

- $2p^+p^- = (p^\perp p^\perp + m^2) > 0$
- $\int d^4p\, \theta(\pm p^+)\theta(\pm p^-)\delta(p^2 - m^2)$

$$= \int d^2p^\perp dp^+ \int dp^- \; \theta(\pm p^+)\theta(\pm p^-)\, \delta(2p^+p^- - \left[m^2 + {p^\perp}^2\right])$$

$$= \int d^2p^\perp dp^+ \theta(p^+)/(2p^+)$$

- $\int d^4p\, \theta(\pm p^0)\delta(p^2 - m^2) = \int d^3\vec{p}/(2E_p); \quad E_p = +\sqrt{\vec{p}^2 + m^2} > 0$

(Srivastava and Sudarshan, 1958).

A distinguishing feature in the case of the LF is thus the appearence of $\theta(p^+)/(2p^+)$ in the phase space factor. Such considerations are relevant, for example, in writing the Fourier transform of the fields and the discussion of chiral boson theory.

Fourier transform of $\psi(x)$ over the complete set of linearly independent plane wave solutions of the free Dirac equation, say, for $p^+ > 0$ may be written as

$$\psi(x) = \frac{1}{\sqrt{(2\pi)^3}} \sum_{r=\pm} \int d^2p^\perp dp^+ \, \theta(p^+) \sqrt{\frac{m}{p^+}} \left[b^{(r)}(p) u^{(r)}(p) e^{-ip.x} + d^{\dagger(r)}(p) \right. \tag{8}$$

A very useful form (Srivastava, 1995) of the solution for the four-spinors in the context of the LF quantization is

$$u^{(r)}(p) = \frac{1}{(\sqrt{2}p^+ m)^{\frac{1}{2}}} \left[\sqrt{2}p^+ \Lambda^+ + (m + \gamma^\perp p_\perp) \Lambda^- \right] \tilde{u}^{(r)} \tag{9}$$

where the constant spinors $\tilde{u}^{(r)}$ satisfy $\gamma^0 \tilde{u}^{(r)} = \tilde{u}^{(r)}$ and $\Sigma_3 \tilde{u}^{(r)} = r \tilde{u}^{(r)}$ with $\Sigma_3 = i\gamma^1\gamma^2$ and $r = \pm$. Its Λ^+ projection is by construction very simple, $u^{(r)}{}_+(p) = (\sqrt{2}p^+/m)^{\frac{1}{2}} (\Lambda^+ \tilde{u}^{(r)})$ and they are eigenstates of Σ_3 as well while the $\tilde{u}^{(r)}$ correspond to the rest frame spinors for which $\sqrt{2}p^\pm = m$.

The anticommutation relations of the spinor field are satisfied if the creation and the annihilation operators are assumed to satisfy the canonical anticommutation relations, with the nonvanishing ones given by: $\{b^{(r)}(p), b^{\dagger(s)}(p')\} = \{d^{(r)}(p), d^{\dagger(s)}(p')\} = \delta_{rs}\delta(p^+ - p'^+)\delta^2(p^\perp - p'^\perp)$.

- **The free propagator**

$$
\begin{aligned}
&\langle 0|\, T(\psi_{+A}(x)\psi^{\dagger}_{+B}(0))\,|0\rangle \\
&= \langle 0|\left[\theta(\tau)\psi_{+A}(x)\psi^{\dagger}_{+B}(0) - \theta(-\tau)\psi^{\dagger}_{+B}(0)\psi_{+A}(x)\right]|0\rangle \\
&= \frac{1}{\sqrt{2}}\frac{\Lambda^{+}_{AB}}{(2\pi)^3}\int d^2q^{\perp}dq^{+}\theta(q^{+})\left[\theta(\tau)e^{-iqx} - \theta(-\tau)e^{iqx}\right]
\end{aligned}
$$

where $A, B = 1.4$ label the spinor components.

The only relevant differences, compared with the case of the scalar field, are, apart from the appearance of the projection operator, the *absence* **of the factor $(1/2q^{+})$ in the integrand and the** *negative sign* **of the second term in the fermionic case. They, however,** *compensate*, **and the standard manipulations to factor out the exponential give rise to the factor $[\theta(q^{+}) + \theta(-q^{+})]$ which may be interpreted as unity in the distribution theory sense, parallel to what we find in the derivation of the scalar field propagator on the LF. The straightforward use of the integral representation $2\pi i\,\theta(\tau)\,e^{-ip\tau} = \int d\lambda\, e^{(-i\lambda\tau)}/(p-\lambda-i\epsilon)$ of $\theta(\pm\tau)$, together with the standard manipulations in the second term to factor out the exponential, leads**

to

$$< 0|T(\psi^i{}_+(x)\psi^{\dagger j}{}_+(0))|0 >= \frac{i\delta^{ij}}{(2\pi)^4}\int d^4q\, \frac{\sqrt{2}q^+\Lambda^+}{(q^2 - m^2 + i\epsilon)}\, e^{-iq.x}, \tag{10}$$

where we have renamed the dummy integration variable originating from the integral representation of $\theta(\pm\tau)$ as q^- and $d^4q = d^2q^\perp dq^+ dq^-$ with all integrations ranging from $-\infty$ to ∞.

The fermionic propagator is causal and contains *no instantaneous term* found traditionally in literature!

Gauge field propagator (Cov. Gauge)

The free abelian gauge theory described by the following Lagrangian density

$$\frac{1}{2}\left[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\right] + B(\partial_+A_- + \partial_-A_+ + \partial_\perp A^\perp) + \frac{\xi}{2}B^2, \tag{11}$$

The canonical momenta are $\pi^+ = 0$, $\pi_B = 0$, $\pi^\perp = F_{-\perp}$, $\pi^- = F_{+-} + B$. In the Dirac procedure the primary constraints are $\pi^+ \approx 0$, $\pi_B \approx 0$ and $\eta \equiv \pi^\perp - \partial_- A_\perp + \partial_\perp A_- \approx 0$ etc. etc. We find

$$H_o{}^{LF} = -\frac{1}{2}\int d^2x^\perp dx^-\; g^{\mu\nu} A_\mu \partial^\perp \partial_\perp A_\nu. \tag{12}$$

SYSTEM IN $x^0 \to$ - INSTANT FORM CONVENTIONAL THEORY $\to$ EQUAL-TIME QUANTIZATIZED THEORY

- LIGHT-FRONT QUANTIZATION IS EQUALLY VALID AS THE CONVENTIONAL ONE ON GENERAL CONSIDERATIONS (SSB, QED$_2$, CHIRAL QED$_2$, CHERN-SIMONS-HIGGS, LF QUANTIZED QCD IN COV. GAUGE, ...)

Quantum Action FOR QCD: in Light-Cone Gauge: $A^a_- = 0$

l.c. gauge

$$\mathcal{L}_{QCD} = \underbrace{-\frac{1}{4}F^{a\mu\nu}F^a{}_{\mu\nu}}_{\mathcal{L}_{gauge}} + \underbrace{B^a A^a{}_-}_{\mathcal{L}_{gf}} + \underbrace{\bar{c}^a \mathcal{D}^{ab}_- c^b}_{\mathcal{L}_{ghost}} + \underbrace{\bar{\psi}^i (i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j}_{\mathcal{L}_q}$$

Spinor field propagator on the LF:

(PR D61, 25013, 2000, OPS + Brodsky)

$$\begin{aligned}\bar{\psi}^i(i\gamma^\mu D^{ij}{}_\mu - m\delta^{ij})\psi^j &= i\sqrt{2}\bar{\psi}^i_+\gamma^0 D^{ij}{}_+\psi_+{}^j + \bar{\psi}^i_+(i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_-{}^j \\ &+ \bar{\psi}^i_-\left[i\sqrt{2}\gamma^0 D^{ij}{}_-\psi_-{}^j + (i\gamma^\perp D^{ij}{}_\perp - m\delta^{ij})\psi_+{}^j\right]\end{aligned}$$

Light-Cone Gauge : $A_0^a = A_3^a$ (Equal-time, x^0 quantization)

- G. Leibbrandt, R.M.P. 59, 1067 ('87)
- "Physical and Nonstandard Gauges", Eds. P. Gaigg et al Lecture Notes # 361, Springer-Verlag, 1990
- A. Bassetto, G. Nardelli & R. Soldati; "Y.M. Theories in Algebraic Non-covariant Gauges", World Scientific, 1991
- G. Leibbrandt, "Non-covariant Gauges, Quantization of YM and Chern-Simons Theory", World Scientific, 1994

Light-Cone Gauge : $A_-^a = 0$ (On the Light-front) (Equal x^+ quantization)

Lepage and Brodsky, P.R. D22, 2157 ('80)
Brodsky, Pauli, Pinsky, Phys. Rep. 301, 299 ('98)
Wilson, Perry, Harindranath, ..., P.R. D49, 6720 ('94)

- Only physical degrees of freedom
-

$A_- = 0$ On the Light-Front:

- mostly old fashioned perturbation theory employed. – Lepage Brodsky (difficult to do higher loop computations)

- attempts made to use Dyson-Wick expansion with massive gauge field. (Y Soper, Kogut, Yan et.al.

- We study massless gauge field.
- Show that Dyson-Wick perturbation theory procedure is least straightforwardly
- It is economical in L.C. gauge
- Simultaneously with $A_-^a = 0$ we also have Lorentz condition $\partial \cdot A^a = 0$
- The gauge field propagator is transverse
- The fermion field propagator is causal with no instantaneous terms

In L.F. coordinates:

$\mathcal{L}_{gauge}$
$$= \frac{1}{2}[F^a{}_{+-}F^a{}_{+-} + 2F^a{}_{+\perp}F^a{}_{-\perp} - F^a{}_{12}F^a{}_{12}] + B^a A^a{}_- + \bar{c}^a\partial_- c^a.$$

$$\mathcal{L}^{(0)}_{gauge} = \int d^2x^\perp dx^- \left\{\frac{1}{2}\Big[(F_{+-})^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp}\Big] + BA_-\right\}$$

- **$A^a{}_+$ like B^a have no kinetic terms and they enter in the action as auxiliary multiplier fields.**

- **Dirac Procedure (Standard):**

Constraints:

$$\pi^+ \approx 0,$$
$$\pi^+ \approx 0,$$
$$\eta^\perp \equiv \pi^\perp - \partial_- A_\perp + \partial_\perp A_- \approx 0,$$

$$\Phi \equiv \partial_-\pi^- + \partial_\perp \pi^\perp \approx 0,$$
$$A_- \approx 0$$

$$\Psi \equiv \pi^- + \partial_- A_+ \approx 0.$$

Add the external gauge-fixing constraint to take care of first class $\pi_B \approx 0$.

- Surviving dynamical variables

- $$A_\perp \qquad \perp = 1,2$$

- A_+ IS DEPENDENT VARIABLE

$$\partial_-(\partial_- A_+ - \partial_\perp A_\perp) = 0$$

- Reduced Hamiltonian H_0^{LF} ON THE LIGHT-FRONT:

$$H_0{}^{LF} = \frac{1}{2}\int d^2x^\perp dx^- \left[(\partial_- A_+)^2 + \frac{1}{2}F_{\perp\perp'}F^{\perp\perp'}\right].$$

- We have retained A_+ for convenience in order to facilitate the comparasion with the the results obtained in the covariant gauge.

- **COMMUTATORS**

$$[A_\perp(\tau, x^-, x^\perp), A_{\perp'}(\tau, y^-, y^\perp)] = -i\frac{\delta_{\perp\perp'}}{4}\epsilon(x^- - y^-)\delta^2(x^\perp - y^\perp)$$

- **THE COMMUTATORS OF A_+ FOLLOW TO BE AS OBTAINED BY SUBSTITUTION:**

$$A_+ \rightarrow \frac{\partial_\perp}{\partial_-}A_\perp$$

- **WHICH IMPLIES: THE LORENTZ CONDITION HOLDS AS AN OPERATOR EQUATION IN THE LF QUANTIZED QCD IN L.C. GAUGE.**

- **THE FREE PROPAGATOR FOR MASSLESS GAUGE FIELD ON THE LF:**

- $$< 0|\, T(A^a{}_\mu(x) A^b{}_\nu(0))\, |0 >= \frac{i\delta^{ab}}{(2\pi)^4} \int d^4k\, e^{-ik.x}\, \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon}$$

- $$D_{\mu\nu}(k) = D_{\nu\mu}(k) = -g_{\mu\nu} + \underbrace{\frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)}}_{\text{usual (in Eq.-time theory)}} \underbrace{- \frac{k^2}{(n \cdot k)^2} n_\mu n_\nu}_{\text{(on the L.F. extra term)}}.$$

$$n_\mu = \delta_\mu{}^+, \qquad n^\mu = \delta^\mu{}_-$$

- TRANSVERSE PROPAGATOR: (off-shell transverse)
(as Landau gauge)

$$D_{\mu\lambda}(k) D^\lambda{}_\nu(k) = D_{\mu\perp}(k) D^\perp{}_\nu(k) \; = \; -D_{\mu\nu}(k),$$
$$k^\mu D_{\mu\nu}(k) = 0, \qquad n^\mu D_{\mu\nu}(k) \equiv D_{-\nu}(k) = 0,$$
$$D_{\lambda\mu}(q)\, D^{\mu\nu}(k)\, D_{\nu\rho}(q) \; = \; D_{\lambda\rho}(q)$$

- **WE MAY CHOOSE THE TWO PHYSICAL POLARIZATION VECTORS AS**

- $$\boxed{E_{(\perp)}{}^{\mu}(k) = E^{(\perp)\mu}(k) = -\, D^{\mu}_{\perp}(k)}$$ $(\perp) = (1), (2)$

- $$\sum_{(\perp)=1,2} E^{(\perp)}{}_{\mu}(k) E^{(\perp)}{}_{\nu}(k) = D_{\mu\nu}(k), \qquad g^{\mu\nu} E^{(\perp)}{}_{\mu}(k) E^{(\perp')}{}_{\nu}(k) = g^{\perp}$$
$$k^{\mu} E^{(\perp)}_{\mu}(k) = 0, \qquad n^{\mu} E^{(\perp)}_{\mu} \equiv E^{(\perp)}_{-} = 0$$

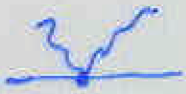
- $$A^{\mu}(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^{\perp} dk^{+} \frac{\theta(k^{+})}{\sqrt{2k^{+}}} \sum_{(\perp)} E_{(\perp)}{}^{\mu}(k) \left[b_{(\perp)}(k^{+}, k^{\perp}) e^{-ik\cdot x} + b^{\dagger}_{(\perp)}(\right.$$

where the l.c. gauge $A_{-} = 0$ along with the Lorentz condition are already incorporated in it through the very construction of the polarization vectors.

LF QCD HAMILTONIAN IN L.C. GAUGE:

- $\mathcal{H}^{LF} = \mathcal{H}_0^{LF} + \mathcal{H}_{int}$

- $$\begin{aligned}
\mathcal{H}_{int} = & -g\,\bar{\psi}^i \gamma^\mu A_\mu{}^{ij} \psi^j \\
& +\frac{g}{2} f^{abc} \left(\partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu\right) A^{b\mu} A^{c\nu} \\
& +\frac{g^2}{4} f^{abc} f^{ade} A_{b\mu} A^{d\mu} A_{c\nu} A^{e\nu} \\
& -\frac{g^2}{2}\, \bar{\psi}^i \gamma^+ \left(\gamma^{\perp'} A_{\perp'}\right)^{ij} \frac{1}{i\partial_-} \left(\gamma^\perp A_\perp\right)^{jk} \psi^k \\
& -\frac{g^2}{2}\, j_a^+ \frac{1}{(\partial_-)^2} j_a^+
\end{aligned}$$

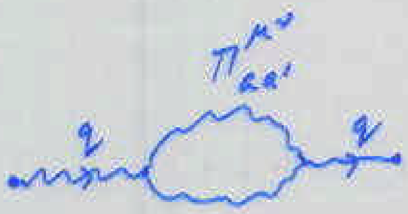


→ Thomson scattering

where

- $$j_a^+ = \bar{\psi}^i \gamma^+ (t_a)^{ij} \psi^j + f_{abc}(\partial_- A_{b\mu}) A^{c\mu}$$

- **Gluon Self-energy corrections:**



$$\Pi^{\mu\nu}_{aa'} = \frac{-ig^2}{2} C_A \delta_{aa'} \Pi^{\mu\nu}(q)$$

$$\Pi^{\mu\nu}(q) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{[k^2 + i0]\,[(k+q)^2 + i0]}\, I^{\mu\nu}(q,k),$$

$$I^{\mu\nu}(q,k) = [-(2k+q)^{\mu} g^{\alpha\beta} + (k-q)^{\beta} g^{\mu\alpha} + (2q+k)^{\alpha} g^{\mu\beta}] D_{\alpha\rho}(k)$$
$$[-(2k+q)^{\nu} g^{\rho\sigma} + (k-q)^{\sigma} g^{\nu\rho} + (2q+k)^{\rho} g^{\nu\sigma}] D_{\sigma\beta}(k+q)$$

SIMPLIFIES TO $(k^{\mu} D_{\mu\nu}(k) = 0)$

$$I^{\mu\nu}(q,k) = [-(2k+q)^{\mu} g^{\alpha\beta} - 2q^{\beta} g^{\mu\alpha} + 2q^{\alpha} g^{\mu\beta}] D_{\alpha\rho}(k)$$
$$[-(2k+q)^{\nu} g^{\rho\sigma} - 2q^{\sigma} g^{\nu\rho} + 2q^{\rho} g^{\nu\sigma}] D_{\sigma\beta}(k+q)$$

For the divergent term we find (suppressing I^{div}):

$$D_{\lambda\mu}(q)\,\Pi^{\mu\nu}(q)\,D_{\nu\delta}(q) = (-\frac{22}{3}q^2 + 16q^+q^-)\,D_{\lambda\delta}(q)$$

Regularization:

- $\boxed{\text{DIMENSIONAL REGULARIZATION}}$

$\oplus$

- MANDELSTAM – LEIBBRANDT PRESCRIPTION FOR $\left(\frac{1}{k^+}\right)$ SINGULARITY

$\boxed{D_{\mu\nu}(k)/k^2}$:

$(3+1)$ dim: TWO SINGULARITIES TO BE HANDLED

$$k^2 = 0 \quad \text{in} \quad \frac{1}{k^2}$$

$$k^+ = 0 \quad \text{in} \quad \frac{1}{k^+}$$

<u>1+1 dim:</u> $k^2 = 2k^+k^-$

Theory quantized on $x^+ =$ const plane has (explicit) $\frac{1}{k^+}$ (non local) singularity

$$\text{Say} \left(\frac{1}{k^2}\right) = \left(\frac{1}{2k^+k^-}\right) \longrightarrow \frac{1}{(k^2 + i\epsilon)} = \frac{1}{2k^+k^- + i\epsilon}$$

$$\leadsto \left(\frac{2k^-}{2k^-k^+}\right) = \left(\frac{2k^-}{k^2}\right) \longrightarrow \frac{2k^-}{2k^+k^- + i\epsilon} = \frac{1}{k^+ + i\epsilon\,\mathrm{Sgn}(k^-)}$$

Both $\left(\frac{1}{k^2}\right)$ and $\left(\frac{1}{k^+}\right)$ are treated by Causal prescription.

e.g.

$$\boxed{\begin{aligned} \frac{1}{k^+} \longrightarrow \frac{1}{k^+ + i\epsilon\,\mathrm{Sgn}(k^-)} \quad & M \\ \text{or} \quad = \frac{2k^-}{(k^2 + i\epsilon)} \quad & L \end{aligned}}$$

- $\Pi^{\mu\nu}$ **IS NOT TRANSVERSE**

$$q_\nu I^{\mu\nu} = 2\,(q^2{+}2k.q)\;[(2k{+}q)^\mu{+}(q^\beta g^{\mu\alpha}{-}q^\alpha g^{\mu\beta})]\;D^\rho{}_\beta(k{+}q)D_{\rho\alpha}(k).$$

$$div\ q_\nu\ \Pi^{\nu\mu}(q) = -8\,q^- q^{+2}\,D^\mu{}_+(q)\,I^{div}.$$

- **Quark-loop gluon self-energy correction:**

$$\Pi^{\mu\nu}_{aa'} = -ig^2 \sum_{ij} t^a_{ij} t^{a'}_{ji} \int \frac{d^d k}{(2\pi)^d}\,\frac{Tr(\not k + m_f)\gamma^\mu(\not k + \not q + m_f)\gamma^\nu}{[k^2 - m_f{}^2]\,[(k+q)^2 - m_f{}^2]}$$

$$div\ \Pi^{\mu\nu}_{aa'} = \delta_{aa'}\,\frac{g^2}{16\pi^2}\,\frac{2}{3}\,n_f\,(q^2 g^{\mu\nu} - q^\mu q^\nu)\;(\frac{2}{\epsilon})$$

- $$D_{\lambda\mu}(q)\ \Pi^{\mu\nu}_{aa'}(q)\ D_{\nu\delta}(q)$$

The contributions from $\Pi^{-\nu}_{aa'}$ or $\Pi^{\mu-}_{aa'}$ are automatically suppressed in view of $D_{-\mu} = D_{\mu-} = 0$, as they should in the l.c. gauge.

$$D_{\lambda\mu}(q)\;(q^2 g^{\mu\nu} - q^\mu q^\nu)\;D_{\nu\delta}(q) = -q^2 D_{\lambda\delta}(q).$$

CONCLUSIONS:

- The canonical quantization of l.c. gauge QCD in the FRONT FORM theory has been discussed employing the Dirac procedure to construct a self-consistent LF Hamiltonian theory resulting from the gauge-fixed BRS invariant quantum action.

- In the context of perturbation theory it incorporates in it the Lorentz gauge condition as an operator equation as well.

- The interaction Hamiltonian, in which the ghosts decouple, is re-expressed in a form closely resembling the one in covariant theory, except for additional instantaneous interactions, which can be treated systematically.

- The propagator of the dynamical ψ_+ part of the free fermionic propagator in the *front form* theory is shown to be causal and not to contain instantaneous terms.

- Transverse gauge field propagator for the massless field is derived using the Dirac method and its properties discussed.

- It differs from the ones found in the literature in the context of equal-time l.c. gauge QCD.

- It is in substance closer to the rules given by Lepage and Brodsky in 1980 in the context of old fashioned perturbation theory on the LF.

- The dimensional regularization is very convenient to handle both the $1/k^2$ and the $1/k^+$ singularities which arise from the noncovariant piece of the gauge field propagator. For the latter the causal ML prescription seems to be naturally suggested if we are using LF components. The power counting rules in l.c. gauge become similar to those of the covariant gauge.

- Electron-muon scattering is considered to illustrate the relevance of the instantaneous terms in the interaction Hamiltonian. It also demonstrates that the apparent lack of rotational invariance in the noncovariant l.c. gauge or even Lorentz invariance is not a problem when we employ the Dyson-Wick expansion; the final result is Lorentz covaraint.

- The fact that in the *front form* theory the classical Thomson scattering limit is obtained from a seagull term at the tree level is significant since, it seems difficult to build on the LF a systematic procedure to obtain semiclassical approximation.

- We have made an *ad hoc* choice of only one (of the family) of the characteristic LF hyperplanes, $x^+ = const.$, in order to quantize the theory.

- The conclusions reached in the l.c. gauge here reconfirm the conjecture made earlier, based on the studies on SSB, Schwinger model, Chiral Schwinger model, LF QCD in covariant gauge, on the irrelevance, in the quantized theory, of the fact that the hyperplanes $x^\pm = 0$ constitute characteristic surfaces of hyperbolic partial differential equation.