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Symmetry Breaking/Restoration in a Non-simply Connected Space

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Interesting features of field theories in non-simply connected spaces:

- ★ *a mechanism of spontaneous SUSY breaking* → *Tachibana's talk*
- ★ *rich & nontrivial phase structure*
- ★ *critical radius*
- ★ *spontaneous breaking of translational invariance*

To demonstrate them, let us consider a simple model of

$$O(N)\ \phi^4 \text{ model in } M^3 \otimes S^1$$

- $O(N)\ \phi^4$ **model on** $M^3 \otimes S^1$

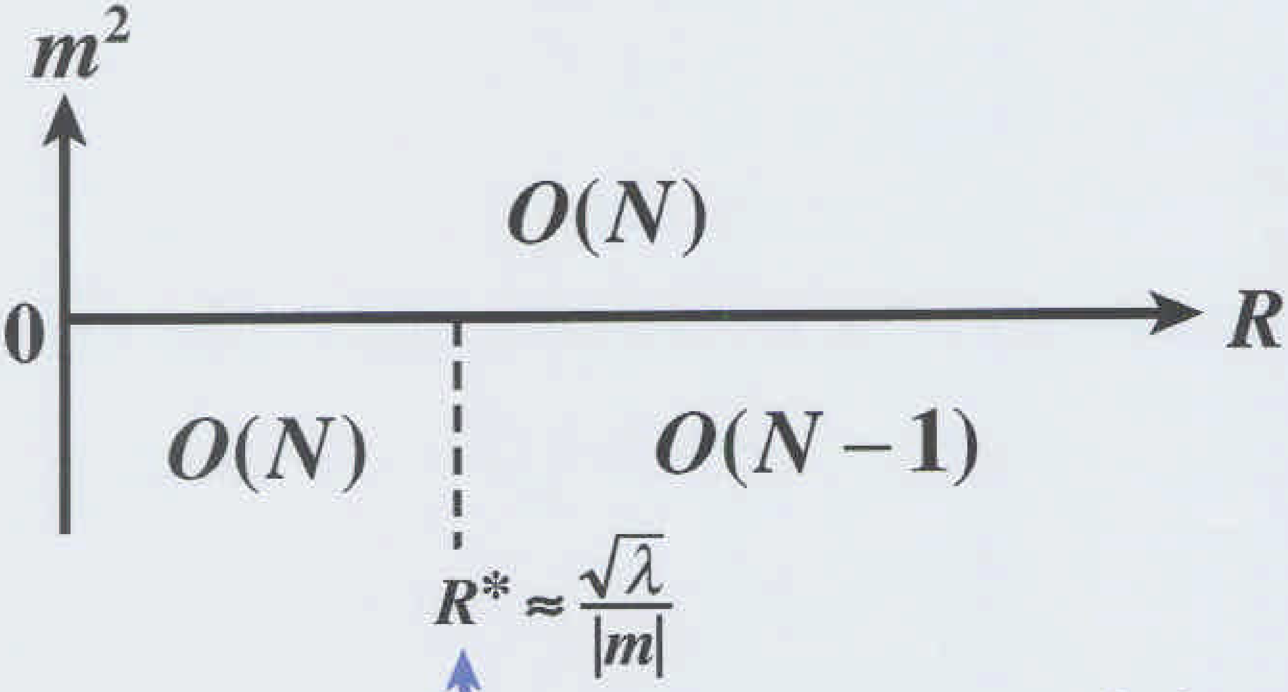
$$S = \int d^3x \int_0^{2\pi R} dy \left\{ \partial_\mu \phi_i \partial^\mu \phi_i - \frac{m^2}{2}\phi_i^2 - \frac{\lambda}{8}\left(\phi_i^2\right)^2 \right\}$$

- **twisted boundary condition**

$$\phi_i(y + 2\pi R) = V_{ij}\,\phi_j(y) \qquad V \in O(N)$$

$V = \mathbb{1}$ **(periodic B.C.)**

phase structure



quantum origin

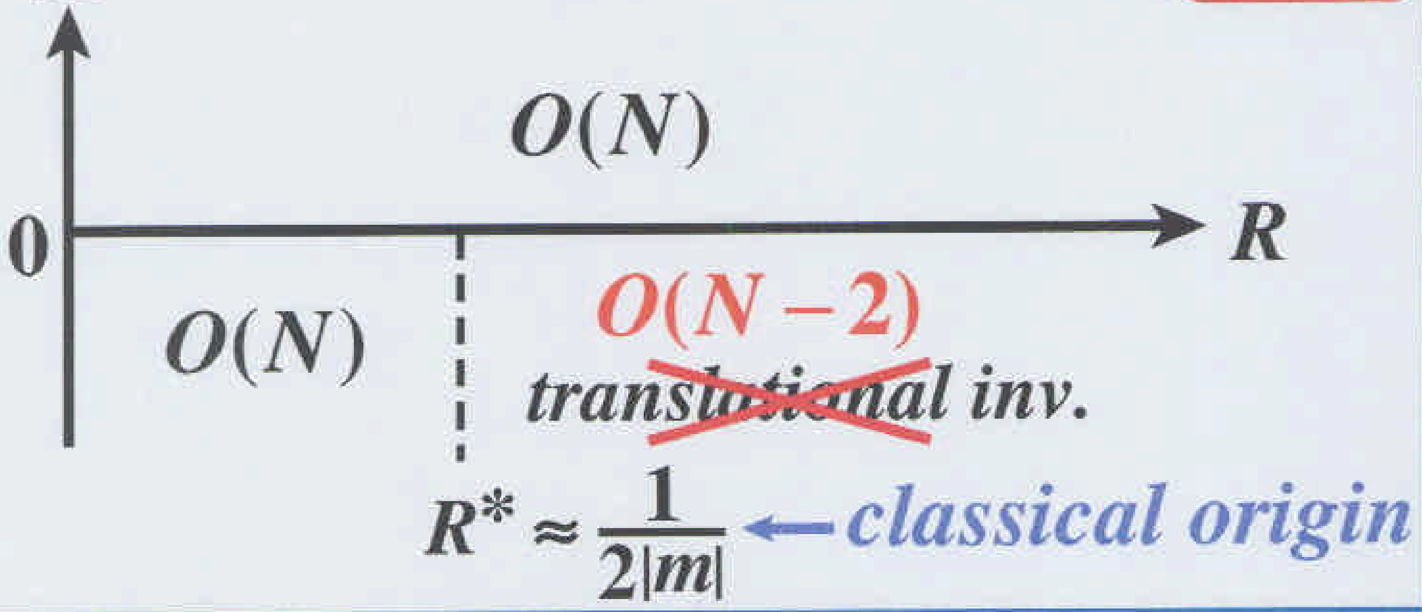
(just like symmetry restoration at high T)

Nothing New!

$V = -\mathbb{1}$ (anti-periodic B.C.)

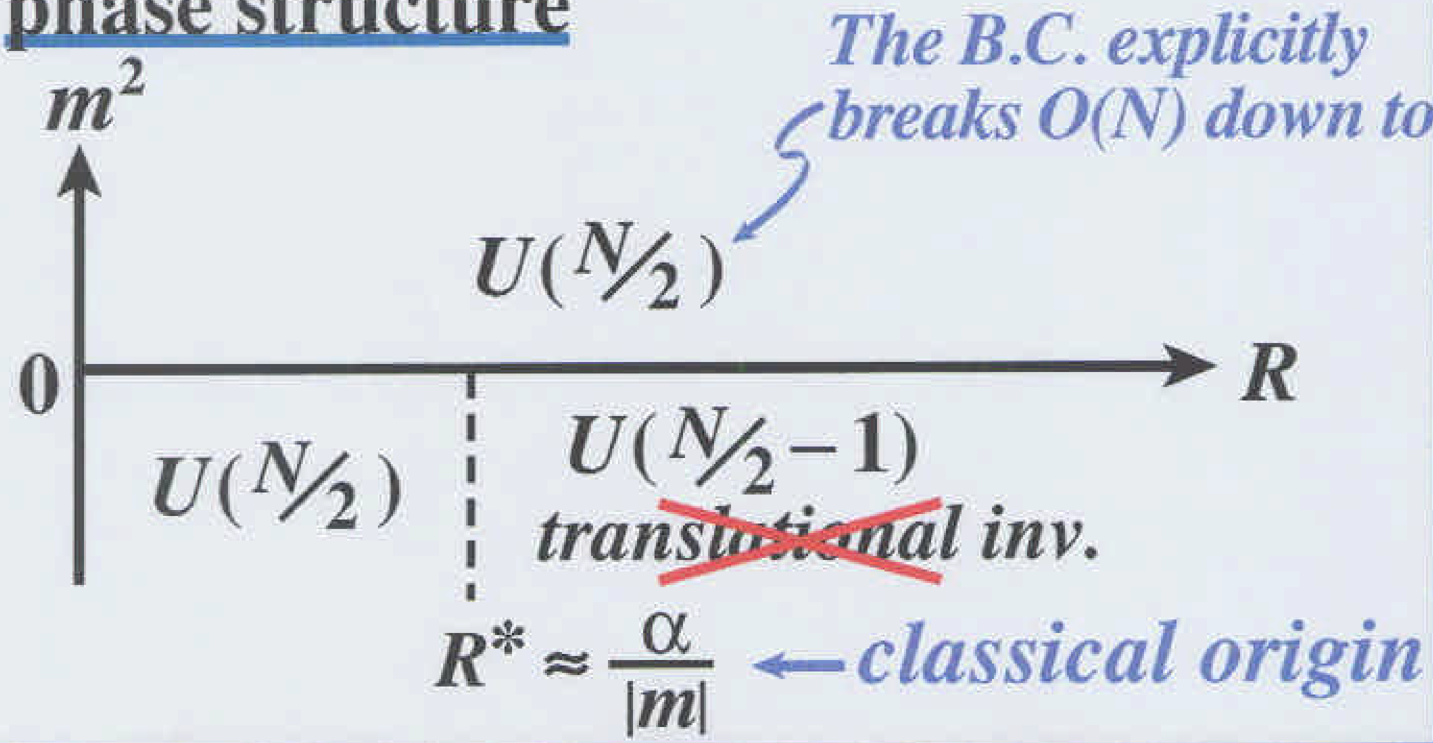
phase structure

→ 5 6



$$V = \begin{pmatrix} r(\alpha) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & r(\alpha) \end{pmatrix} \qquad r(\alpha) \equiv \begin{pmatrix} \cos(2\pi\alpha) & -\sin(2\pi\alpha) \\ \sin(2\pi\alpha) & \cos(2\pi\alpha) \end{pmatrix}$$

phase structure



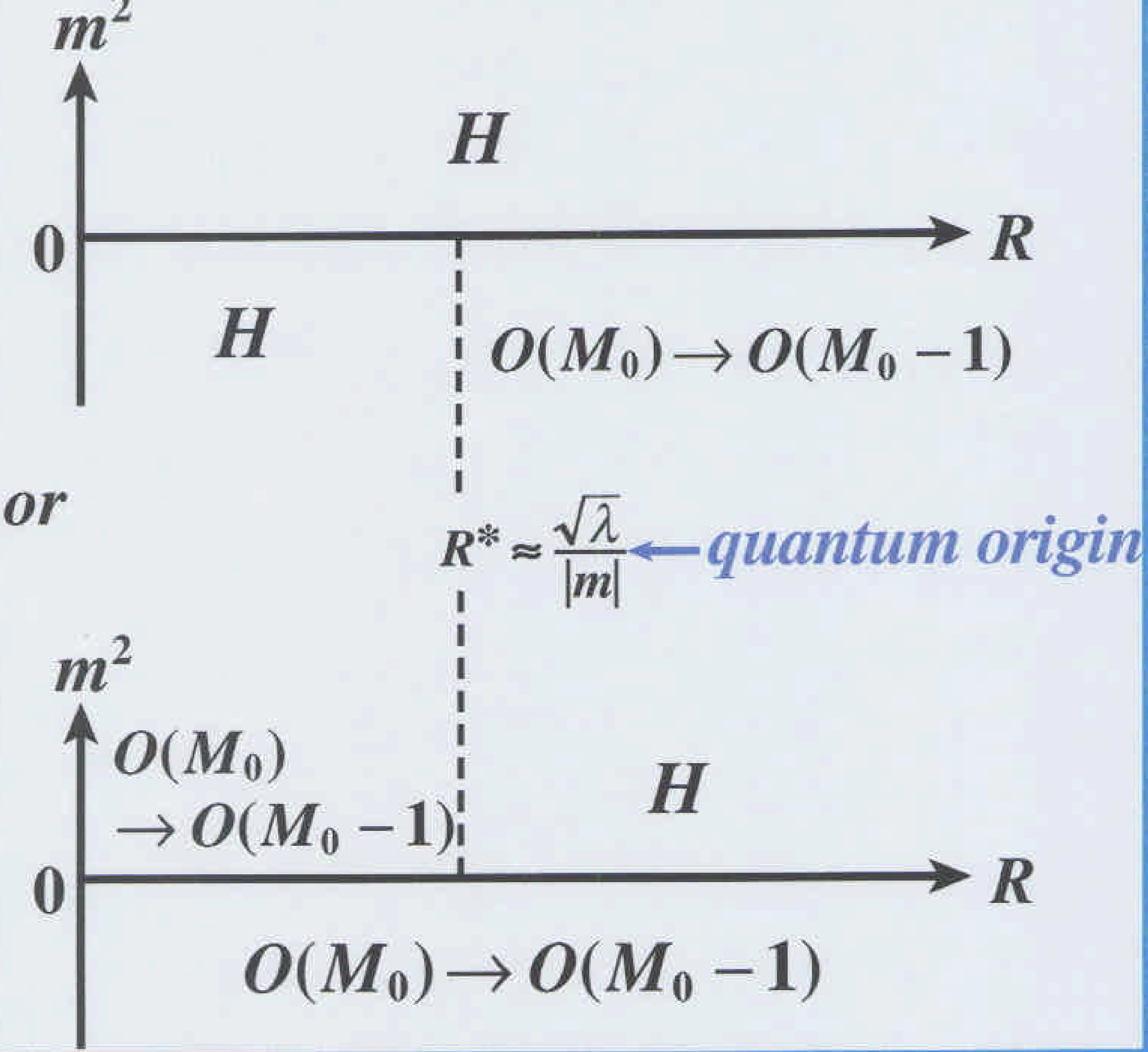
genaral twisted B.C. $V \in O(N)$

The B.C. generally breaks $O(N)$ to

$$H \equiv O(M_0) \times U(M_\alpha/2) \times \cdots \times U(M_{\alpha'}/2) \times O(M_{1/2})$$

with $M_0 + M_\alpha + \cdots + M_{\alpha'} + M_{1/2} = N$.

There are two types of phase structures which depend on M_α and α.



Why does sym. restoration occur for $R \leq R^*$? Why is translational inv. broken for $R > R^*$?

- $\langle \phi_i(y+2\pi R)\rangle = -\langle \phi_i(y)\rangle$

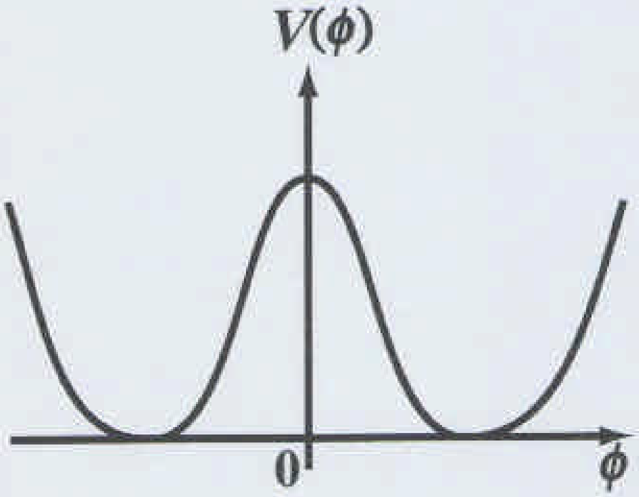
↓

If $\langle \phi_i(y)\rangle \neq 0$,then = y-dependent!

- **y-dep. configuration of $\langle \phi_i(y)\rangle$**

↓

kinetic energy $\propto \frac{1}{R^2}$



- **For $R \to \infty$,**

  $\langle \phi_i(y)\rangle \neq 0$ is preferable because the contribution from K.E. is small.

↓

$O(N)$ symmetry, translational inv.) are spontaneously broken

- **For $R \to 0$,**

  $\langle \phi_i(y)\rangle = 0$ is preferable because the contribution from K.E. is large.

Why is $O(N)$ symmetry broken to $O(N-2)$?

▸Vacuum configuration(at tree level)

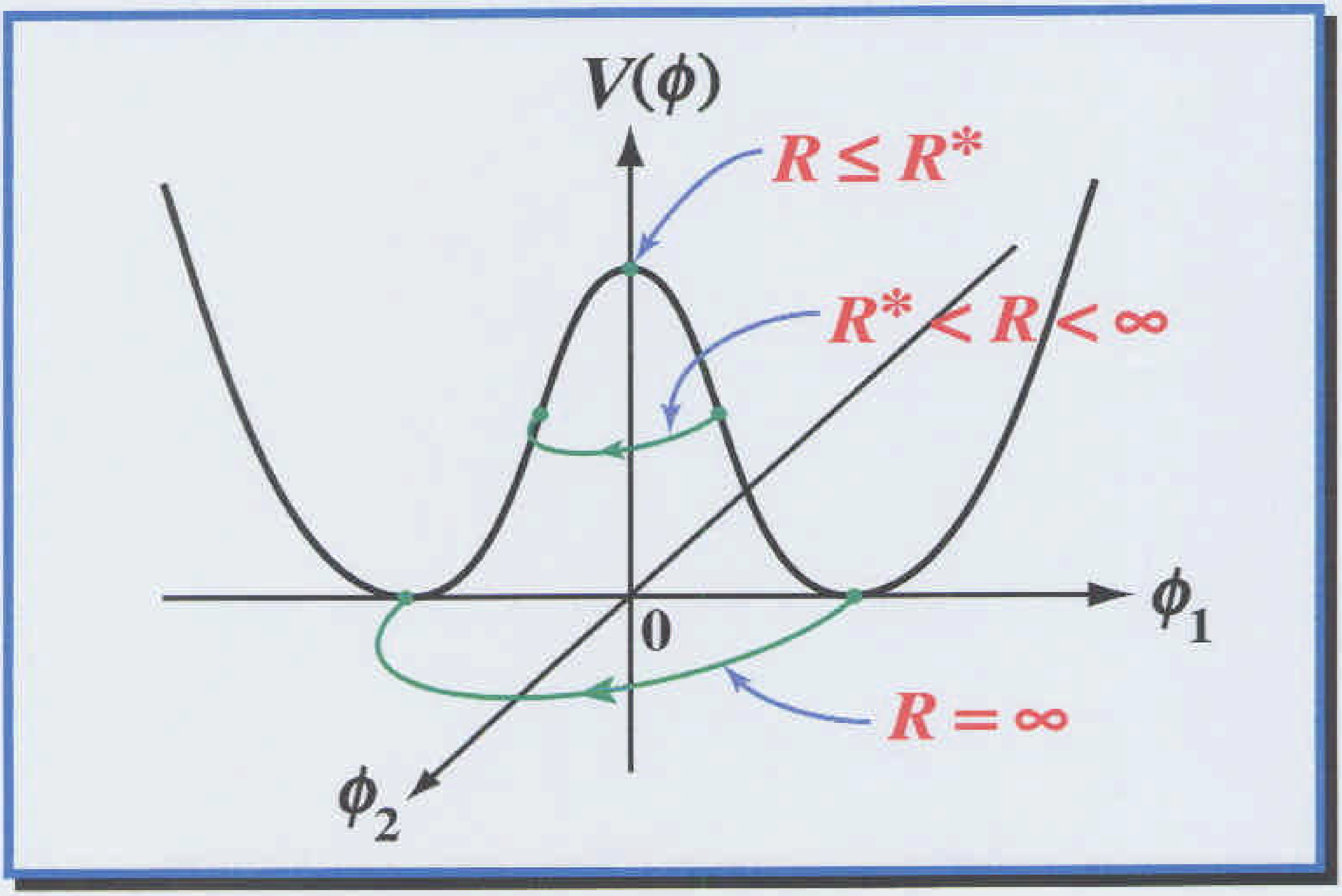
For $R > R^* = \frac{1}{2|m|}$,

$$\langle \phi_i(y) \rangle = \left(v\cos\left(\frac{y}{2R}\right), v\sin\left(\frac{y}{2R}\right), 0, \cdots, 0 \right)$$

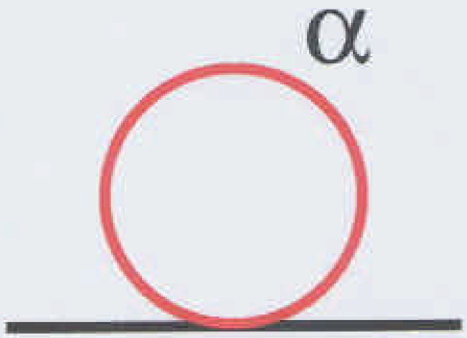
$$v \equiv \sqrt{\frac{2}{\lambda}\left(\mu^2 - \frac{1}{4R^2}\right)} \qquad (\mu^2 \equiv -m^2)$$

$$\Rightarrow O(N) \to O(N-2)$$

For $R \leq R^* = \frac{1}{2|m|}$, $\quad \langle \phi_i(y) \rangle = 0$.



one-loop mass correction



$$\overset{R \ll \frac{1}{m}}{\sim} \quad (1-6\alpha+6\alpha^2)\frac{\lambda}{R^2}$$

$$1-6\alpha+6\alpha^2 \begin{cases} \geq 0 & \text{for } 0 \leq \alpha \leq (3-\sqrt{3})/6 \\ < 0 & \text{for } (3-\sqrt{3})/6 < \alpha \leq 1/2 \end{cases}$$

negative!

▸ Fourier expansion

no zero mode!

$$\hat{\phi}(x,y)=\frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{\infty}\varphi^{(n+\alpha)}(x)\exp\left\{i\left(\frac{n+\alpha}{R}\right)y\right\}$$

▸ critical raidius R^*

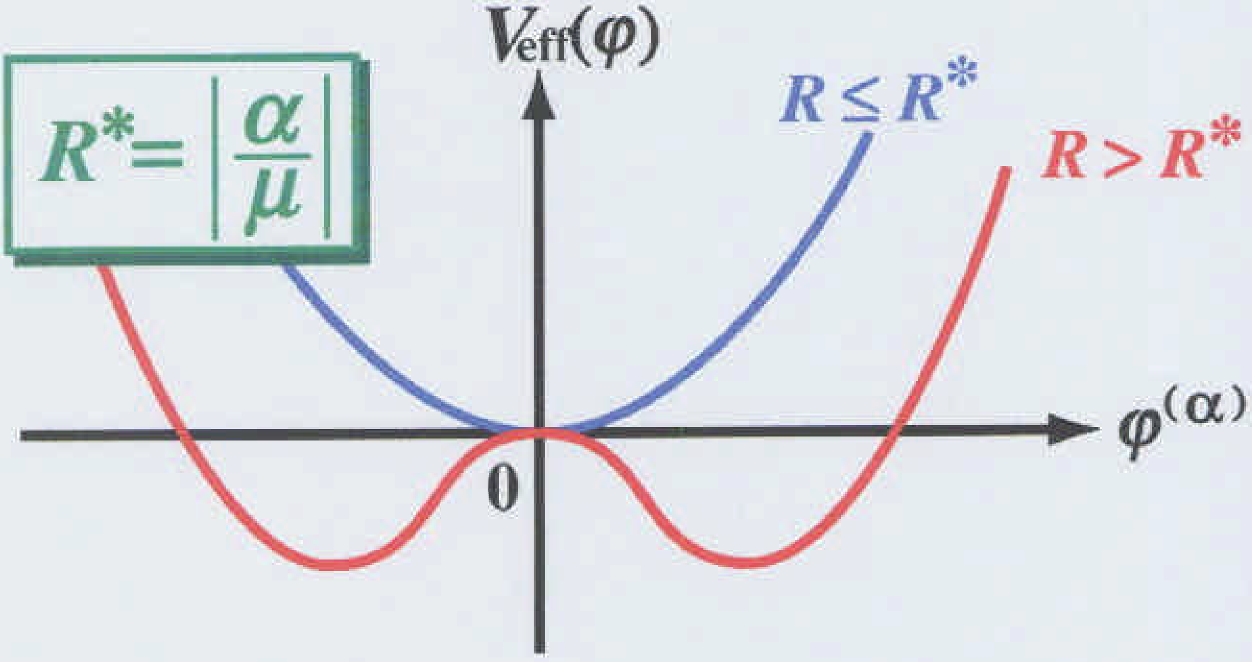
"effective" potential in D-dim.

$$V_{\rm eff}(\varphi)\equiv\int_0^{2\pi R}dy\left\{\,|\partial_y\hat{\phi}|^2+V(\hat{\phi})\,\right\}$$

$$=\sum_{n=-\infty}^{\infty}\left[\left(\frac{n+\alpha}{R}\right)^2-\mu^2\right]|\varphi^{(n+\alpha)}|^2+\cdots$$

$$=\left[\left(\frac{\alpha}{R}\right)^2-\mu^2\right]|\varphi^{(\alpha)}|^2+\cdots$$

$$R^*=\left|\frac{\alpha}{\mu}\right|$$



$$\Rightarrow\ \langle\hat{\phi}(y)\rangle=\begin{cases}0 & \text{for } R\le R^*\\ \text{y-dependent} & \text{for } R>R^*\end{cases}$$