ICHEP 2000
July 27

Abelian dominance in low-enery gluodynamics due to dynamical mass generation

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based on
K.-I. K. & T. Shinohara, hep-th/0004158

§ Introduction

I want to discuss a possible mechanism of gluon mass generation in gluodynamics and its implications to low-energy QCD, especially, quark confinement.

We start from the Yang-Mills theory

$$\mathcal{L}_{YM} = -\frac{1}{4} \mathcal{F}^A_{\mu\nu} \mathcal{F}^{\mu\nu A}$$

$A=1, \cdots, N_c^2-1$ for $G=SU(N_c)$

with the gauge-fixing (GF) and Faddeev-Popov (FP) ghost term

$$\mathcal{L}_{GF+FP} = -i\, \delta_{BRST}\left(\bar{\mathcal{C}}^A \left(F^A + \frac{\alpha}{2} B^A \right) \right)$$

for the gauge-fixing condition.

$$F^A[\mathcal{A}] = 0.$$

The typical gauge is the Lorentz gauge

$$F^A \equiv \partial^\mu \mathcal{A}^A_\mu$$

In this talk, we adopt the

maximal Abelian (MA) gauge

$$F^a \equiv D^{ab}_\mu[a] A^b_\mu = [\partial_\mu \delta^{ab} - g f^{abi} a^i_\mu] A^b_\mu$$

Partial gauge fixing $G \to H$ (G/H : fixed.)

$$\mathcal{A}_\mu \equiv \mathcal{A}_\mu^A T^A = a_\mu^i T^i + A_\mu^a T^a$$

a_μ^i: diagonal, A_μ^a: off-diagonal

$$G = SU(2) \Rightarrow a_\mu^3 \qquad A_\mu^1, A_\mu^2$$

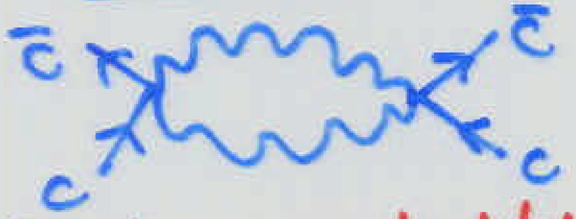
The GF+FP term for the naive MA gauge is

$$\mathcal{L}_{GF+FP} = -i\delta\left[\bar{C}^a\left(D_\mu[a]A^\mu + \frac{\alpha}{2}B\right)^a\right]$$

$$(G=SU(2)) = B^a\left(D_\mu[a]A^\mu\right)^a + \frac{\alpha}{2}B^aB^a$$
$$+ i\bar{C}^a D_\mu^{ab}[a] D^{\mu bc}[a] C^c$$
$$- ig^2 \varepsilon^{ab}\varepsilon^{cd}\bar{C}^a C^b A^{\mu c} A_\mu^d \Rightarrow$$

This choice spoils the renormalizability.

$\because$) Radiative corrections induce the 4-ghost interaction.



MA gauge
= non linear gauge

For the renormalizability of YM theory in MA gauge, we need 4-ghost interaction from the beginning.

The renormalizability of YM theory in MA gauge with 4-ghost interaction was proved to all orders in perturbation theory by

- Zinn-Justin
- Min, Lee & Pac

We have proposed to use
a modified MA gauge.
K.-I. K., PRD58, 105019 (1998)

$$\mathcal{L}_{\text{GF+FP}} = i\,\delta\,\bar{\delta}\left[\frac{1}{2}A_\mu^a A^{\mu a} - \frac{\alpha}{2} i\, C^a \bar{C}^a\right]$$

with characteristic features

- BRST invariant $\quad \delta^2 = 0$
- anti-BRST invariant $\quad \bar{\delta}^2 = 0$
- supersymmetric
  OSp(4|2) invariance for the super multiplet $(\mathcal{A}_\mu, C, \bar{C})$ on the super space $(x_\mu, \theta, \bar{\theta})$
  → dimensional reduction ($\alpha \neq 0$)
  4D $\mathcal{L}_{\text{GF+FP}}$ → 2D coset nonlinear sigma model
- $\underbrace{\text{FP}}_{\text{ghost}}$ conjugate invariant

$$\begin{aligned} C^A &\to \pm \bar{C}^A \\ \bar{C}^A &\to \mp C^A \\ B^A &\to -\bar{B}^A \\ \mathcal{A}_\mu^A &\to \mathcal{A}_\mu^A \end{aligned}$$

(C and $\bar{C}$ are treated on a equal footing)

- renormalizable ⇒ See explicit form

§ Modified MA gauge

$$S_{GF+FP} = \int d^4x\; i\,\delta\bar{\delta}\left[\frac{1}{2}A^a_\mu(x)A^{\mu a}(x) - \frac{\alpha}{2} i\, C^a(x)\bar{C}^a(x)\right]$$

off-diagonal only

$$= \boxed{-\int d^4x\; i\,\delta\left[\bar{C}^a\left\{D_\mu[a]A^\mu + \frac{\alpha}{2}B\right\}^a\right.} \leftarrow \text{naive MA gauge}$$

$$\left. - i\frac{\alpha}{2} f^{abi}\,\bar{C}^a\bar{C}^b C^i - i\frac{\alpha}{4} g f^{abc} C^a\bar{C}^b\bar{C}^c\right]$$

- $G = SU(2)$ $\quad f^{abc} = 0 \quad (a,b,c = 1,2.)$

$$S_{GF+FP} = \int d^4x \left\{ B^a(D_\mu[a]A^\mu)^a + \frac{\alpha}{2}B^aB^a \right.$$

$$+ i\,\bar{C}^a D^{ac}_\mu[a]D^{\mu cb}[a]C^b$$

$$\begin{pmatrix} \varepsilon^{ab} = \varepsilon^{ab3} \\ \text{structure constant of } SU(2) \end{pmatrix} \quad - i g^2 \varepsilon^{ad}\varepsilon^{cb}\,\bar{C}^a C^b A^{\mu c}A^d_\mu$$

$$+ \frac{\alpha}{4} g^2 \varepsilon^{ab}\varepsilon^{cd}\,\bar{C}^a\bar{C}^b C^c C^d$$

$$+ i\,\bar{C}^a g\,\varepsilon^{ab}(D_\mu[a]A_\mu)^b C^3$$

$$\left. - \alpha g\,\varepsilon^{ab}\, i\, B^a\bar{C}^b C^3 \right\}$$

Integrating out Nakanishi-Lautrup field B^a,

$$S'_{GF+FP} = \int d^4x \left\{ -\frac{1}{2\alpha}(D^{ab}_\mu[a]A^{\mu b})^2 \right.$$

$$+ i\,\bar{C}^a D_\mu[a]D^\mu[a]C^b$$

$$- i g^2 \varepsilon^{ad}\varepsilon^{cb}\,\bar{C}^a C^b A^{\mu c}A^d_\mu$$

a new term in the modified MA gauge $\rightarrow$ $$+ \frac{\alpha}{4} g^2 \varepsilon^{ab}\varepsilon^{cd}\,\bar{C}^a\bar{C}^b C^c C^d$$

$$+ \underbrace{(1-\alpha/\alpha)}_{\to 0}\, i\,\bar{C}^a g\,\varepsilon^{ab}(D_\mu[a]A^\mu)^b C^3$$

§ Coleman-Weinberg type derivation

- Introduce the auxiliary field φ

$$\frac{\xi}{4}g^2\varepsilon^{ab}\varepsilon^{cd}\bar{c}^a\bar{c}^b c^c c^d$$

$$\to \frac{-1}{2\xi g^2}\varphi^2 - \varphi\, i\varepsilon^{ab}\bar{c}^a c^b$$

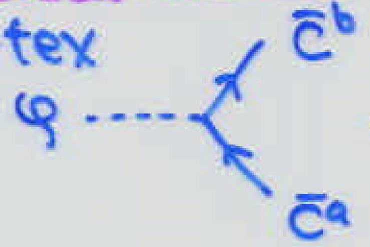
$$\left(\begin{aligned}\varepsilon^{ab}\varepsilon^{cd}\bar{c}^a\bar{c}^b c^c c^d &= 2(i\varepsilon^{ab}\bar{c}^a c^b)^2\\ &= 2(i\bar{c}^a c^a)^2\\ &= -4\bar{c}^1 c^1 \bar{c}^2 c^2\end{aligned}\right)$$

$$\mathcal{L}_{GF+FP} = i\bar{c}^a\partial_\mu\partial^\mu c^a - \varphi\, i\varepsilon^{ab}\bar{c}^a c^b - \frac{1}{2\xi g^2}\varphi^2 - ig^2\varepsilon^{ad}\varepsilon^{cb}\bar{c}^a c^b A^{\mu c}A_\mu^d + \cdots$$

A_μ, c, $\bar{c}$ are massless at tree level.

We want to see the effect of radiative corrections.

- vertex

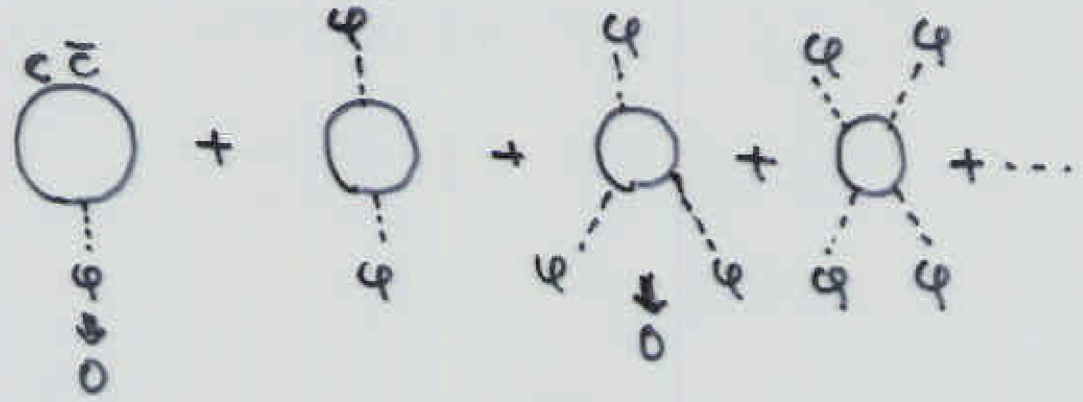


$-i\varepsilon^{ab}$.

- propagator

$\longrightarrow$ ghost propagator $\frac{1}{k^2}$

$\cdots\cdots$ φ propagator ξg^2



$$-\int \frac{d^D k}{i(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{-i\varphi}{-k^2}\right)^{2n}$$

$$= \int \frac{d^D k}{i(2\pi)^D} \ln\left[1+\frac{\varphi^2}{(-k^2)^2}\right]$$

$$= \int \frac{d^D k}{i(2\pi)^D} \left\{\ln(-k^2+i\varphi)+\ln(-k^2-i\varphi)-\ln[(-k^2)^2]\right\}$$

dimensional regularization

$$= -\frac{\Gamma(-D/2)}{(4\pi)^{D/2}}\left[(i\varphi)^{D/2}+(-i\varphi)^{D/2}\right] \qquad \varepsilon \equiv 2-\frac{D}{2}$$

$$= -\frac{\Gamma(\varepsilon-2)}{(4\pi)^2}(4\pi)^{\varepsilon}\left[(e^{i\frac{\pi}{4}}\varphi)^{2-\varepsilon}+(e^{-i\frac{\pi}{4}}\varphi)^{2-\varepsilon}\right]$$

minimal subtraction

$$= -\frac{1}{(4\pi)^2}\varphi^2\left(\ln 4\pi - \ln\frac{\varphi}{\hat{\mu}^2}\right) = \frac{1}{32\pi^2}\varphi^2 \ln\left(\frac{\varphi}{4\pi\hat{\mu}^2}\right)^2$$

$$\therefore \boxed{V(\varphi) = \frac{1}{2\xi g^2}\varphi^2 + \frac{1}{32\pi^2}\varphi^2\left[\ln\left(\frac{\varphi}{4\pi\mu^2}\right)^2 + C\right]}, \quad C = 2\gamma - 3$$

for MS

§ Dynamical mass generation

- In order to see whether the QCD vacuum favors a non-trivial φ ($\varphi \neq 0$) or not ($\varphi = 0$), we consider the effective potential

$$V(\varphi)\int d^4x = \int d^4x \frac{1}{2\xi}\varphi^2 + i \ln \det(\partial_\mu \gamma^\mu \delta^{ab} - g\varphi \varepsilon^{ab})$$

$$\varphi \equiv \varphi(x) = \text{const.}$$

$$\therefore V(\varphi) = \frac{1}{2\xi}\varphi^2 - \int \frac{d^4k}{i(2\pi)^4} \ln[(-k^2)^2 + g^2\varphi^2]$$

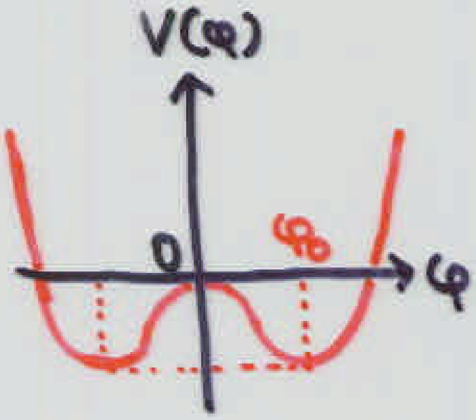
- The stationary point is given by the zero of the gap eq.

$$V'(\varphi) \equiv \varphi\left[\frac{1}{\xi} - 2g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(-k^2)^2 + g^2\varphi^2}\right] = 0$$

- The minimal subtraction (MS) scheme of dimensional regularization leads to

$$V(\varphi) = \frac{1}{2\xi}\varphi^2 + \frac{1}{32\pi^2} g^2\varphi^2 \left[\ln\left(\frac{g}{4\pi\mu}\varphi\right)^2 + 2\gamma - 3\right]$$

($\gamma = 0.577\ldots$ Euler's constant)



$$v \equiv g\varphi_0 = 4\pi\mu^2 e^{1-\gamma} \exp\left\{\frac{-8\pi^2}{\xi g^2(\mu)}\right\} > 0$$

The gap eq. has a non-trivial solution at $\varphi = \pm\varphi_0$. (besides a trivial one $\varphi = 0$)

$\varphi = \pm\varphi_0$ … global minima of $V(\varphi)$

$$V(\varphi_0) = -\frac{1}{32\pi^2} v^2 (< 0)$$

QCD vacuum prefers a ghost − anti-ghost condensation (as far as $\xi g^2 \neq 0$). $\varphi_0 \sim \xi g \langle i\epsilon^{ab}\bar{c}^a c^b\rangle$

- Off-diagonal ghost propagator

$(\partial_\mu \partial^\mu \delta^{ab} - g\varphi\epsilon^{ab})^{-1}$

$$\langle c^a(x)\bar{c}^b(y)\rangle = i\int \frac{d^4k}{i(2\pi)^4}\frac{-k^2\delta^{ab} - v\epsilon^{ab}}{(-k^2)^2+v^2}\, e^{ik\cdot(x-y)}$$

$$\Rightarrow \langle i\bar{c}^a(x)\, c^a(x)\rangle = \int \frac{d^4k}{i(2\pi)^4}\frac{-2k^2}{(-k^2)^2+v^2} = \frac{v}{16\pi} > 0$$

$v = g\varphi_0 \sim \xi g^2\langle i\epsilon^{ab}\bar{c}^a c^b\rangle$

$\langle i\epsilon^{ab}\bar{c}^a c^b\rangle \neq 0 \Leftrightarrow \langle \bar{c}^a c^a\rangle \neq 0$

In the condensed vacuum ($v \neq 0$)

$-ig^2\epsilon^{ad}\epsilon^{cb}\,\bar{c}^a c^b A^{\mu c}A^d_\mu$

$\to -ig^2\epsilon^{ad}\epsilon^{cb}\langle\bar{c}^a c^b\rangle A^{\mu c}A^d_\mu = \frac{1}{2}g^2\langle i\bar{c}^c c^c\rangle A^{\mu a}A^a_\mu$

⊙ The off-diagonal gluon acquires a mass

$$m_A^2 = g^2\langle i\bar{c}^a c^a\rangle = \frac{g^2 v}{16\pi} > 0$$

[The introduction of the explicit mass term

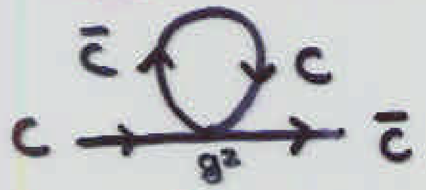
$\frac{1}{2}m^2 A^a_\mu A^{\mu a}$

spoils the renormalizability.]

⊙ The 4-ghost interaction can give a mass for the ghost.

$$\frac{\zeta}{4} g^2 \varepsilon^{ab} \varepsilon^{cd} \bar{c}^a \bar{c}^b c^c c^d$$
$$= \frac{\zeta}{2} g^2 (i \varepsilon^{ab} \bar{c}^a c^b)^2 = \frac{\zeta}{2} g^2 (i \bar{c}^a c^a)^2$$

$\to \zeta g^2 \langle i \bar{c}^a c^a \rangle i \bar{c}^b c^b$ (Hartree-Fock)



⊙ The off-diagonal ghost acquires a mass

$$m_c^2 = \zeta g^2 \langle i \bar{c}^a c^a \rangle = \zeta g^2 \frac{v}{16\pi}$$

The self-consistent investigation is desired. → e.g. Schwinger–Dyson equation

- $\bar{c}^a c^a$, $\varepsilon^{ab} \bar{c}^a c^b$ … residual U(1) invariant

Even in the presence of the ghost condensation, $\langle \bar{c}^a c^a \rangle \neq 0$, $\langle \varepsilon^{ab} \bar{c}^a c^b \rangle \neq 0$, the residual U(1) invariance is not broken spontaneously.

⇒ The diagonal gluon a_μ remains massless.

⊙ Massiveness of off-diagonal gluons and ghost gives an evidence of <u>Abelian dominance</u>. in low-energy gluodynamics (or QCD)

§ RG

$$V(\varphi)=\frac{1}{2\xi g^2}\varphi^2+\frac{1}{32\pi^2}\varphi^2\left[\ln\left(\frac{\varphi}{4\pi\mu^2}\right)^2+C\right]$$

$$V'(\varphi)=\varphi\left\{\frac{1}{\xi g^2}+\frac{1}{16\pi^2}\left[\ln\left(\frac{\varphi}{4\pi\mu^2}\right)^2+C+1\right]\right\}$$

$$V''(\varphi)=\frac{1}{\xi g^2}+\frac{1}{16\pi^2}\left[\ln\left(\frac{\varphi}{4\pi\mu^2}\right)^2+C+3\right]$$

- <u>renormalization condition</u>

$$\frac{d^2V(\varphi)}{d\varphi^2}\Bigg|_{\varphi=4\pi\mu^2 e^{-\frac{C+3}{2}}}=\frac{1}{(\xi g^2)(\mu)}$$

$$V(\varphi)_\mu-V(\varphi)_{\tilde\mu}=\left[\frac{1}{\xi g^2}-\widetilde{\left(\frac{1}{\xi g^2}\right)}+\frac{1}{8\pi^2}\ln\frac{\tilde\mu^2}{\mu^2}\right]\frac{\varphi^2}{2}$$

$$V(\varphi)_\mu=V(\varphi)_{\tilde\mu}\ \Rightarrow\ \frac{1}{\xi g^2}-\frac{1}{4\pi^2}\ln\mu=\widetilde{\left(\frac{1}{\xi g^2}\right)}-\frac{1}{4\pi^2}\ln\tilde\mu$$

= invariant

$$\therefore\ \xi g^2(\mu)=\frac{\xi g^2}{1+\xi g^2\frac{1}{4\pi^2}\ln\frac{\mu}{\mu_0}}$$ running coupling

$$\mu\frac{d}{d\mu}(\xi g^2)=-\frac{1}{4\pi^2}(\xi g^2)^2$$ RG eq. for ξg^2

From $\boxed{\mu\frac{dg^2}{d\mu}=-\frac{b_0}{8\pi^2}g^4}$ Asymptotic freedom.

$\Rightarrow\ \boxed{\mu\frac{d\xi}{d\mu}=\frac{-g^2}{4\pi^2}\xi\left(\xi-\frac{b_0}{2}\right)}$ RG eq. for ξ

$\xi=0,\ \xi=\frac{b_0}{2}$ fixed point

$$\xi=\frac{b_0}{2}=\frac{1}{2}\frac{11}{3}\cdot 2=\frac{11}{3}$$

$V(\varphi)$ is μ-independent, $\mu\frac{d}{d\mu}V(\varphi)=0$, i.e. RG eq.

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_\xi(g)\frac{\partial}{\partial\xi}-\gamma_\varphi(g)\,\varphi\frac{\partial}{\partial\varphi}\right]V(\varphi)=0$$

where
$$\beta(g)\equiv\mu\frac{\partial g}{\partial\mu}$$
$$\gamma_\xi(g)\equiv\mu\frac{\partial\xi}{\partial\mu}$$
$$\gamma_\varphi(g)\equiv\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_\varphi \quad , \varphi_R=Z_\varphi^{-1/2}\varphi$$

Substituting the expression of $V(\varphi)$ into the RG eq.,

$$\gamma_\varphi(g)=-\frac{b_0 g^2}{16\pi^2} \quad , \beta(g)=g\gamma_\varphi(g)=-\frac{b_0}{16\pi^2}g^3$$
$$\gamma_\xi(g)=-\frac{g^2}{4\pi^2}\xi\left(\xi-\frac{b_0}{2}\right)$$

◎ For $\xi=\frac{b_0}{2}$

$$v=4\pi e^{1-\gamma}\mu^2\exp\left\{-\frac{16\pi^2}{b_0 g^2(\mu)}\right\} \propto \langle i\varepsilon^{ab}\bar{c}^a c^b\rangle$$
$$=4\pi e^{1-\gamma}\Lambda_{QCD}^2 \qquad \text{RG invariant}$$

$$m_A=\sqrt{\frac{g^2 v}{16\pi}}=\frac{g}{2}e^{(1-\gamma)/2}\Lambda_{QCD}=(\pi\alpha_s)^{1/2}e^{(1-\gamma)/2}\Lambda_{QCD}$$
$$m_c=\sqrt{\frac{b_0}{2}\frac{g^2 v}{16\pi}}=\sqrt{\frac{11}{12}\alpha_s v}=\sqrt{\frac{11\pi}{3}\alpha_s}\,e^{(1-\gamma)/2}\Lambda_{QCD}$$

c.f. Lattice simulation

$m_A\simeq 1.2\,\text{GeV}$ (Amemiya & Suganuma)

§ Conclusion

1. We have proposed a <u>modified MA gauge</u>. to study the non-perturbative properties of Yang-Mills theory.
   This gauge includes <u>4-ghost interaction</u> which is needed for the <u>renormalizability</u>.

2. The attractive 4-ghost interaction leads to <u>ghost – anti-ghost condensation</u>

   $$\langle i\varepsilon^{ab} c^a \bar{c}^b \rangle \neq 0$$

   This provides the mass for the off-diagonal <u>gluons</u>

   $$m_A^2 = g^2 \langle i\bar{c}^a c^a \rangle = \frac{g^2}{16\pi} v$$

   the off-diagonal <u>ghosts</u>

   $$m_c^2 \cong \zeta g^2 \langle i\bar{c}^a c^a \rangle$$

   RG argument indicates

   $$\zeta = \frac{b_0}{2}$$

   is a fixed point. Then

   $$v \propto \mu^2 \exp\left\{-\frac{16\pi^2}{b_0 g^2(\mu)}\right\} = \Lambda_{QCD}^2$$

   $$\beta(g) = -\frac{b_0}{16\pi^2} g^3 + O(g^5) \qquad b_0 = \frac{11}{3} N_c$$

3. The dynamical mass generation for off-diagonal fields strongly supports the Abelian dominance in low-energy (or long-distance) QCD.
The mass provides the scale in QCD and is comparable to Λ_{QCD}.

4. At least for $G=SU(2)$, QCD in the modified MA gauge has a novel global symmetry $SL(2,R)$ and the mass generation can be considered as a spontaneous breaking of the (continuous) global symmetry $SL(2,R)$. (dynamical Higgs mechanism)
The associated NG boson can be confined in the unphysical state space with zero-norm and can not be observed.
($Q_B|0\rangle = 0 = Q_c|0\rangle$ is assumed)

5. For $G=SU(3)$, the ghost condensation scenario for mass generation of off-diagonal fields leads to two different masses for off-diagonal gluons, two off-diagonal gluons are heavier than the remaining four off-diagonal gluons, e.g.,
$$m_{A^1} = m_{A^2} = \sqrt{2}\, m_{A^4} = \sqrt{2}\, m_{A^5} = \sqrt{2}\, m_{A^6} = \sqrt{2}\, m_{A^7}$$
(up to Weyl symmetry)

6. We can write down the low-energy effective field theory which is written in terms of only the diagonal fields.
K.-I.K. PRD57, 7467 (1998)
K.-I.K & T. Shinohara, hep-th/0005125