# Thermal forward scattering amplitudes in temporal gauges

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## Some properties of the temporal gauge at finite temperature

- The response of the QCD plasma depends only on the gluon self-energy (the chromoelectric field is linear in the gauge potential)[a].
- Natural choice at finite temperature, since the Lorentz covariance is already broken by the heat bath.
- Effectively ghost-free.
- Calculations are complicated by the presence of extra poles at $q \cdot n = q_0 = 0$ (the tensor structure of the propagator is also much more involved than in the Feynman gauge). It is necessary to employ some *prescription*[b].

[a] K. Kajantie and J. Kapusta, Ann. Phys. **160**, 477 (1985)
[b] G. Leibbrandt and M. Staley, Nucl. Phys. **B428**, 469 (1994)

# The forward scattering amplitudes in Thermal Field Theories

Some typical one-loop thermal Green´s functions containing 1, 2 and 3 vertices, in the *imaginary time formalism*,


$$\longrightarrow \frac{s}{(2\pi)^{D-1}} \int d^{D-1}\vec{q}\, W_1,$$

where

$$W_1 = T \sum_{n=-\infty}^{\infty} \frac{1}{q_0^2 - \vec{q}^{\,2}}\, t(q); \; ^a \quad q_0 = i\,\omega_n^\sigma$$

$$\omega_n^\sigma = \pi T\,(2n+\sigma); \begin{cases} \sigma = 0 & (Bosons) \\ \sigma = 1 & (Fermions) \end{cases}$$

$t(q)$ results from the structures of vertices and propagators.

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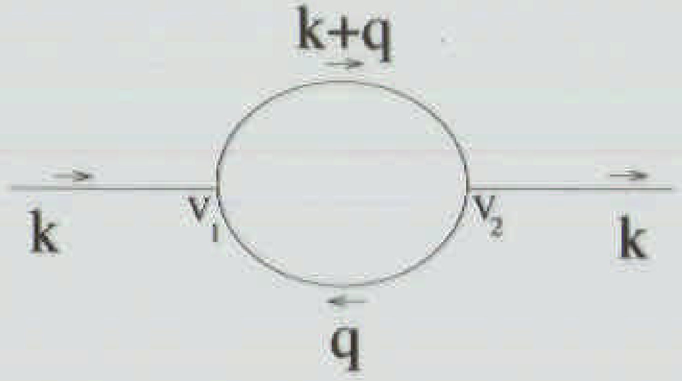
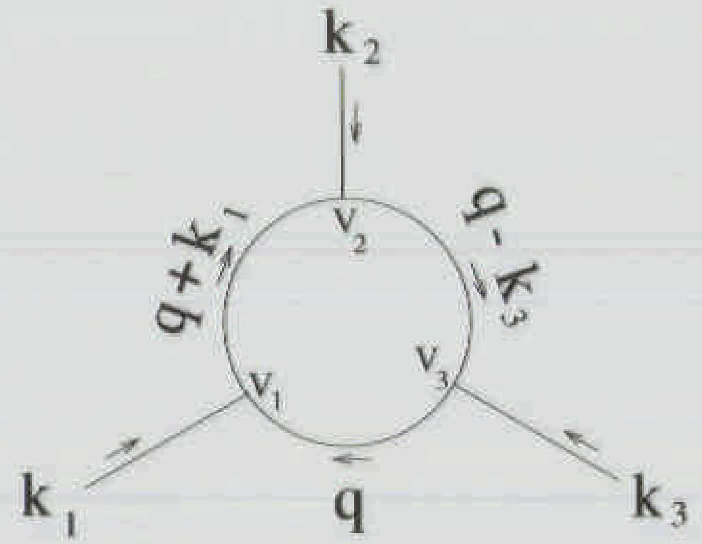
[a] $\sum_{q_0} \frac{1}{q_0^2 - q^2} = -\frac{1}{2qT}\left[\coth\left(\frac{q}{2T}\right)\right]^{\pm 1}$




$$\longrightarrow \frac{s}{(2\pi)^{D-1}} \int d^{D-1}\vec{q}\, W_2.$$

$$W_2 = T \sum_n \frac{1}{q_0^2 - \vec{q}^{\,2}} \frac{1}{(q_0 + k_0)^2 - \vec{p}^{\,2}}\, t(q,p),$$

where $p = q + k$.




$$\longrightarrow \frac{s}{(2\pi)^{D-1}} \int d^{D-1}\vec{q}\, W_3.$$

$$W_3 = T \sum_{q_0} \frac{1}{q_0^2 - \vec{q}^2} \frac{1}{(q_0 + k_{1_0})^2 - \vec{p}^2} \frac{1}{(q_0 - k_{3_0})^2 - \vec{r}^2} t(q, p, r),$$

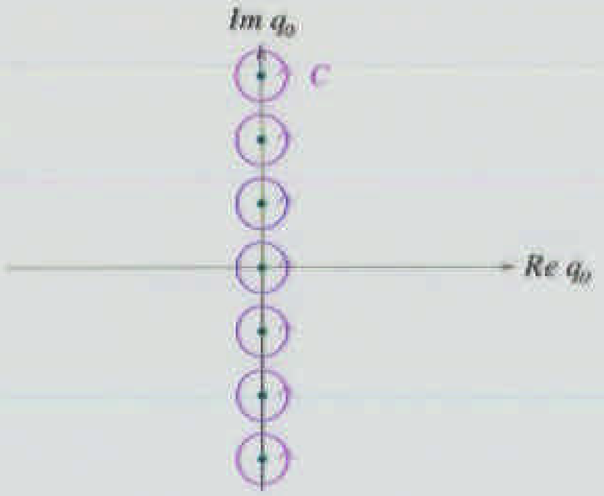
where $p = q + k_1$ and $r = q - k_3$.

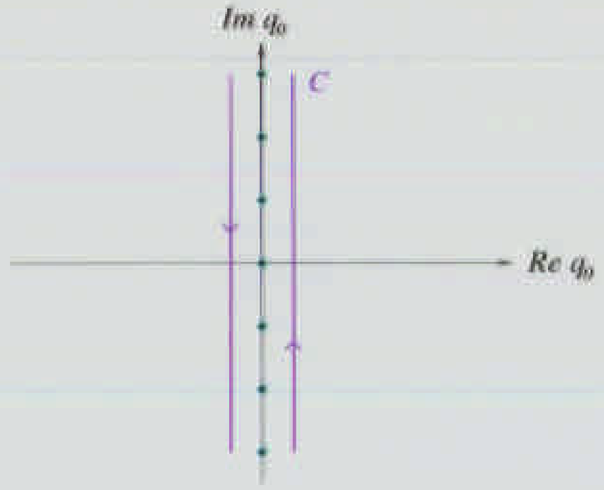
## Summing over the Matsubara frequencies

*If* the function $f(q_0)$ does not have poles on the imaginary axis

$$T \sum_{n=-\infty}^{\infty} f(q_0 = i\,\omega_n^\sigma) = \frac{1}{2\pi i} \oint_C dq_0 f(q_0) \frac{1}{2} \left\{ \begin{array}{c} \tanh\left(\frac{1}{2}\beta q_0\right) \\ \coth\left(\frac{1}{2}\beta q_0\right) \end{array} \right\},$$

where $\beta \equiv 1/T$.

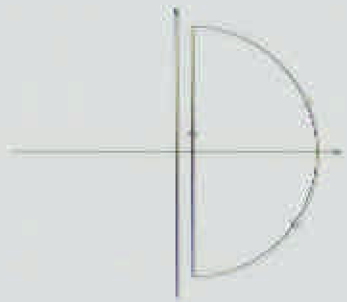


$$
\begin{aligned}
T \sum_{n=-\infty}^{\infty} f(q_0 = i\,\omega_n^\sigma) &= \frac{1}{2\pi i}\int_{i\infty-\delta}^{-i\infty-\delta} dq_0 f(q_0)\frac{1}{2}\left\{ \begin{array}{c} \coth\left(\frac{1}{2}\beta q_0\right) \\ \tanh\left(\frac{1}{2}\beta q_0\right) \end{array} \right\} \\
&+ \frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta} dq_0 f(q_0)\frac{1}{2}\left\{ \begin{array}{c} \coth\left(\frac{1}{2}\beta q_0\right) \\ \tanh\left(\frac{1}{2}\beta q_0\right) \end{array} \right\} \\
&= \frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta} dq_0 [f(q_0) + f(-q_0)] N_{b\,f}(q_0) + \\
&\pm \frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta} dq_0 \frac{1}{2}[f(q_0) + f(-q_0)], \qquad (1)
\end{aligned}
$$

where $N_{b\,f}(q_0) = \dfrac{1}{e^{q_0/T} \mp 1}$ are respectively the *Bose-Einstein* or *Fermi-Dirac* distributions.

Closing the contour in the righthand side complex plane

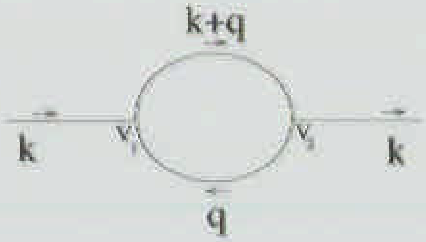


performing appropriated shifts and using the property $N(q_0 \pm k_0) = N(q_0 \pm i\,2\pi\, n) = N(q_0)$, we obtain the following representations in terms of *forward scattering amplitudes of on-shell thermal particles*

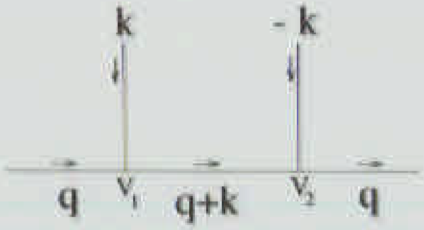

$$\rightarrow -\frac{s}{(2\pi)^{D-1}} \int \frac{d^{D-1}\vec{q}}{2|\vec{q}|} N(|\vec{q}|) \left\{ \quad + \quad \right\}_{q^2=0}$$



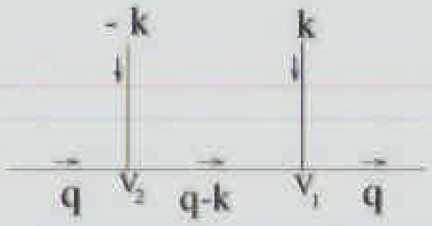
+(contributions from prescription poles)




$$= -\frac{s}{(2\pi)^{D-1}} \int \frac{d^{D-1}\vec{q}}{2|\vec{q}|} N(|\vec{q}|) \Bigg\{$$

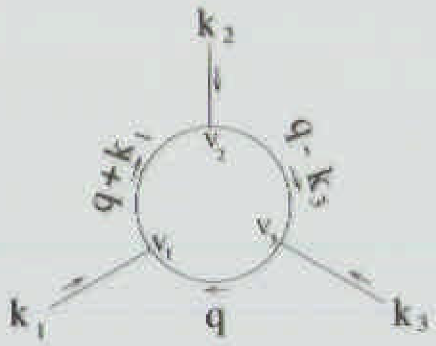



$$+$$




$$+ (q \leftrightarrow -q) \Bigg\}_{q^2=0}$$

+(contributions from prescription poles)




$$= -\frac{s}{(2\pi)^{D-1}} \int \frac{d^{D-1}\vec{q}}{2|\vec{q}|} N(|\vec{q}|) \Bigg\{$$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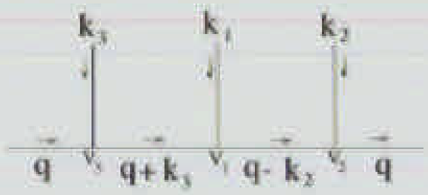
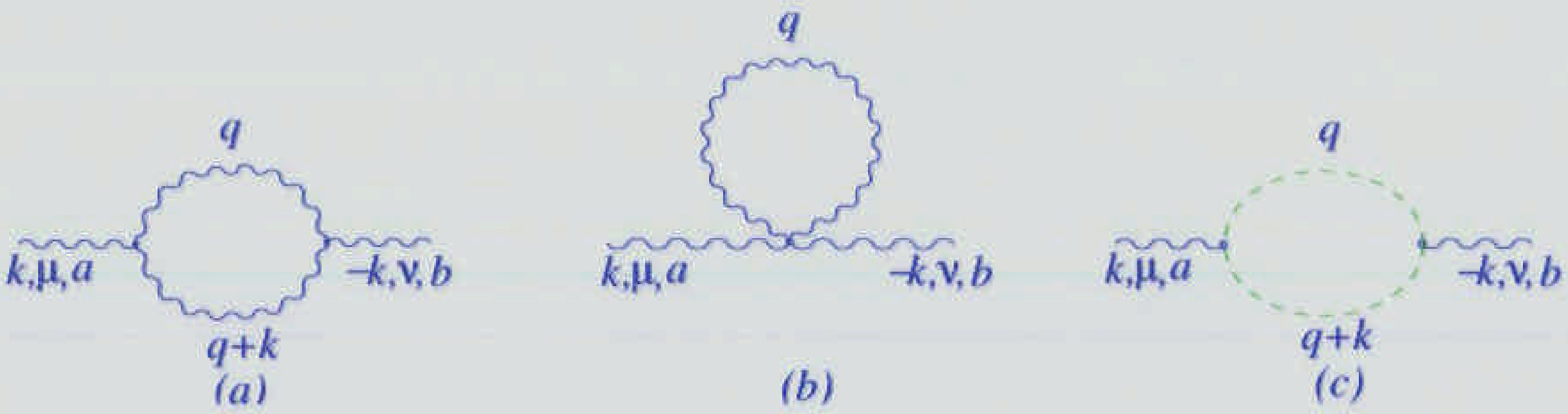


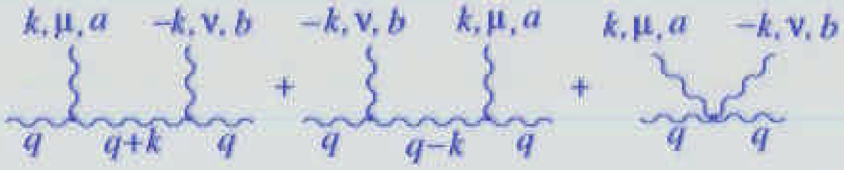

$$+ (q \leftrightarrow -q) \Bigg\}_{q^2=0}$$

+(contributions from prescription poles)

## The gluon self-energy




$$\Pi^{ab}_{\mu\nu}\Big|_{FS} = -\frac{1}{(2\pi)^3}\int \frac{d^3\vec{q}N(|\vec{q}|)}{2|\vec{q}|}\frac{1}{2}$$

$\times \Big\{$  $+\, q \leftrightarrow -q \Big\}_{q_0=|\vec{q}|}$ .

# Feynman rules in the temporal gauge

## Propagators

Gluon: $i\frac{\delta^{ab}}{k^2}\left[g_{\mu\nu} - \frac{1}{k\cdot n}(k_\mu n_\nu + k_\nu n_\mu) + \frac{k_\mu k_\nu}{(k\cdot n)^2}\left(\alpha k^2 + n^2\right)\right]$

Ghost: $i\frac{\delta^{ab}}{n\cdot q}$

## Vertices

Gluon-ghost: $gf^{abc}n_\mu$

Three gluons: $gf^{abc}[(q_\alpha - p_\alpha)g_{\mu\nu} + (p_\nu - k_\nu)g_{\mu\alpha} + (k_\mu - q_\mu)g_{\nu\alpha}]$

Four gluons:

$$-ig^2 f^{gab} f^{gcd}(g_{\alpha\nu}g_{\mu\beta} - g_{\alpha\beta}g_{\mu\nu}) + f^{gac}f^{gbd}(g_{\alpha\mu}g_{\nu\beta} - g_{\alpha\beta}g_{\mu\nu})$$
$$+ f^{gbc}f^{gad}(g_{\alpha\mu}g_{\nu\beta} - g_{\beta\mu}g_{\alpha\nu})$$

## Corrections to the FS amplitude result

The simplest correction to the FS amplitude comes from the *ghost loop diagram*

$$\int d^3\vec{q} \sum_{q0} \left[ \frac{t_{\mu\nu}}{n \cdot q\, n \cdot (q+k)} + q \leftrightarrow -q \right],$$

where $t_{\mu\nu}$ is a momentum independent quantity. Using partial fractions

$$\frac{1}{n \cdot q\, n \cdot (q+k)} = \frac{1}{n \cdot k} \left[ \frac{1}{n \cdot q} - \frac{1}{n \cdot (q+k)} \right]$$

and performing a shift $q \to q - k$ in the second term, we can easily see that the ghosts *effectively decouple*. This is a very simple illustration of a property which is shared with some other *tensor components* of the self-energy. The careful identification of such cancellations avoids the unnecessary computation of many residues of *prescription poles*.

## General structure of the gluon self-energy

$$\Pi^{ab}_{\mu\nu} = \delta^{ab}(\Pi_T P^T_{\mu\nu} + \Pi_L P^L_{\mu\nu} + \Pi_C P^C_{\mu\nu} + \Pi_D P^D_{\mu\nu}), \quad (2)$$

where

$$\begin{aligned} P^T_{\mu\nu} &= g_{\mu\nu} - P^L_{\mu\nu} - P^D_{\mu\nu}\,, \\ P^L_{\mu\nu} &= -\frac{\left(u\cdot k k_\mu - k^2 u_\mu\right)\left(u\cdot k k_\nu - k^2 u_\nu\right)}{k^2|\vec{k}|^2}\,, \\ P^C_{\mu\nu} &= \frac{2k\cdot u k_\mu k_\nu - k^2(k_\mu u_\nu + k_\nu u_\mu)}{k^2|\vec{k}|}\,, \\ P^D_{\mu\nu} &= \frac{k_\mu k_\nu}{k^2}\,, \end{aligned} \quad (3)$$

- $k^\mu P^{T,L}_{\mu\nu} = 0$; $k^i P^T_{i\nu} = 0$; $k^i P^L_{i\nu} \neq 0$ $(i = 1, 2, 3)$,
- $k^\mu\, P^{C,D}_{\mu\nu} \neq 0$,
- $\frac{1}{2} P^T_{\mu\nu} P^{\mu\nu\ T} = P^L_{\mu\nu} P^{\mu\nu\ L} = -\frac{1}{2} P^C_{\mu\nu} P^{\mu\nu\ C} = P^D_{\mu\nu} P^{\mu\nu\ D} = 1$; $P^T_{\mu\nu} P^{\mu\nu\ L} = P^T_{\mu\nu} P^{\mu\nu\ C} = P^T_{\mu\nu} P^{\mu\nu\ D} = P^L_{\mu\nu} P^{\mu\nu\ C} = P^L_{\mu\nu} P^{\mu\nu\ D} = P^C_{\mu\nu} P^{\mu\nu\ D} = 0.$

## Leading and sub-leading high temperature contributions

The results for the leading and sub-leading contributions, in a high-temperature expansion can be easily computed projecting the FS amplitude on the tensor basis.

$$\Pi_T^{htl}\Big|_{FS} = -g^2 N_C \left\{ \frac{T^2}{12|\vec{k}|^2} \left[ \frac{k^2 k_0}{|\vec{k}|} \ln\left( \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} \right) - 2k_0^2 \right] - \frac{k^2}{12\pi^2} \left( 11 - 4\frac{\alpha}{n_0^2} k^2 \right) \int \frac{d|\vec{q}|}{|\vec{q}|} N(|\vec{q}|) \right\}$$

$$\Pi_L^{htl}\Big|_{FS} = g^2 N_C \left\{ \frac{T^2}{6|\vec{k}|^2} \left[ \frac{k^2 k_0}{|\vec{k}|} \ln\left( \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} \right) - 2k^2 \right] + \frac{k^2}{12\pi^2} \left( 11 - 4\frac{\alpha}{n_0^2} k_0^2 \right) \int \frac{d|\vec{q}|}{|\vec{q}|} N(|\vec{q}|) \right\} .$$

There are no longitudinal components: $\Pi_C\big|_{FS} = \Pi_D\big|_{FS} = 0$

## Contributions from prescription poles

- Explicit calculation of the contribution from prescription poles shows that $\Pi_T$ and $\Pi_L$ are not modified in the leading and sub-leading orders.
- In the case of the longitudinal structures, $\Pi_C$ and $\Pi_D$, all the tersm containing denominators such as $q \cdot n$ or $(q+k) \cdot n$ are shown to cancel *before* using the contour integration formula. The remaining terms (which are expressible as FS amplitudes) vanish after performing shifts in the loop momentum.

## Conclusions

The *full tensor structure* of the gluon self-energy can be consistently computed in the temporal gauge and has the following properties:

- The leading $T^2$ is the known gauge invariant result obtained previously in the Feynman and general covariant gauges.
- The sub-leading contributions exhibits a general property which is shared with all covariant gauges, namely the identity between the ultraviolet poles which arises at $T = 0$ and the contributions proportional to $\ln(1/T)$.
- The one-loop calculation shows that the thermal self-energy *is transverse*. This result was also extended to higher orders, using BRS identities[a].

[a] F. T. Brandt, J. Frenkel and F. R. Machado, Phys. Rev. **D61**, 125014 (2000)