# Improved SUSY QCD corrections to Higgs boson decays into quarks and squarks

Youichi Yamada (Tohoku Univ.)

with

Helmut Eberl (HEPHY Wien)
Keisho Hidaka (Tokyo Gakugei Univ.)
Sabine Kraml (HEPHY Wien)
Walter Majerotto (HEPHY Wien)

hep-ph/9912463
to appear in PRD

# Contents

- Higgs boson decays into $b$ and $\tilde{b}$ in the MSSM with large $\tan\beta$, and large SUSY QCD corrections in the on-shell scheme expansion

- $\mathcal{O}(\alpha_s \tan\beta)$ gluino correction to $m_b$ and Higgs-bottom couplings

- Appropriate choices of Higgs-$b$ and Higgs-$\tilde{b}$ tree-level couplings

- Improved numerical results of $\mathcal{O}(\alpha_s)$ corrected decay widths

# Higgs boson decays in the MSSM with large $\tan\beta$

MSSM has two Higgs scalar doublets $(H_1, H_2)$.

VEVs: $\langle H_i \rangle = v_i/\sqrt{2}$.
$m_W^2 = g_2^2 \bar{v}^2/4 \equiv g_2^2(v_1^2 + v_2^2)/4$, $\tan\beta \equiv v_2/v_1$.

Mass eigenstates $(h^0, H^0, A^0, H^\pm)$.

Couplings to quarks

$$\mathcal{L}_{\text{int}} = -h_b \bar{b}_R q_L H_1 - h_t \bar{t}_R q_L H_2 + \text{(h.c.)}.$$

$$q_L = (t_L, b_L)$$

$$h_b = \frac{\sqrt{2} m_b}{\bar{v}\cos\beta} = \frac{h_b(\text{SM})}{\cos\beta}, \quad h_t = \frac{\sqrt{2} m_t}{\bar{v}\sin\beta} = \frac{h_t(\text{SM})}{\sin\beta}$$

Large $h_b$ for large $\tan\beta$

# Higgs decays in large $\tan\beta$ case

$(h^0, H^0, A^0) \to b\bar{b}$, $H^+ \to t\bar{b}$ are usually dominant.

large decay widths by enhanced Yukawa coupling $h_b$

$(h^0, H^0, A^0) \to \tilde{b}_i\tilde{b}_j^*$, $H^+ \to \tilde{t}_i\tilde{b}_j^*$ $(i, j = 1, 2)$ can be also dominant if kinematically allowed and left-right squark mixing are large.

Light $(\tilde{t}, \tilde{b})$ by RGE flow and $\tilde{q}_L - \tilde{q}_R$ mixing

$g_{H\tilde{b}\tilde{b}} \sim (h_b^2, h_b A_b, h_b \mu)$

$\tilde{b}_L$-$\tilde{b}_R$ mixing parameters

Radiative corrections to above decays: important in SUSY phenomenology

$\mathcal{O}(\alpha_s)$ SUSY QCD corrections
in the "on-shell scheme"

(lowest order Higgs-(s)quark couplings are given in terms of pole $M_q$, $M_{\tilde{q}_i}$, and on-shell $\theta_{\tilde{q}}$)

Often $|\Delta\Gamma^{(\mathrm{QCD})}_{\mathrm{OS}}| > \Gamma^0$
givivng unreliable numerical results

$\mathcal{O}(\alpha_s \ln \frac{m_\phi}{m_b})$ gluon corrections:
Absorved by using (non-SUSY) QCD running coupling at tree-level

$$g_{\phi bb}(Q) \propto m_b(Q)_{\mathrm{SM}}, \quad Q \sim m_\phi$$

However, gluino loop corrections can be also very large.

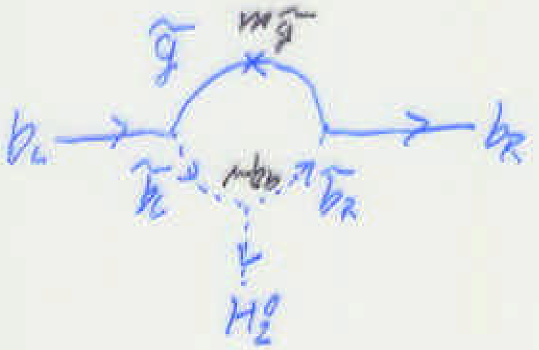
# gluino-squark loop corrections

$$\Delta\Gamma^{(\tilde{g})}((H^0, A^0) \to b\bar{b}, H^+ \to t\bar{b}) \sim \mathcal{O}\left(\frac{\alpha_s}{\pi}\tan\beta\right)$$

$\gg \frac{\alpha_s}{\pi}$ for large ($\tan\beta \gg 1$, $m_{\tilde{g}}$, $\mu$)
$|\Delta\Gamma^{(\tilde{g})}| > \Gamma^0$ possible.

Origin: effective $\bar{b}bH_2$ coupling

Babu-Kolda
Carena-Mrenna-Wagner
Chankowski-Pokorski



$$\to -h_b\Delta_b\bar{b}_R b_L H_2^{0*}$$

$\Delta_b = 0$ if SUSY is exact.

$$\Delta_b \sim \frac{\alpha_s}{\pi}\frac{m_{\tilde{g}}\mu}{m_{\tilde{b}}^2} \ll 1$$

(no danger in perturbation)

c.f. other loops

$\Delta_b \sim \frac{h_t^2 \mu A_t}{m_{\tilde{t}}^2}$

Effective Higgs-bottom couplings after squarks are integrated out.

$$\mathcal{L}_{\text{int}}^{\text{eff}} = -h_b \bar{b}_R (b_L H_1^0 - t_L H_1^-)$$

$$-h_b \Delta_b \bar{b}_R (b_L H_2^{0*} + t_L H_2^-) + (\text{h.c.}).$$

$$\begin{aligned} m_b(Q)_{\text{MSSM}} &= h_b \bar{v} \cos\beta / \sqrt{2}, \\ m_b(Q)_{\text{SM}} &= \frac{h_b \bar{v}}{\sqrt{2}} \cos\beta [1 + \Delta_b \tan\beta] \\ &= m_b(Q)_{\text{MSSM}} + \delta m_b^{(\tilde{g})} (\text{leading}) \end{aligned}$$

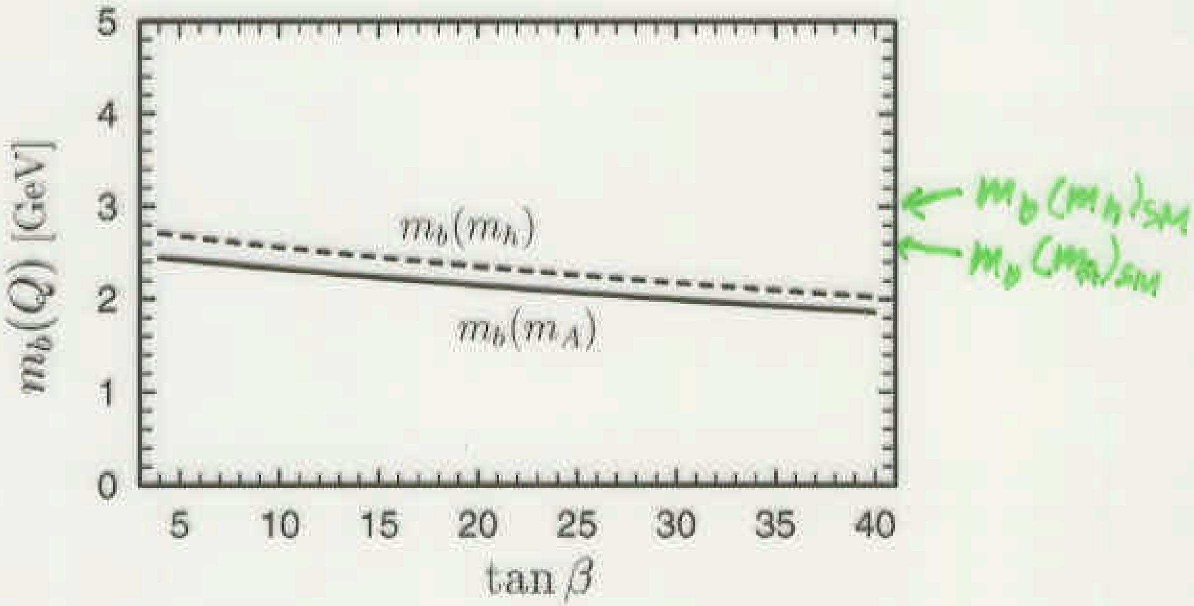
Large counterterm despite [redacted] $|\Delta_b| \ll 1$

Hempfling
Hall-Ratazzi-Sarid
Carena et al.
Pierce et al.

$\text{sgn}(\Delta_b) = \text{sgn}(\mu)$

SUSY threshold correction to $b$-$\tau$ Yukawa unification

# SUSY QCD running $m_b(Q)$




Parameters:

$M_b = 5$ GeV, $m_b(m_h, m_A)_{\text{SM}} \sim (3.0, 2.6)$ GeV,

$(M_{\tilde{Q}}(\tilde{t}), M_{\tilde{U}}, M_{\tilde{D}}) = (300, 270, 330)$ GeV,

$A_t = 150$ GeV, $A_b(Q) = -700$ GeV (where $Q = m_h$ or $m_A$),

$m_{\tilde{g}} = 350$ GeV, $\mu = 260$ GeV,

$m_A = 800$ GeV, $m_h = 93 - 103$ GeV.

# Higgs-bottom effective couplings:

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{int}} &= -\frac{h_b \bar{v}}{\sqrt{2}}[\cos\beta + \Delta_b \sin\beta]\,\bar{b}b \\
&\quad -\frac{h_b}{\sqrt{2}}[\cos\alpha + \Delta_b \sin\alpha]\,H^0\bar{b}b \\
&\quad +\frac{h_b}{\sqrt{2}}[\sin\alpha - \Delta_b \cos\alpha]\,h^0\bar{b}b \\
&\quad +\frac{ih_b}{\sqrt{2}}[\sin\beta - \Delta_b \cos\beta]\,A^0\bar{b}\gamma_5 b \\
&\quad +h_b[\sin\beta - \Delta_b \cos\beta]\,H^-\bar{b}_R t_L + (\text{h.c.}).
\end{aligned}
$$

Different contributions of $\Delta_b$ to $m_b$ and couplings: Origin of large $\mathcal{O}(\alpha_s \tan\beta)$ gluino corrections to $\Gamma$ renormalized by $M_b$ or $m_b(Q)_{\text{SM}}$.

Example: $A^0\bar{b}b$ coupling

$$
\begin{aligned}
g_{Abb} &= \frac{i\,h_b(Q\sim m_A)}{\sqrt{2}}[\sin\beta - \Delta_b \cos\beta] \\
&= \frac{i\,m_b(Q)_{\text{SM}}}{\bar{v}}\tan\beta\left[1 - \frac{1}{\sin\beta\cos\beta}\frac{\Delta_b}{1+\Delta_b\tan\beta}\right] \\
&= \frac{i\,m_b(Q)_{\text{MSSM}}}{\bar{v}}\tan\beta[1 - \Delta_b \cot\beta]
\end{aligned}
$$

$m_b(Q)_{\text{MSSM}}$ is appropriate for tree-level couplings of $A^0$, $H^0$, and $H^0$(with $m_A \gg m_Z$) to $b$ quark.
($m_b(Q)_{\text{SM}}$ is better for $h^0 b\bar{b}$ coupling.)

# Higgs-squark couplings

Depend on $m_q$, $\tilde{q}_L$-$\tilde{q}_R$ mixing angle $\theta_{\tilde{q}}$, trilinear soft SUSY breaking $A_q$, ....

Example:

$$G(A^0\tilde{b}_i^*\tilde{b}_j) = \frac{ig_2}{2m_W}\begin{pmatrix} 0 & m_b(A_b\tan\beta+\mu) \\ -m_b(A_b\tan\beta+\mu) & 0 \end{pmatrix}$$

Large SUSY corrections ($> \mathcal{O}(1)$) to $\Gamma(\phi \to \tilde{b}\tilde{b}^*,\ H^+ \to \tilde{t}\tilde{b}^*)$ appear in the on-shell scheme for squarks.

Origin: large counterterms for $(m_b,\ m_bA_b)$

Appropriate tree-level couplings are given in terms of $\overline{\text{DR}}$ running $(m_q(Q),\ A_q(Q))$ and on-shell $\theta_{\tilde{q}}$

* no enhanced vertex corrections
* $\delta\theta_{\tilde{q}}$ cancels large off-diagonal squark wave function corrections

# Procedure of numerical calculation of the corrected widths

## Input parameters:

on-shell: $M_t = 175$ GeV, $M_b = 5$ GeV,
$(M_{\tilde{Q}}(\tilde{t}), M_{\tilde{U}}, M_{\tilde{D}}) = (300, 270, 330)$ GeV,
$A_t = 150$ GeV,
running: $A_b(Q)(Q = m_\phi)$,
tree: $\tan\beta$, $m_{\tilde{g}} = 350$ GeV, $\mu$, $m_A = 800$ GeV

$\Longrightarrow$ obtain running parameters ($m_t(Q)$, $m_b(Q)$, $A_t(Q)$) and on-shell ($m_{\tilde{q}_i}$, $\theta_{\tilde{q}}$)

$$m_b(Q)_{\text{MSSM}} = m_b(Q)_{\text{SM}} + \delta m_b^{(\tilde{g})}$$

$\delta m_b^{(\tilde{g})}$ depends on the $\tilde{b}$ parameters.

$$\begin{aligned}\delta m_b^{(\tilde{g})} &= -\frac{\alpha_s}{3\pi}\Big[m_b(B_1(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}_1}^2) + B_1(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}_2}^2) + 1) \\ &\quad + m_{\tilde{g}} \sin 2\theta_{\tilde{b}}(B_0(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}_1}^2) - B_0(m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}_2}^2))\Big].\end{aligned}$$

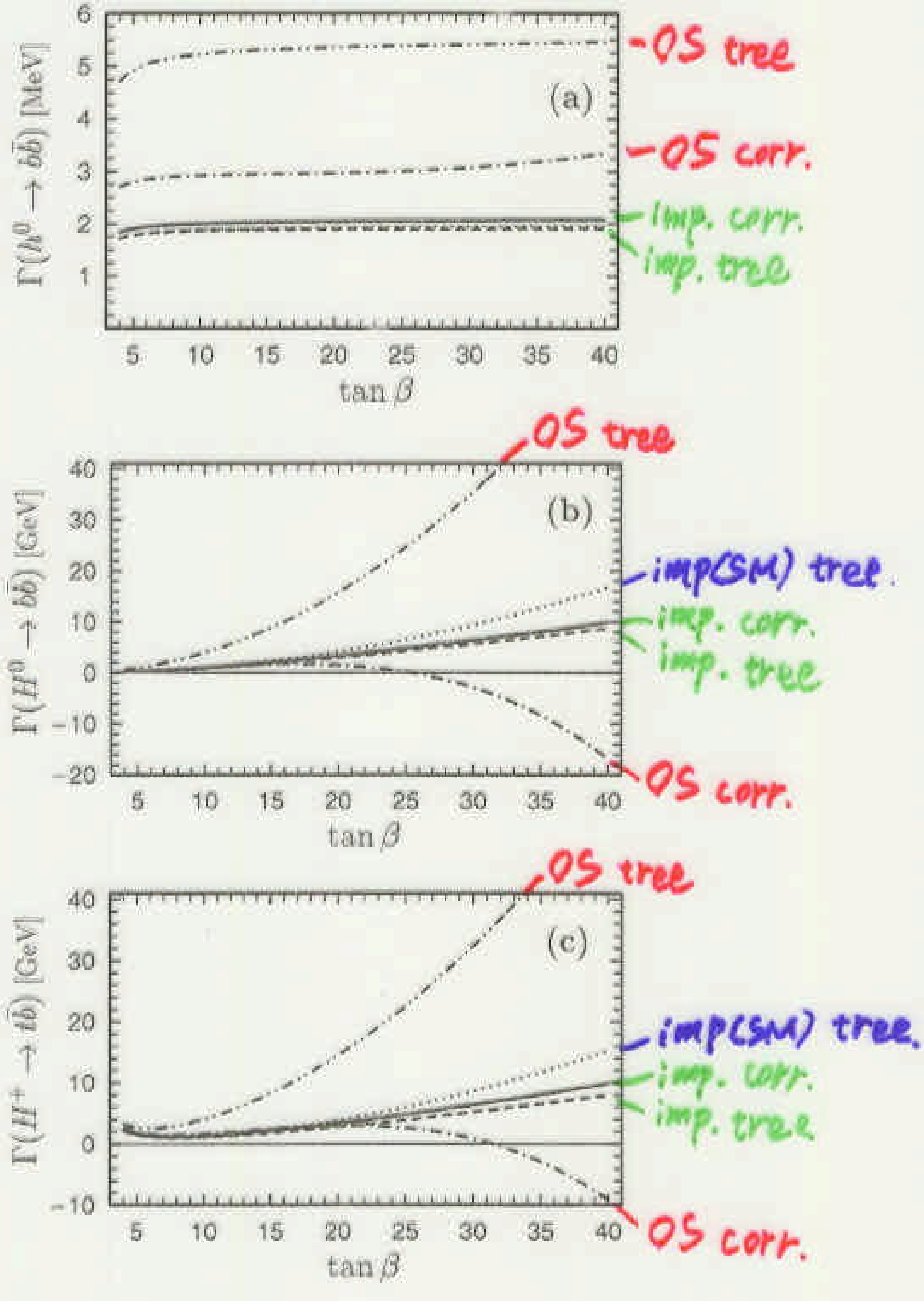
We calculate $\delta m_b^{(\tilde{g})}$ by using running $m_{\tilde{b}_i}$, $\theta_{\tilde{b}}$, and $m_b(Q)_{\text{MSSM}}$ at $Q \sim m_{\tilde{b}}$ (with iteration). All large higher-order gluino corrections are then resummed.

Rigorous proof in Carena et al., NPB 577 (2000) 88

# Decay widths into quarks



significantly improved perturbation expansion

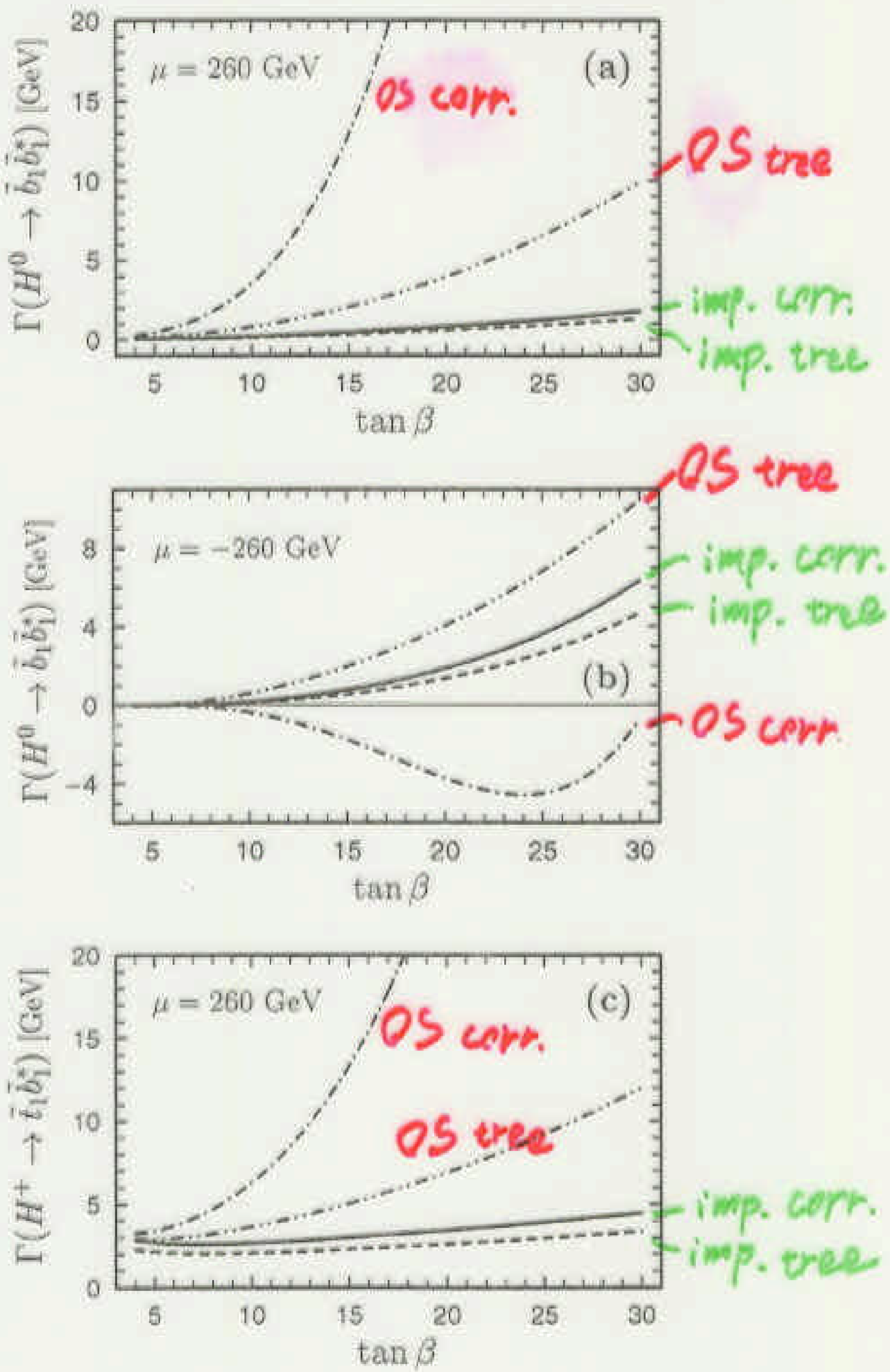



$A_b = -700$ GeV, $\mu = +260$ GeV

# Decay widths into squarks



significantly improved perturbation expansion




$A_b = -700$ GeV

# Summary

- The $\mathcal{O}(\alpha_s)$ SUSY QCD corrections to the decay widths of the MSSM Higgs bosons into $b$ and $\tilde{b}$ are very large for large $\tan\beta$, when the on-shell scheme for quarks and squarks is adopted, which makes the numerical results unreliable.

- We improved the above corrections by defining appropriate tree-level couplings of the Higgs bosons to quarks and squarks, in terms of running $m_q(Q)_{\rm SM}$, $m_q(Q)_{\rm MSSM}$, $A_q(Q)$, and on-shell $\theta_{\tilde{q}}$.

- Numerical convergence of the corrected decay widths was greatly improved.

- This method of improvement is also useful in studying other processes with Higgs-(s)quark couplings.