FLAVOR SYMMETRY
NATURALNESS OF BI-MAXIMAL MIXING in NEUTRINO OSCILLATIONS

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ICHEP2000

Osaka, Japan, 27 July 2000

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INTRODUCTION

The Greatest Success of Standard Model (MS) is the Gauge Symmetry Structure:

$$SU(3)_c \times SU_L(2) \times U_Y(1)$$

with Three Massless Neutrinos:

$$m_{\nu_{L\ e}} = 0, \quad m_{\nu_{L\ \mu}} = 0, \quad m_{\nu_{L\ \tau}} = 0\ .$$

Experimental Evidences:

- LEP Collaboration: Only 3 Light Neutrinos

$$\nu_{L\ e}, \quad \nu_{L\ \mu}, \quad \nu_{L\ \tau}$$

- S-K Collaboration: Neutrinos are Massive

$$m_{\nu_{L\ e}} \neq 0, \quad m_{\nu_{L\ \mu}} \neq 0, \quad m_{\nu_{L\ \tau}} \neq 0$$

Strong Indication:

NEW PHYSICS BEYOND SM

BASIC ASSUMPTIONS & THE MODEL

- **Only Three Light Neutrinos:** $\nu_{L\ e}$, $\nu_{L\ \mu}$, $\nu_{L\ \tau}$
- **Gauged SO(3) Flavor Symmetry:** $A^i_\mu(x)$, $\varphi^{(i)}$ – Barbieri, Hall, Kane, Ross et. al.

Gauge Symmetry of Model:

$$SO(3)_F \times SU(3)_c \times SU_L(2) \times U_Y(1)$$

– Caldwell & Mohapatra

The effective Yukawa couplings with $SO(3)_F \times SU(3)_c \times SU_L(2) \times U_Y(1)$ symmetry, – Barr & Babu

$$\frac{\varphi_i \varphi_j}{M^2} \bar{L}_i \phi_1 e_{R\ j} + \frac{\varphi_i \varphi_j^*}{M^2\ M_N} \bar{L}_i \phi_2 \phi_2^T L_j^c + h.c.$$

– Tanimoto et. al.

A realistic model:

$$\begin{aligned}
\mathcal{L} = \mathcal{L}_{SM} &+ \frac{1}{2} g_3' A^k_\mu (\bar{L}_i \gamma^\mu (t^k)_{ij} L_j + \bar{e}_{Ri} \gamma^\mu (t^k)_{ij} e_{Rj}) \\
&+ (c_1 \varphi_i \varphi_j \chi + c_1' \varphi_i' \varphi_j' \chi' + c_1'' \varphi_i'' \varphi_j'' \chi'') \bar{L}_i \phi_1 e_{R\ j} + H.c. \\
&+ (c_0 \varphi_i \varphi_j^* + c_0' \varphi_i' \varphi_j^{'*} + c_0'' \varphi_i'' \varphi_j^{''*} + c \delta_{ij}) \bar{L}_i \phi_2 \phi_2^T L_j^c + H.c. \\
&+ D_\mu \varphi^* D^\mu \varphi + D_\mu \varphi^{'*} D^\mu \varphi' + D_\mu \varphi^{''*} D^\mu \varphi'' - V_\varphi
\end{aligned}$$

After the symmetry is broken down to the $U(1)_{em}$ symmetry, the mass matrices of the neutrinos and charged leptons are given by

$$\begin{aligned}
(M_e)_{ij} &= m_1 \frac{\hat{\sigma}_i \hat{\sigma}_j}{\sigma^2} + m_1' \frac{\hat{\sigma}_i' \hat{\sigma}_j'}{\sigma'^2} + m_1'' \frac{\hat{\sigma}_i'' \hat{\sigma}_j''}{\sigma''^2} \\
(M_\nu)_{ij} &= m_0 \frac{\hat{\sigma}_i \hat{\sigma}_j^* + \hat{\sigma}_j \hat{\sigma}_i^*}{2\sigma^2} + m_0' \frac{\hat{\sigma}_i' \hat{\sigma}_j^{'*} + \hat{\sigma}_j' \hat{\sigma}_i^{'*}}{2\sigma'^2} \\
&+ m_0'' \frac{\hat{\sigma}_i'' \hat{\sigma}_j^{''*} + \hat{\sigma}_j'' \hat{\sigma}_i^{''*}}{2\sigma''^2} + m_\nu \delta_{ij}
\end{aligned}$$

The constants $\hat{\sigma}_i =< \varphi_i(x) >$, $\hat{\sigma}'_i =< \varphi'_i(x) >$ and $\hat{\sigma}''_i =< \varphi''_i(x) >$ are the vacuum expectation values (VEVs) of the three Higgs triplets with $\sigma^2 = \sum_{i=1}^3 |\hat{\sigma}_i|^2$, $\sigma'^2 = \sum_{i=1}^3 |\hat{\sigma}'_i|^2$ and $\sigma''^2 = \sum_{i=1}^3 |\hat{\sigma}''_i|^2$. The Higgs potential for the $SO(3)_F$ Higgs triplets is given by

$$
\begin{aligned}
V_\varphi =& \frac{1}{2}\mu^2(\varphi^\dagger\varphi) + \frac{1}{2}\mu'^2(\varphi'^\dagger\varphi') + \frac{1}{2}\mu''^2(\varphi''^\dagger\varphi'') \\
+& \frac{1}{4}\lambda(\varphi^\dagger\varphi)^2 + \frac{1}{4}\lambda'(\varphi'^\dagger\varphi')^2 + \frac{1}{4}\lambda''(\varphi''^\dagger\varphi'')^2 \\
+& \frac{1}{2}\kappa_1(\varphi^\dagger\varphi)(\varphi'^\dagger\varphi') + \frac{1}{2}\kappa'_1(\varphi^\dagger\varphi)(\varphi''^\dagger\varphi'') \\
+& \frac{1}{2}\kappa''_1(\varphi'^\dagger\varphi')(\varphi''^\dagger\varphi'') + \frac{1}{2}\kappa_2(\varphi^\dagger\varphi')(\varphi'^\dagger\varphi) \\
+& \frac{1}{2}\kappa'_2(\varphi^\dagger\varphi'')(\varphi''^\dagger\varphi) + \frac{1}{2}\kappa''_2(\varphi'^\dagger\varphi'')(\varphi''^\dagger\varphi') .
\end{aligned}
$$

In general, a complex Higgs triplet can be reparameterized in terms of the form

$$
\begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{pmatrix} = e^{i\eta_i(x)t^i}\frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1(x) \\ i\rho_2(x) \\ \rho_3(x) \end{pmatrix}
$$

After spontaneous symmetry breaking with $< \rho_i(x) >= \sigma_i$, $< \rho'_i(x) >= \sigma'_i$ and $< \rho''_i(x) >= \sigma''_i$, the following equations are obtained from minimizing the Higgs potential

$$
\begin{aligned}
\omega^2\sigma_i + \kappa_2\sum_{j=1}^3(\sigma_j\sigma'_j)\sigma'_i + \kappa'_2\sum_{j=1}^3(\sigma_j\sigma''_j)\sigma''_i = 0 \\
\omega'^2\sigma'_i + \kappa_2\sum_{j=1}^3(\sigma_j\sigma'_j)\sigma_i + \kappa''_2\sum_{j=1}^3(\sigma'_j\sigma''_j)\sigma''_i = 0 \\
\omega''^2\sigma''_i + \kappa'_2\sum_{j=1}^3(\sigma_j\sigma'_j)\sigma_i + \kappa''_2\sum_{j=1}^3(\sigma'_j\sigma''_j)\sigma'_i = 0
\end{aligned}
$$

with $\omega^2 = \mu^2 + \lambda\sigma^2 + \kappa_1\sigma'^2 + \kappa'_1\sigma''^2$, $\omega'^2 = \mu'^2 + \lambda'\sigma'^2 + \kappa_1\sigma^2 + \kappa''_1\sigma''^2$ and $\omega''^2 = \mu''^2 + \lambda''\sigma''^2 + \kappa'_1\sigma^2 + \kappa''_1\sigma'^2$.

Solving these equations, we obtain the following Solutions

$$\sigma_1'/\sigma_2' = \sigma_1/\sigma_2, \qquad \sigma_{12}'/\sigma_3' = -\sigma_3/\sigma_{12}$$
$$\sigma_1''/\sigma_2'' = -\sigma_2/\sigma_1, \quad \sigma_3'' = 0$$

Thus the mass matrices of the neutrinos and charged leptons can be reexpressed as

$$M_e = m_1 \begin{pmatrix} s_1^2 s_2^2 & i c_1 s_1 s_2^2 & s_1 c_2 s_2 \\ i c_1 s_1 s_2^2 & -c_1^2 s_2^2 & i c_1 c_2 s_2 \\ s_1 c_2 s_2 & i c_1 c_2 s_2 & c_2^2 s_2^2 \end{pmatrix}$$
$$+ m_1' \big((c_2 \leftrightarrow -s_2) \big) + m_1'' \big((c_1 \leftrightarrow -s_1 \ , \ c_2 = 0 \big)$$

and

$$M_\nu = m_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_0 \begin{pmatrix} s_1^2 s_2^2 & 0 & s_1 c_2 s_2 \\ 0 & c_1^2 s_2^2 & 0 \\ s_1 c_2 s_2 & 0 & c_2^2 s_2^2 \end{pmatrix}$$
$$+ m_0' \big((c_2 \leftrightarrow -s_2) \big) + m_0'' \big((c_1 \leftrightarrow -s_1 \ , \ c_2 = 0 \big)$$

with

$$s_1 = \sin\theta_1 = \sigma_1/\sigma_{12}, \quad s_2 = \sin\theta_2 = \sigma_{12}/\sigma$$

and

$$\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2}, \qquad \sigma = \sqrt{\sigma_{12}^2 + \sigma_3^2}$$

It is remarkable that the three mass matrices in M_e can be simultaneously diagonalized by a unitary matrix U_e via $D_e = \hat{U}_e^\dagger M_e \hat{U}_e^*$

$$U_e^\dagger = \begin{pmatrix} ic_1 & -s_1 & 0 \\ c_2 s_1 & -ic_1 c_2 & -s_2 \\ s_1 s_2 & -ic_1 s_2 & c_2 \end{pmatrix}$$

with

$$D_e = \begin{pmatrix} m_e = -m_1'' & 0 & 0 \\ 0 & m_\mu = m_1' & 0 \\ 0 & 0 & m_\tau = m_1 \end{pmatrix}$$

The neutrino mass matrix is diagonalized by an orthogonal matrix O_ν via $O_\nu^T M_\nu O_\nu$

$$O_\nu = \begin{pmatrix} c_\nu & 0 & s_\nu \\ 0 & 1 & 0 \\ -s_\nu & 0 & c_\nu \end{pmatrix}$$

with

$$\tan 2\theta_\nu = 2\delta_- \sin 2\theta_2 / (\Delta_+ - \Delta_- s_1^2)$$

Thus the CKM-type lepton mixing matrix U_{LEP} appearing in the interaction $\mathcal{L}_W = \bar{e}_L \gamma^\mu U_{LEP} \nu_L W_\mu^- + h.c.$ is given by $U_{LEP} = U_e^\dagger O_\nu$

$$U_{LEP} = \begin{pmatrix} ic_1 c_\nu & -s_1 & ic_1 s_\nu \\ c_2 s_1 c_\nu + s_2 s_\nu & -ic_1 c_2 & c_2 s_1 s_\nu - s_2 c_\nu \\ s_1 s_2 c_\nu - c_2 s_\nu & -ic_1 s_2 & s_1 s_2 s_\nu + c_2 c_\nu \end{pmatrix}$$

NEARLY BI-MAXIMAL MIXING

Notice that at the minimizing point, the Higgs potential is given by

$$V_\varphi|_{min} = -\sigma^4(\lambda + \lambda'\xi^2 + \lambda''\xi'^2 \\ + 2\kappa_1\xi + 2\kappa_1'\xi' + 2\kappa_1''\xi\xi')/4$$

with $\sigma'^2 = \xi\sigma^2$ and $\sigma''^2 = \xi'\sigma^2$. When requiring the model to have a

global minimum potential energy $V_\varphi|_{min}$, we arrive at the relations

$$\begin{aligned} &\sigma_1' = \sqrt{\xi}\sigma_1,\ \sigma_2' = \sqrt{\xi}\sigma_2,\ \sigma_3' = -\sqrt{\xi}\sigma_3, \\ &\sigma_1'' = \sqrt{2\xi'}\sigma_1,\ \ \sigma_2'' = -\sqrt{2\xi'}\sigma_2,\ \ \sigma_3'' = 0, \\ &\sigma_3^2 = \sigma_1^2 + \sigma_2^2 = 2\sigma_1^2 = \sigma^2/2, \\ &\ i.e.,\ \theta_1 = \theta_2 = \pi/4 \end{aligned}$$

In this case, the VEVs are completely determined by the Higgs potential! The mass matrices of the neutrinos and charged leptons are greatly simplified to the following nice forms

$$M_e = \frac{m_1}{2}\begin{pmatrix} \frac{1}{2} & \frac{1}{2}i & \frac{1}{\sqrt{2}} \\ \frac{1}{2}i & -\frac{1}{2} & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}i & 1 \end{pmatrix} + \frac{m_1'}{2}\begin{pmatrix} \frac{1}{2} & \frac{1}{2}i & -\frac{1}{\sqrt{2}} \\ \frac{1}{2}i & -\frac{1}{2} & -\frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}i & 1 \end{pmatrix} + \frac{m_1''}{2}\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$M_\nu = \hat{m}_\nu \begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}}\hat{\delta}_- \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}}\hat{\delta}_- & 0 & 1 + \hat{\Delta}_- \end{pmatrix}$$

with $\hat{m}_\nu = m_\nu(1 + \Delta_+)$, $\hat{\delta}_- = \delta_-/(1 + \Delta_+)$ **and** $\hat{\Delta}_- = \Delta_-/(1 + \Delta_+)$. **Here** $\Delta_\pm = (\delta_+ \pm \delta_0)/2$, $\delta_0 = m_0''/m_\nu$ **and** $\delta_\pm = (m_0 \pm m_0')/2m_\nu$.

It is of interest to note that the mass matrix M_e is now diagonalized by

a unitary bi-maximal mixing matrix U_e via $D_e = U_e^\dagger M_e U_e^*$ with

$$U_e^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}i & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2}i & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2}i & \frac{1}{\sqrt{2}} \end{pmatrix}$$

• Barger
Pakvasa
Weiler
Whisnant
⋮

The CKM-type lepton mixing matrix is simplified to be

$$U_{LEP} = \begin{pmatrix} \frac{1}{\sqrt{2}}ic_\nu & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}is_\nu \\ \frac{1}{2}c_\nu + \frac{1}{\sqrt{2}}s_\nu & -\frac{1}{2}i & \frac{1}{2}s_\nu - \frac{1}{\sqrt{2}}c_\nu \\ \frac{1}{2}c_\nu - \frac{1}{\sqrt{2}}s_\nu & -\frac{1}{2}i & \frac{1}{2}s_\nu + \frac{1}{\sqrt{2}}c_\nu \end{pmatrix}.$$

The three neutrino masses are found to be

$$\begin{aligned} m_{\nu_e} &= \hat{m}_\nu[1 - (\sqrt{\hat{\Delta}_-^2 + 2\hat{\delta}_-^2} - \hat{\Delta}_-)/2\ ] \\ m_{\nu_\mu} &= \hat{m}_\nu \\ m_{\nu_\tau} &= \hat{m}_\nu[1 + \hat{\Delta}_- + (\sqrt{\hat{\Delta}_-^2 + 2\hat{\delta}_-^2} - \hat{\Delta}_-)/2\ ]. \end{aligned}$$

The similarity between the Higgs triplets $\varphi(x)$ and $\varphi'(x)$ naturally motivates us to consider an approximate permutation symmetry between them, which implies that

$$m_0 \simeq m_0', |\hat{\delta}_-| << 1\ ; \quad \varphi(x) \leftrightarrow \varphi'(x)$$
$$2s_\nu \simeq \tan 2\theta_\nu = \sqrt{2}\hat{\delta}_-/\hat{\Delta}_- << 1$$

NEUTRINO OSCILLATIONS

To a good approximation, the mass-squared differences are given by

$$\Delta m^2_{\mu e} \equiv m^2_{\nu_\mu} - m^2_{\nu_e} \simeq \hat{m}^2_\nu \hat{\Delta}_- (\hat{\delta}_- / \hat{\Delta}_-)^2 \ ,$$
$$\Delta m^2_{\tau\mu} \equiv m^2_{\nu_\tau} - m^2_{\nu_\mu} \simeq \hat{m}^2_\nu \hat{\Delta}_- (2 + \hat{\Delta}_-) \ .$$

which leads to the following approximate relation

$$\frac{\Delta m^2_{\mu e}}{\Delta m^2_{\tau\mu}} \simeq \left(\frac{\hat{\delta}_-}{\sqrt{2}\hat{\Delta}_-} \right)^2 \simeq s^2_\nu = 2|U_{e3}|^2 << 1 \ .$$

The fitted range of the ratio is

$$\frac{\Delta m^2_{\mu e}}{\Delta m^2_{\tau\mu}} \simeq \begin{array}{ll} 10^{-1} \sim 10^{-2} & MSW - LMA \\ 10^{-4} \sim 10^{-5} & MSW - LOW \end{array}$$

with the fitted result for $\Delta m^2_{\tau\mu}$ from atmospheric neutrino oscillations

$$\Delta m^2_{\tau\mu} \simeq 3 \times 10^{-3} \ eV^2$$

which implies that

$$|U_{e3}| \simeq \begin{array}{ll} 0.2 \sim 0.09 & MSW - LMA \\ 0.02 \sim 0.002 & MSW - LOW \end{array}$$

and

$$\hat{m}_\nu \sqrt{2\hat{\Delta}_-} \simeq 0.055 \ eV$$

DOUBLE β DECAYS & HOT DARK MATTER

When going back to the weak gauge and charged-lepton mass basis, the neutrino mass matrix gets the following interesting form

$$\frac{M_\nu}{\hat{m}_\nu} \simeq \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ \frac{1}{\sqrt{2}}i & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}}i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \frac{\hat{\delta}_-}{2} \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}i & \frac{1}{\sqrt{2}}i \\ -\frac{1}{\sqrt{2}}i & -1 & 0 \\ \frac{1}{\sqrt{2}}i & 0 & 1 \end{pmatrix} + \frac{\hat{\Delta}_-}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}.$$

As

$$(M_\nu)_{ee} = 0$$

Thus the neutrinoless double beta decay is forbiden in the model and the neutrino masses can be approximately degenerate and large enough !

$$m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq \hat{m}_\nu = O(1)eV$$

The relation between the total neutrino mass $m(\nu)$ and the fraction Ω_ν of critical density that neutrinos contribute is

$$\Omega_\nu = 0.06 \left(\frac{m(\nu)}{2eV}\right)\left(\frac{0.6}{h}\right)^2 \simeq 0.18 \left(\frac{m_0}{2eV}\right)\left(\frac{0.6}{h}\right)^2$$

The almost degenerate neutrino masses allow to be large enough to play a significant cosmological role in this bi-maximal mixing scenario.

$SO(3)_F$ GAUGE INTERACTIONS

The mass matrix of gauge fields A^i_μ is found to be

$$M_F^2 = \frac{m_F^2}{3}\begin{pmatrix} 2(\xi_+ + \xi') & 0 & -\sqrt{2}\xi_- \\ 0 & 3\xi_+ + \xi' & 0 \\ -\sqrt{2}\xi_- & 0 & 3\xi_+ + \xi' \end{pmatrix}$$

with

$$m_F^2 = 3g_3'^2\sigma^2/8 \ , \quad \xi_\pm = (1 \pm \xi)/2$$

This mass matrix is diagonalized by an orthogonal matrix O_F via $O_F^T M_F^2 O_F$. Denoting the physical gauge fields as F^i_μ, we then have $A^i_\mu = O_F^{ij} F^j_\mu$, i.e.,

$$\begin{pmatrix} A^1_\mu \\ A^2_\mu \\ A^3_\mu \end{pmatrix} = \begin{pmatrix} c_F & 0 & -s_F \\ 0 & 1 & 0 \\ s_F & 0 & c_F \end{pmatrix} \begin{pmatrix} F^1_\mu \\ F^2_\mu \\ F^3_\mu \end{pmatrix}$$

with

$$s_F \equiv \sin\theta_F, \quad \tan 2\theta_F = 2\sqrt{2}\xi_-/(\xi_+ - \xi')$$

Masses of the three physical gauge bosons F^i_μ are given by

$$\begin{aligned} m_{F_1}^2 &= m_F^2(5\xi_+ + 3\xi' - \sqrt{(\xi_+ - \xi')^2 + 8\xi_-^2}\ )/6 \\ m_{F_2}^2 &= m_F^2(\xi_+ + \xi'/3) \\ m_{F_3}^2 &= m_F^2(5\xi_+ + 3\xi' + \sqrt{(\xi_+ - \xi') + 8\xi_-^2}\ )/6 \end{aligned}$$

In the physical mass basis, the gauge interactions are given by

$$\mathcal{L}_F = \frac{g_3'}{2} F_\mu^i \left(\bar{\nu}_L t^j O_F^{ji} \gamma^\mu \nu_L + \bar{e}_L V_e^i \gamma^\mu e_L - \bar{e}_R V_e^{i*} \gamma^\mu e_R \right)$$

with $V_e^i = U_e^\dagger t^j U_e O_F^{ji}$. Explicitly, we find

$$V_e^1 = \begin{pmatrix} c_F & i\frac{1}{2}s_F & -i\frac{1}{2}s_F \\ -i\frac{1}{2}s_F & \frac{1}{2}c_F + \frac{1}{\sqrt{2}}s_F & \frac{1}{2}c_F \\ i\frac{1}{2}s_F & \frac{1}{2}c_F & \frac{1}{2}c_F - \frac{1}{\sqrt{2}}s_F \end{pmatrix}$$

$$V_e^2 = \begin{pmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & 0 & i\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -i\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

$$V_e^3 = \begin{pmatrix} -s_F & i\frac{1}{2}c_F & -i\frac{1}{2}c_F \\ -i\frac{1}{2}c_F & -\frac{1}{2}s_F + \frac{1}{\sqrt{2}}c_F & -\frac{1}{2}s_F \\ i\frac{1}{2}c_F & -\frac{1}{2}s_F & -\frac{1}{2}s_F - \frac{1}{\sqrt{2}}c_F \end{pmatrix} .$$

The aproximate permutation symmetry $\varphi \leftrightarrow \varphi'$ implies that

$$\xi_{=} 1 - \xi << 1 \ , \quad \sin\theta_F << 1$$

Thus

$$m_{F_1} < m_{F_2} \ < \simeq \ m_{F_3}$$

LEPTON FLAVOR VIOLATIONS & GAUGE BOSON MASS

The $SO(3)_F$ gauge interactions allow lepton flavor violating process

$$\mu \to 3e$$

its branch ratio is found

$$Br(\mu \to 3e) = \left(\frac{v}{\sigma}\right)^4 \frac{2\xi_-^2}{[(3\xi_+ + \xi')(\xi_+ - \xi') - \xi_-^2]^2}$$
$$= \left(\frac{v\sqrt{\tan 2\theta_F}}{\sqrt{2}\sigma}\right)^4 \frac{32}{[2\sqrt{2}(3\xi_+ + \xi') - \xi_- \tan 2\theta_F]^2}$$

with $v = 246$GeV. Once

$$\sigma \sim 10^3 v\ \sqrt{\tan 2\theta_F}/\sqrt{2}$$

The branch ratio could be very close to the present experimental upper bound

$$Br(\mu \to 3e) < 1 \times 10^{-12}$$

Thus for

$$g_3' \sim g_W\ , \quad \sin\theta_F \sim 10^{-4}$$

i.e. taking the coupling constant g_3' for the $SO(3)_F$ gauge bosons to be at the same order of magnitude as those for the electroweak gauge bosons, and the mixing angle θ_F to be small, we have

$$\sigma \sim 1\, TeV\ , \quad m_{F_1} < m_{F_2} < \simeq m_{F_3} < 1\, TeV$$

which could be detectable at LHC or LC.

CONCLUSIONS

- 3 LIGHT NEUTRINOS
- SO(3) GAUGE SYMMETRY
- 3 HIGGS TRIPLETS
- GLOBAL MINIMAL POTENTIAL ENERGY
- APPROX. PERMUTATION SYMMETRY

NEARLY BI-MAXIMAL MIXING
IN NEUTRINO OSCILLATIONS
&
LEPTON-FLAVOR VIOLATING
GAUGE INTERACTIONS

GENERALIZED STANDARD MODEL

$$SO(3)_F \times U(1)_Y \times SU(2)_L \times SU(3)_c$$

TeV SCALE NEW PHYSICS