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Neutrino Mixing from the CKM Matrix in SUSY $SO(10) \times U(2)_F$

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Our model based on $SO(10) \times U(2)_F$

Require: Symmetric mass textures
(natural for $SU(4) \times SU(2)_L \times SU(2)_R$)

In $SO(10)$

up quarks $\leftrightarrow$ Dirac neutrinos

down quarks $\leftrightarrow$ charged leptons

Phenomenology with symmetric (real) mass textures.

Naively 6 texture zeroes for quarks

Ramond, Roberts & Ross: 5 texture zeroes. Arrived at 5 sets of up- and down-quark textures.

Our analysis with present expt'l data: One set is viable.

At GUT scale:

$$M_u = \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & 1 \end{pmatrix} d, \quad M_d = \begin{pmatrix} 0 & e & 0 \\ e & f & 0 \\ 0 & 0 & 1 \end{pmatrix} h,$$

$a \simeq b \ll c \ll 1$ $\qquad$ $e \ll f \ll 1$

G-J Relations:

$$\left.\begin{array}{l} m_b = m_\tau \\ m_s \simeq \frac{1}{3} m_\mu \\ m_d \simeq 3 m_e \end{array}\right\} \Longrightarrow \quad M_e = \begin{pmatrix} 0 & e & 0 \\ 0 & -3f & 0 \\ 0 & 0 & 1 \end{pmatrix} h$$

$$\mathcal{M}_u = U^{+}_{uL} M_u U_{uR} = \mathrm{Diag}\{m_u, m_c, m_t\}$$

$$\mathcal{M}_d = U^{+}_{dL} M_d U_{dR} = \mathrm{Diag}\{m_d, m_s, m_b\}$$

$$V_{CKM} = U^{+}_{uL} U_{dL}$$

In SO(10) at GUT scale Dirac neutrinos are related to up-quark.
To get small neutrino mass we use seesaw mechanism.

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \quad m_R \gg m_D$$

$$m_1 \simeq m_D^2/m_R$$
$$m_2 = m_R.$$

$M_{\nu RR} = ?$

$M_{\nu LL} \to$ low energy Majorana mass matrix

Need $m_{\nu_\tau} \gg m_{\nu_\mu}, m_{\nu_e}$
Also large mixing of ν_μ, ν_τ
Choose
$$M_{\nu LL} \sim \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \lambda$$

This gives large mixing in both $\nu_e - \nu_\mu$ & $\nu_\mu - \nu_\tau$ sectors (bimaximal mixing) for $0 \lesssim t \lesssim 1$

$$\mathcal{M}_{\nu LL} = U^{+}_{\nu LL} M_{\nu LL} U_{\nu LL} \overset{\text{Dig}(m_1, m_2, m_3)}{=}$$

$$M_{\nu LR} = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \beta & \gamma \\ \alpha & \gamma & 1 \end{pmatrix} \eta$$
$\alpha \simeq \beta \ll \gamma < 1$

To get $M_{\nu LL}$ as above
$$M_{\nu RR} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R$$

$\delta_1, \delta_2, \delta_3 \ll 1$

$\delta_i = f(\alpha, \beta, \gamma, t)$

$$M_{\nu LL} = - M^T_{\nu LR} M^{-1}_{\nu RR} M_{\nu LR}$$

Neutrino mixing matrix

$$U_{MNS} = U_{eL} U_{\nu LL}^{\dagger}$$

$$= \begin{pmatrix} U_{e\nu_e} & U_{e\nu_\mu} & U_{e\nu_\tau} \\ U_{\mu\nu_e} & U_{\mu\nu_\mu} & U_{\mu\nu_\tau} \\ U_{\tau\nu_e} & U_{\tau\nu_\mu} & U_{\tau\nu_\tau} \end{pmatrix}$$

U_{eL} is almost diagonal

$$U_{\nu LL} = \begin{pmatrix} \frac{1}{\sqrt{2}} - O(t) & -\frac{1}{2} - O(t) & \frac{1}{2} - O(t) \\ -\frac{1}{\sqrt{2}} - O(t) & -\frac{1}{2} + O(t) & \frac{1}{2} + O(t) \\ \frac{t}{2\sqrt{2}} & \frac{1}{\sqrt{2}} - O(t^2) & \frac{1}{\sqrt{2}} + O(t^2) \end{pmatrix}$$

$$|\Delta m_{12}^2| = |m_{\nu_e}^2 - m_{\nu_\mu}^2| = O(t^3)$$

$$|\Delta m_{23}^2| = |m_{\nu_\tau}^2 - m_{\nu_\mu}^2| = O(1)$$

$$|\Delta m_{23}^2| \gg |\Delta m_{12}^2|$$

Flavor sym $U(2)_F$ (Barbieri et al) . Because 3rd family is very much heavier of 1 & 2:

$$\psi_a \oplus \psi_3 = 2 \oplus 1 \, , \quad a = 1,2$$

In the sym limit only 3rd family has non-vanishing Yukawa couplings

$$U(2) \xrightarrow{\epsilon M} U(1) \xrightarrow{\epsilon' M} \text{nothing}$$

Froggatt-Nielsen mechanism
— Heavy matter fields, mass due to tree level interactions
— light matter fields, mass due higher order with Higgs & vector-like heavy matter fields.

$$\sim \frac{\langle \theta \rangle}{M}$$

Generic $W = H_{(2)} \left(\psi_3 \psi_3 + \psi_3 \frac{\phi^a}{M_{(3)}} \psi_a + \psi_a \frac{S^{ab}}{M} \psi_a \right)$

$$\frac{\langle \phi \rangle}{M} \sim O\begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix}, \quad \frac{\langle S^{ab} \rangle}{M} = O\begin{pmatrix} \epsilon' & \epsilon' \\ \epsilon' & \epsilon \end{pmatrix}$$

$$W = W_{Dirac} + W_{\nu RR}$$

$$W_{Dirac} = \psi_3 \psi_3 T_1 + \frac{1}{M} \psi_3 \psi_a \left(T_2 \phi_{(1)} + T_3 \phi_{(3)} \right) + \frac{1}{M} \psi_a \psi_b \left(T_4 + \overline{C} \right) S_{(2)} + \frac{1}{M} \psi_a \psi_b T_5 S_{(1)}$$

$$W_{\nu RR} = \psi_3 \psi_3 \overline{C}_1 + \frac{1}{M} \psi_3 \psi_a \Phi \overline{C}_2 + \frac{1}{M} \psi_a \psi_b \Sigma \overline{C}_2$$

$\psi_a \sim (16, 2)$ $a = 1, 2$

$\psi_3 \sim (16, 1)$

$(10, 1)$: T_1, T_2, T_3, T_4, T_5

$(\overline{126}, 1)$: $\bar{C}, \bar{C}_1, \bar{C}_2$ R-parity conserving.

Flavon fields : Nonperturbative above M_{GUT}

$(1, 2)$: $\phi_{(1)}, \phi_{(2)}, \Phi$

$(1, 3)$: $S_{(1)}, S_2, \Sigma$

a, b, c, e, f ... are fns of ϵ, ϵ' and ratios of VEV of Higgs.

Determine these using up-quark & charged lepton masses and Cabibbo angle.

Determine δ_i & M_R to produce $M_{\nu LL}$ & Δm^2

7 + 4 = 11 parameters

physical quantities

$\underbrace{6 + 6 + 3 + 3}_{18} + (3)$ ← rt handed neutrino masses.

($\tan\beta = 10$)

$a = 0.00226$, $b = 0.00381$, $c = 0.0392$

$d = 0.572$, $e = 0.00403$, $f = 0.0195$

$h = 0.0678$

$\delta_1 = 0.0016$, $\delta_2 = 3.32 \times 10^{-5}$, $\delta_3 = 0.0152$

$M_R = 1.32 \times 10^{14}$ GeV. $(t = 1 \times 10^{-3})$

	data at M_z	predictions at M_{GUT}	predictions at M_z
m_u	$2.33^{+0.42}_{-0.45} MeV$	4.065×10^{-6}	$1.917 MeV$
m_c	$677^{+36}_{-61} MeV$	0.001566	$738.7 MeV$
m_t	$181^{+}_{-}13 GeV$	0.5729	$184.3 MeV$
$\frac{m_d}{m_s}$	$17 \sim 25$	3.961×10^{-2}	22.5
m_s	$93.4^{+11.8}_{-13.0} MeV$	0.001374	$83.15 GeV$
m_b	$3.00^{+}_{-}0.11 GeV$	0.06779	$3.0141 GeV$
m_e	$0.486847 MeV$	1.880×10^{-5}	$0.486 MeV$
m_μ	$102.75 MeV$	0.003979	$102.8 MeV$
m_τ	$1.7467 GeV$	0.06779	$1.744 GeV$

The quark mixing matrix V_{CKM} at M_z is predicted to be

$$|V_{CKM,prediction}| = \begin{pmatrix} 0.9751 & 0.2215 & 0.003541 \\ 0.2215 & 0.9745 & 0.03695 \\ 0.004735 & 0.03681 & 0.9993 \end{pmatrix}$$

They are to be compared with the experimental results extrapolated to M_z [20]

$$|V_{CKM,exp}| = \begin{pmatrix} 0.9745-0.9757 & 0.219-0.224 & 0.002-0.005 \\ 0.218-0.224 & 0.9736-0.9750 & 0.036-0.046 \\ 0.004-0.014 & 0.034-0.046 & 0.9989-0.9993 \end{pmatrix}$$

Our model predicts the three light Majorana neutrino masses to be

$$m_{\nu_e} = 2.0052 \times 10^{-4} eV, \qquad m_{\nu_\mu} = 2.0123 \times 10^{-4} eV, \qquad m_{\nu_\tau} = 0.05574 eV$$

and the resulting squared mass differences are

$$\Delta m_{23}^2 = 3.11 \times 10^{-3} eV^2, \qquad \Delta m_{12}^2 = 2.87 \times 10^{-10} eV^2$$

exp $10^{-2} - 10^{-3}$ VO: $10^{-9} - 10^{-11}$

The lepton mixing matrix is given by

$$|U_{MNS,prediction}| = \left| U_{e_L} U_{\nu_{LL}}^\dagger \right| = \begin{pmatrix} 0.6710 & 0.7396 & 0.0527 \\ 0.5410 & 0.4397 & 0.7169 \\ 0.5070 & 0.5096 & 0.6952 \end{pmatrix}$$

This translates into

$$\sin^2 2\theta_{atm} \equiv 4|U_{\mu\nu_\tau}|^2(1 - |U_{\mu\nu_\tau}|^2) = 0.9992$$
$$\sin^2 2\theta_{\odot} \equiv 4|U_{e\nu_\mu}|^2(1 - |U_{e\nu_\mu}|^2) = 0.9912.$$

exp $> .82$ best fit ≈ 1

VO: $\gtrsim 0.67$ ≈ 1

These values agree with the Super-Kamiokande atmospheric neutrino oscillation data [1, 21], and the solar VO solution [22]. And the $(1,3)$ element of U_{MNS} is given by $|U_{e\nu_\tau}| = 0.0527$ which is far below the bound by the CHOOZ experiment $|U_{e\nu_\tau}| \lesssim 0.16$ [23]. The three eigenvalues of the right-handed neutrino Majorana mass matrix are given by

$$m_1 \simeq 2.963 \times 10^7 GeV, \qquad m_2 \simeq 2.643 \times 10^{10} GeV, \qquad m_3 \simeq 1.319 \times 10^{14} GeV$$

It is possible to have the LAMSW solution with

$$\delta_1 = 0.001082, \qquad \delta_2 = 0.0009870, \qquad \delta_3 = 0.02238$$

$$M_R = 2.415 \times 10^{12} GeV$$

$(t = 0.1)$

These change the predictions of $m_{u,c,t}$ by less than 1% but have no observable effects on down-quark and charged lepton masses, and the CKM matrix remains essentially the same [3]. In the neutrino sector, we get

$$m_{\nu_e} = 0.01089 eV, \qquad m_{\nu_\mu} = 0.01206 eV, \qquad m_{\nu_\tau} = 0.09999 eV$$

and the squared mass differences are

$$\Delta m^2_{23} = 9.851 \times 10^{-3} eV^2, \qquad \Delta m^2_{12} = 2.752 \times 10^{-5} eV^2$$

The lepton mixing matrix is given by

$$|U_{MNS,prediction}| = |U_{e_L} U^\dagger_{\nu_{LL}}| = \begin{pmatrix} 0.6439 & 0.7486 & 0.1580 \\ 0.6045 & 0.3712 & 0.7049 \\ 0.4690 & 0.5494 & 0.6915 \end{pmatrix}$$

$\sin^2 2\theta_{\mu} = 0.9854$

exp: best fit ≈ 1

The element $|U_{e\nu_\tau}|$ is predicted to be 0.1580 which is right at the experimental bound $|U_{e\nu_\tau}| \lesssim 0.16$ [23]. The three right-handed neutrino eigenvalues are given by

$$m_1 \simeq 5.732 \times 10^6 GeV, \qquad m_2 \simeq 1.177 \times 10^9 GeV, \qquad m_3 \simeq 2.417 \times 10^{12} GeV$$

We note that a $|U_{e\nu_\tau}|$ value of less than 0.1580 would lead to $\Delta m^2_{23} > 10^{-2} eV^2$ leading to the elimination of the LAMSW solution in our model.