# Supersymmetry versus precision experiments revisited

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*based on*

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# 1. Introduction

## MSSM

··· leading candidate for the theory beyond the SM

- an elegant solution to stabilize the EW scale against the GUT or Planck scale
- radiative EW symmetry breaking
- "unification" of gauge couplings

However, no experimental evidence has not been found yet at direct search experiments such as LEP-2, Tevatron

$$
\begin{aligned}
m_{\tilde{q}} &\gtrsim 212\mathrm{GeV},\\
m_{\tilde{g}} &\gtrsim 173\mathrm{GeV}\\
m_{\tilde{\chi}_1^-} &\gtrsim 93\mathrm{GeV},\\
&\vdots
\end{aligned}
$$

comprehensive study of the constraints on the MSSM parameters from EW precision data (LEP-1, SLC, etc) may be useful for the future collider experiments

# Precision Electroweak Data

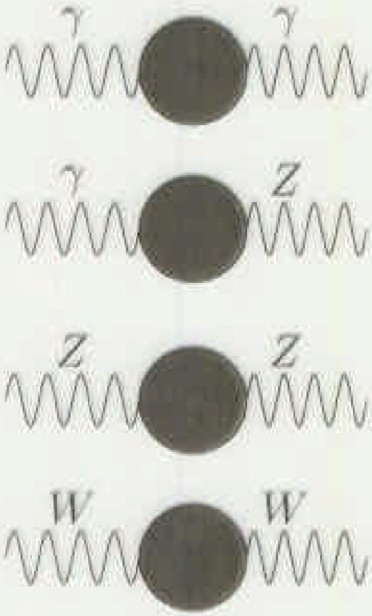
(CERN-EP/99-15)

| | | data | SM* | $\frac{\text{(data)} - \text{SM}}{\text{(error)}}$ |
|---|---|---|---|---|
| **LEP-1** | | | | |
| line-shape & FB asym.: | | | | |
| | $m_Z$(GeV) | $91.1867 \pm 0.0021$ | — | — |
| | $\Gamma_Z$(GeV) | $2.4939 \pm 0.0024$ | 2.4972 | −1.4 |
| | $\sigma_h^0$(nb) | $41.491 \pm 0.058$ | 41.474 | 0.3 |
| | $R_e$ | $20.783 \pm 0.052$ | 20.747 | 0.7 |
| | $R_\mu$ | $20.789 \pm 0.034$ | 20.747 | 1.3 |
| | $R_\tau$ | $20.764 \pm 0.045$ | 20.747 | −0.7 |
| | $A_{FB}^{0,e}$ | $0.0153 \pm 0.0025$ | 0.0165 | −0.5 |
| | $A_{FB}^{0,\mu}$ | $0.0164 \pm 0.0013$ | 0.0165 | −0.1 |
| | $A_{FB}^{0,\tau}$ | $0.0183 \pm 0.0017$ | 0.0165 | 1.1 |
| $\tau$ polarization: | | | | |
| | $A_\tau$ | $0.1431 \pm 0.0045$ | 0.1484 | −1.2 |
| | $A_e$ | $0.1479 \pm 0.0051$ | 0.1484 | −0.1 |
| heavy flavor: | | | | |
| | $R_b$ | $0.21656 \pm 0.00074$ | 0.21566 | 1.2 |
| | $R_c$ | $0.1735 \pm 0.0044$ | 0.1721 | 0.3 |
| | $A_{FB}^{0,b}$ | $0.0990 \pm 0.0021$ | 0.1040 | −2.4 |
| | $A_{FB}^{0,c}$ | $0.0709 \pm 0.0044$ | 0.0744 | −0.8 |
| **SLC** | | | | |
| | $A_{LR}^0$ | $0.1510 \pm 0.0025$ | 0.1484 | 1.0 |
| | $A_b$ | $0.867 \pm 0.035$ | 0.935 | −1.9 |
| | $A_c$ | $0.647 \pm 0.040$ | 0.668 | −0.5 |
| **Tevatron + LEP-2** | $m_W$ | $80.410 \pm 0.044$ | 80.402 | 0.18 |

* SM predictions for
$m_t$=175GeV, $m_H$=100GeV, $\alpha_s(m_Z)$=0.118 and $1/\alpha(m_Z^2) = 128.90$

# Radiative corrections in the $SU(2)_L \times U(1)_Y$ theory

## Gauge boson propagator corrections




$$\sim \bar{e}^2(q^2) = \hat{e}^2[1 - \mathrm{Re}\overline{\Pi}_{T,\gamma}^{\gamma\gamma}(q^2)]$$

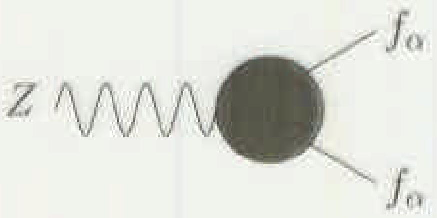
$$\sim \bar{s}^2(q^2) = \hat{s}^2[1 + \frac{\hat{c}}{\hat{s}}\mathrm{Re}\,\overline{\Pi}_{T,\gamma}^{\gamma Z}(q^2)]$$

$$\sim \bar{g}_Z^2(q^2) = \hat{g}_Z^2[1 - \mathrm{Re}\overline{\Pi}_{T,Z}^{ZZ}(q^2)]$$

$$\sim \bar{g}_W^2(q^2) = \hat{g}^2[1 - \mathrm{Re}\overline{\Pi}_{T,W}^{WW}(q^2)]$$

Hagiwara-Haidt-Kim-Matsumoto(1994)

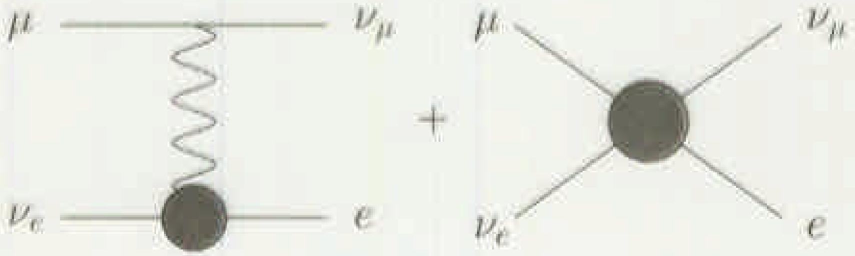
## $Zf_\alpha\overline{f_\alpha}$ vertex corrections




$\sim \Delta g_\alpha^f$

12 couplings for $\left\{ \begin{array}{lcl} f & = & u, d, b, e, \tau, \nu_e, \nu_\tau \\ \alpha & = & L, R \end{array} \right.$

## Vertex/Box corrections on the $\mu$-decay




$\sim \Delta\bar{\delta}_G$

## 2. Formalism

$Z$-pole observables:

$$\Gamma_f \propto |g_L^f + g_R^f|^2 C_{fV} + |g_L^f - g_R^f|^2 C_{fA}, \quad A^f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

$\cdots$ etc

The $Z \to f_\alpha \bar{f}_\alpha$ amplitude:

$$g_\alpha^f = (\text{SM reference value}) + \underbrace{(\cdots)\Delta\bar{g}_Z^2 + (\cdots)\Delta\bar{s}^2}_{\text{oblique corrections}} + \Delta g_\alpha^f$$

↑ Zff vertex correction

the SM reference point

$m_t = 175\text{GeV}$ $\quad m_H = 100\text{GeV}$

$\alpha_s(m_Z) = 0.118$ $\quad 1/\alpha(m_Z^2) = 128.90$

$\Delta\bar{g}_Z^2, \Delta\bar{s}^2$ $\cdots$ oblique corrections

(gauge boson propagator corrections)

$\bar{g}_Z^2(m_Z^2) = 0.55635 + \Delta\bar{g}_Z^2$

$\bar{s}^2(m_Z^2) = 0.23035 + \Delta\bar{s}^2$

$\Delta g_\alpha^f$ $\cdots$ $Z \to f_\alpha \bar{f}_\alpha$ vertex corrections

(non-oblique corrections)

They are related to the $S$, $T$, $U$- parameters (Peskin-Takeuchi, 1990)

$$\frac{1}{\bar{g}_Z^2(0)} = \frac{1+\bar{\delta}_G - \alpha T}{4\sqrt{2}G_F m_Z^2}$$

$$\bar{s}^2(m_Z^2) = \frac{1}{2} - \sqrt{\frac{1}{4} - \bar{\alpha}(m_Z^2)\left(\frac{4\pi}{\bar{g}_Z^2(0)} + \frac{S}{4}\right)}$$

$$\frac{4\pi}{\bar{g}_W^2(0)} = \frac{\bar{s}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{1}{4}(S+U)$$

we express $\Delta\bar{g}_Z^2, \Delta\bar{s}^2$ by

$$\Delta\bar{g}_Z^2 = 0.00412\Delta T_Z$$

$$\Delta\bar{s}^2 = 0.00360\Delta S_Z - 0.00241\Delta T_Z$$

where

$$S_Z \equiv S + R - 0.064 x_\alpha$$

$$T_Z \equiv T + 1.49R - \frac{\Delta\bar{\delta}_G}{\alpha}$$

$x_\alpha = \frac{1/\alpha(m_Z^2) - 128.90}{0.03}$

$\frac{\Delta\bar{\delta}_G}{\alpha}$: correction to $\mu$-decay

The shift $R$

$$\frac{4\pi}{\bar{g}_Z^2(m_Z^2)} - \frac{4\pi}{\bar{g}_Z^2(0)} \equiv -\frac{1}{4}R = -\frac{1}{4}(1.1879 + \Delta R)$$

accounts for the difference between $T$ and $T_Z$ ~


## The third oblique parameter: $\Delta m_W$

$$m_W = 80.402 + \Delta m_W$$

$$\Delta m_W = -0.288\Delta S + 0.418\Delta T + 0.337\Delta U + 0.012 x_\alpha - 0.126\frac{\Delta\bar{\delta}_G}{\alpha}$$

## Oblique corrections
## (Gauge boson propagator corrections)

| Cho-Hagiwara | Peskin-Takeuchi |
|---|---|
| $S_Z$,$T_Z$,$m_W$ | $S, T, U$ |
| $S_Z = S + R - 0.064 x_\alpha$<br>$T_Z = T + 1.49R - \frac{\Delta\bar{\delta}_G}{\alpha}$<br>$m_W = m_W(S, T, U)$ | |
| no correlation between $(S_Z, T_Z)$ and $m_W$ | analysis should be done on the 3-dimensional space$(S, T, U)$ |

## Non-oblique corrections
## (process specific vertex/box corrections)

$Z \to ff$ : $\Delta g^f_\alpha$ $\qquad f = u, d, b, e, \tau, \nu_e, \nu_\tau$

$\qquad \alpha = L, R$

$\mu$-decay : $\Delta\bar{\delta}_G$

# 3. Quantum corrections in the MSSM

New particles:

- sfermions (squarks, sleptons)
- Higgs bosons $(h, H, A, H^-)$
- charginos$(\chi^-)$/neutralinos$(\chi^0)$ (gluon → $Zq\bar{q}$ vertex)

oblique corrections ··· sum of their individual contributions
dominant if either sfermions or charginos/neutralinos are heavy

The best fit:

$$\left.\begin{aligned} \Delta S_Z - 33.7\Delta g_L^b &= -0.070 \pm 0.113 \\ \Delta T_Z - 60.3\Delta g_L^b &= -0.183 \pm 0.137 \end{aligned}\right\} \rho = 0.89$$

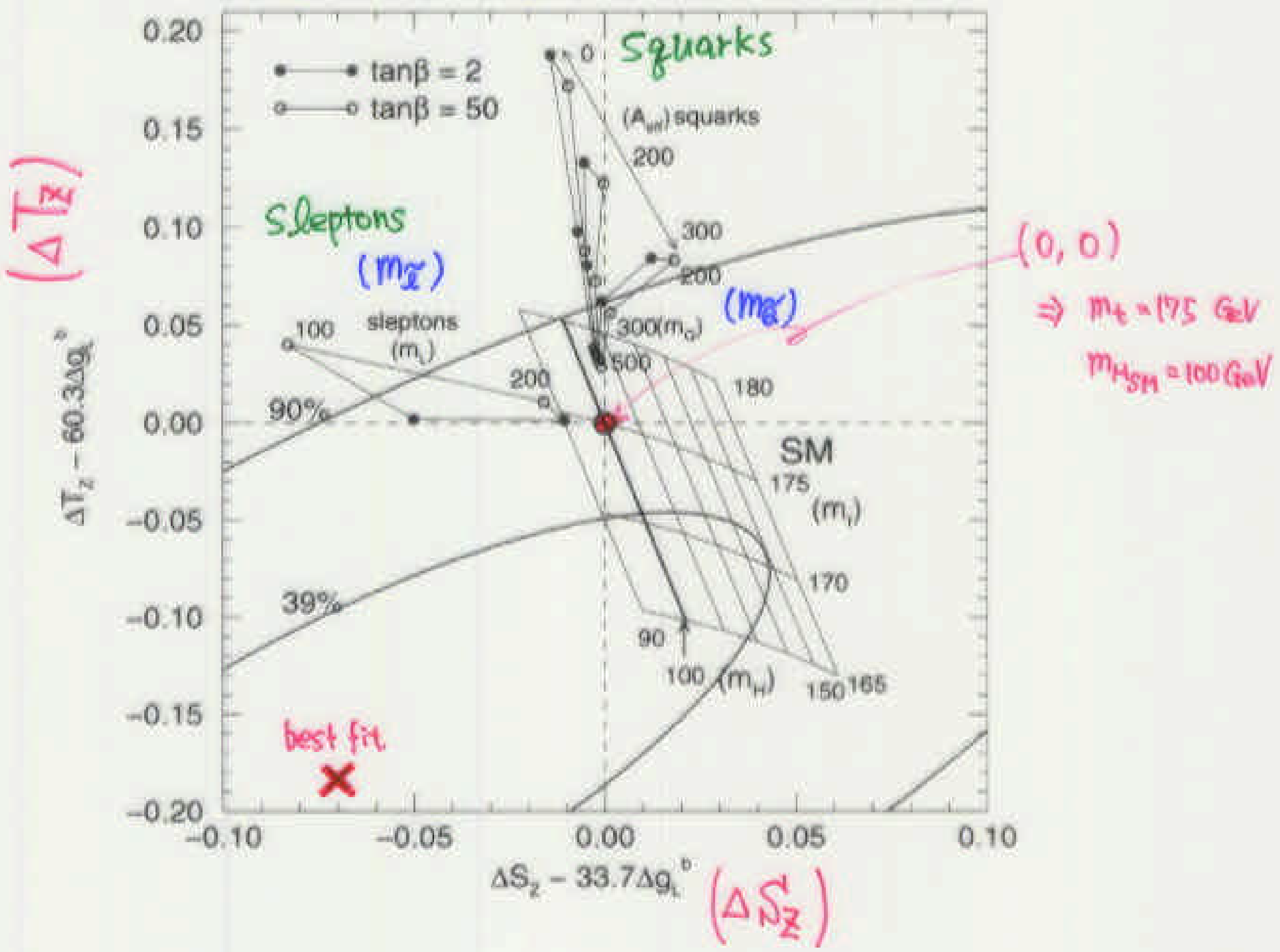
$$\Delta m_W = \quad 0.008 \pm 0.046$$

$$\chi^2_{\text{min}} = 15.4 + \left(\frac{\Delta g_L^b + 0.00086}{0.00077}\right)^2$$

where d.o.f $= 19 - 5 = 14$

[19=$Z$-pole(17)+$m_W$+$\alpha_s$]
[5=$\Delta S_Z, \Delta T_Z, \Delta g_L^b, \Delta m_W, \alpha_s$]

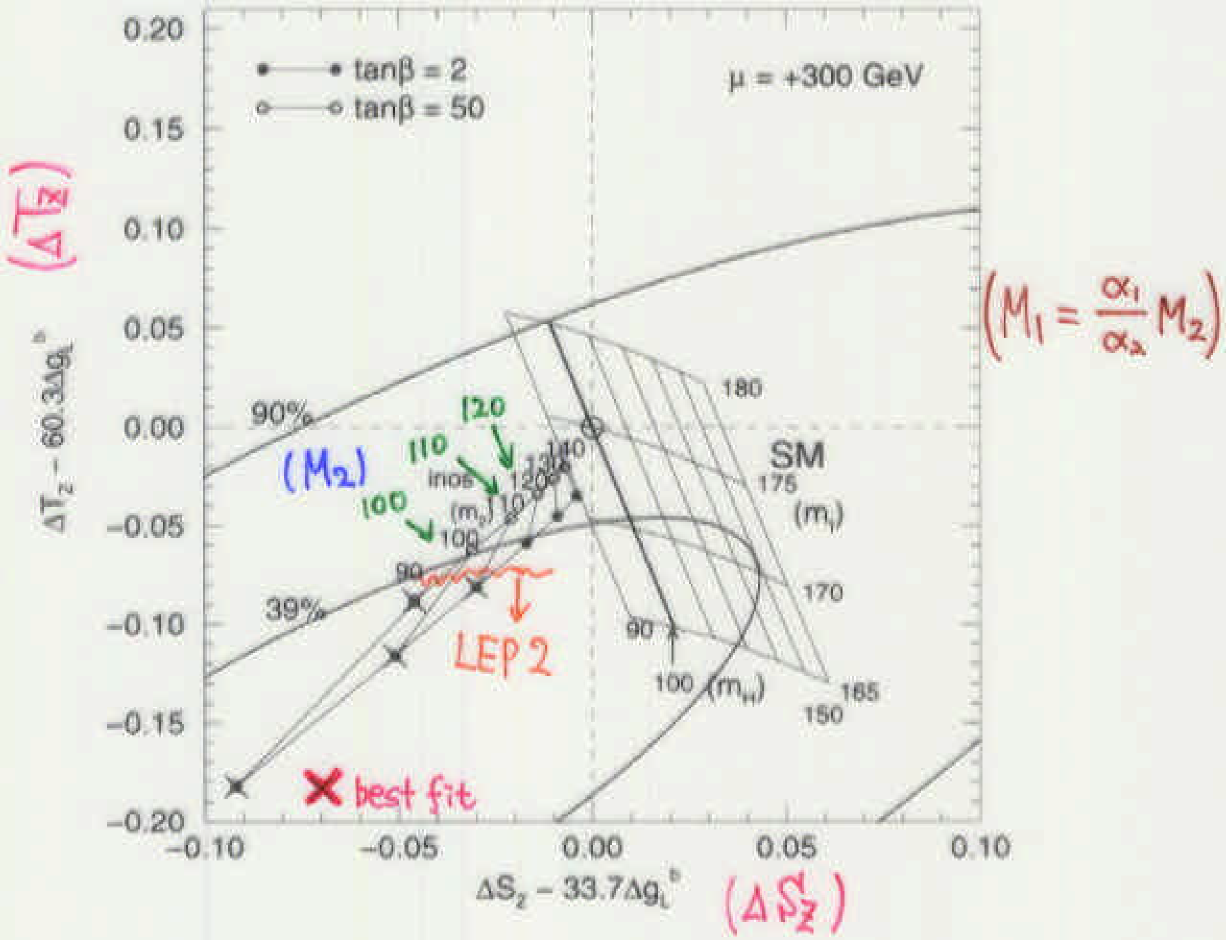
# Sfermion contribution




- $\Delta S_Z, \Delta T_Z$: dominant contributions come from the left-handed sfermions
- $\Delta T_Z$ : constructive interference with the SM (negative $\Delta T_Z$ is favored from the data)
- $A_{\rm eff}$: suppress the contribution to $\Delta T_Z$ through the $\tilde{t}_L$-$\tilde{t}_R$ mixing

⇒ light sfermions ($\sim 100$GeV) make the fit worse (mainly due to the $T_Z$-paramter)

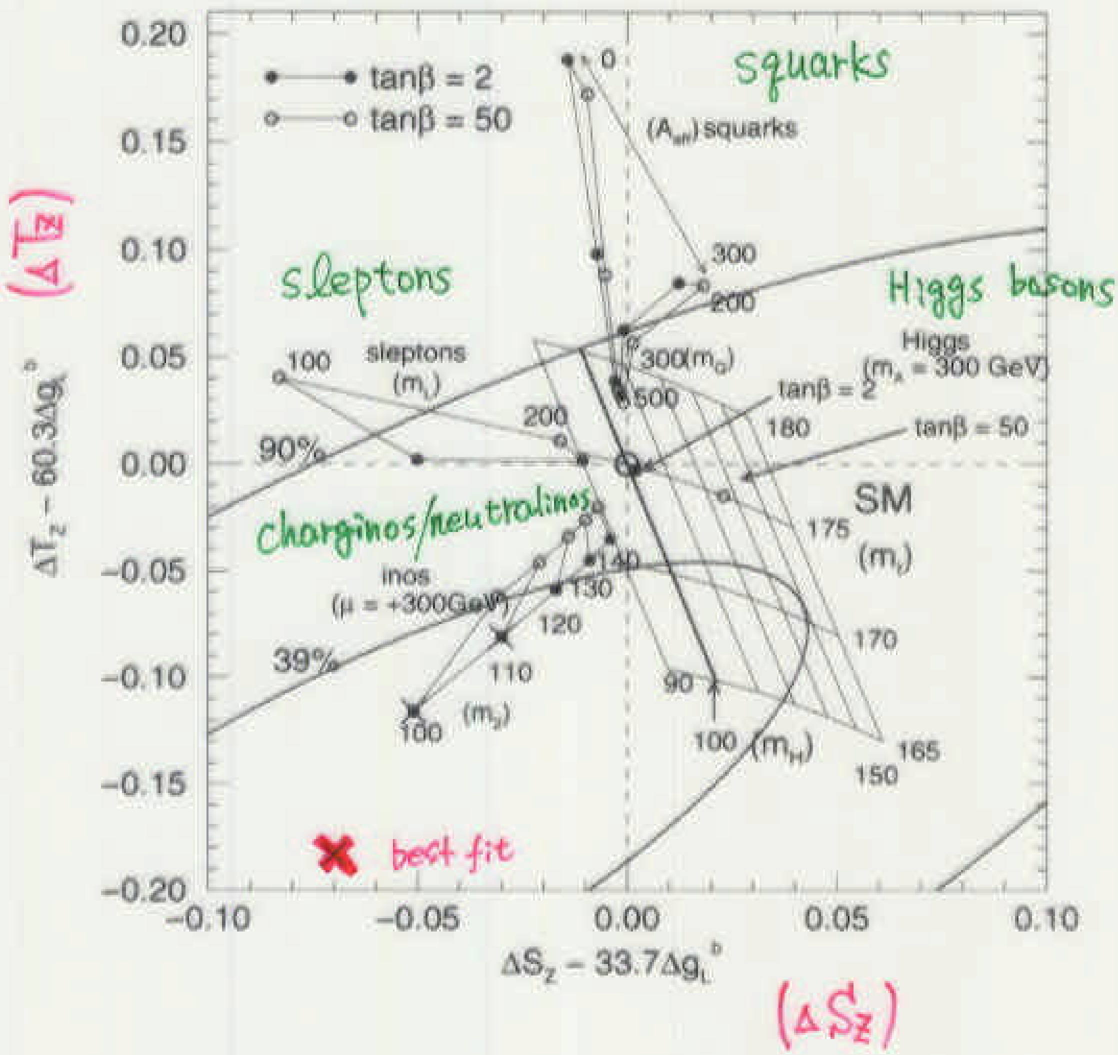
# chargino/neutralino contribution




- contribution to $\Delta R$ ($Z$-boson propagator correction) leads to the cancellation with the $T$-parameter (note: $T_Z = T + 1.49R + \cdots$)
- symbol($\times$) : excluded from direct search ($m_{\tilde{\chi}_1^-} \lesssim 90$GeV)

$\Rightarrow$ light chargino ($\sim 100$GeV) may improve the fit!!
(due to negative $T_Z$)

# Summary of the oblique corrections in the MSSM




- light (left-handed) sfermions always make the fit worse than the SM
- light chargino/neutralino improve the fit

⇒ light chargino ($\sim 100$GeV) and heavy ($\sim 1$TeV) left-handed sfermions may be favored from the data

# Non-oblique corrections in the MSSM

sizable contributions may be possible through

- SUSY QCD correction to the $Zf\bar{f}$ vertex
  sizable when $\tilde{q}, \tilde{g}$ are light

- scalar-top contribution to $R_b$
  when $\tilde{t}_R, \tilde{\chi}^-$ are light and $\tan\beta \sim 1$

- Higgs contribution to the $Zbb$, $Z\tau\tau$ couplings
  when $m_A$ is light and $\tan\beta \gg 1$

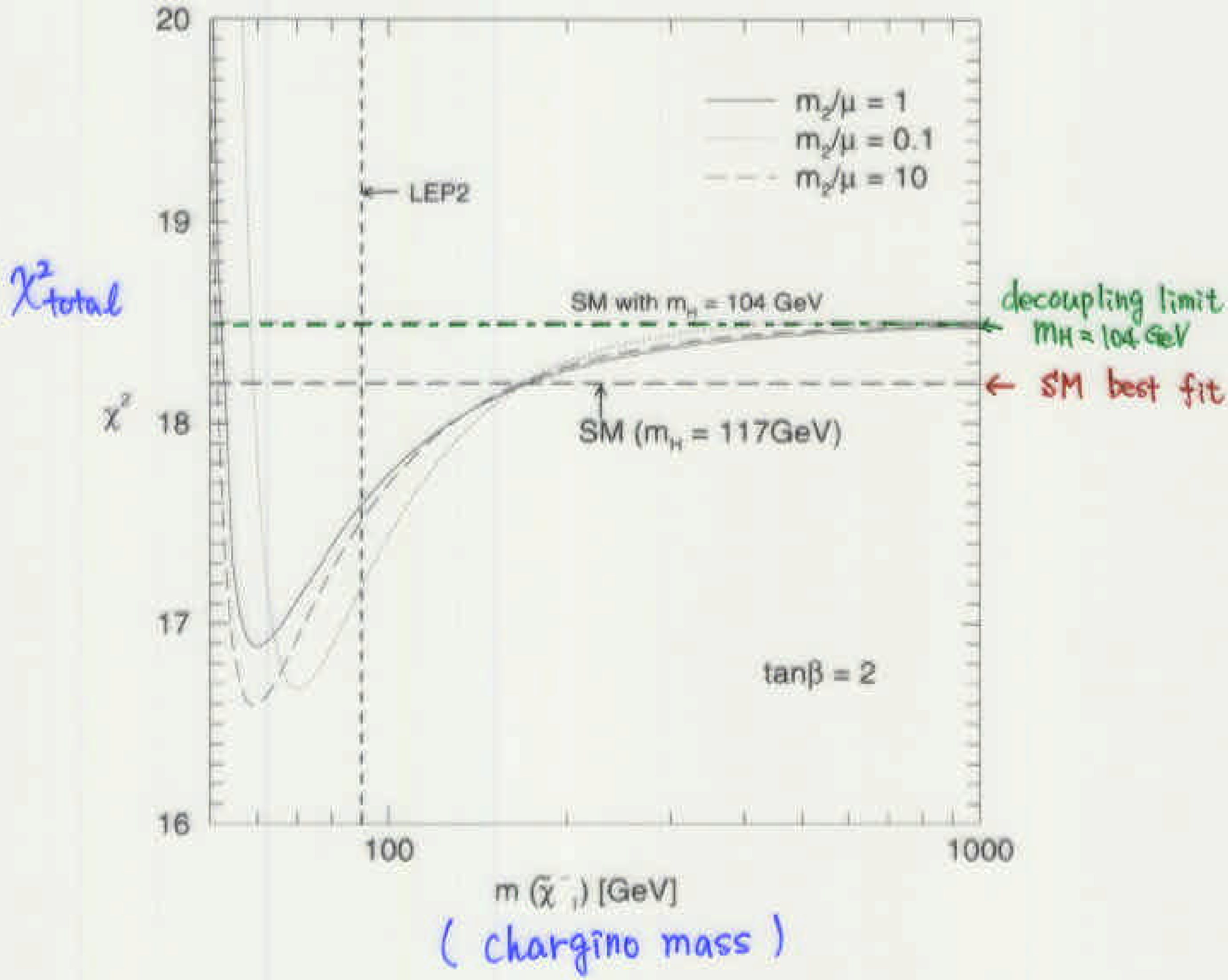
- slepton contribution to the $\mu$-decay
  (vertex & box): $\Delta\bar{\delta}_G$

  (note) $T_Z = T + 1.49R - \dfrac{\Delta\bar{\delta}_G}{\alpha}$

However $\cdots$
No improvement of the fit!!

improvement of the fit is expected from the oblique corrections
$\Rightarrow$ light chargino & heavy sfermions

all the sfermions, pseudo/charged Higgs bosons $\Rightarrow$ 1TeV:




$m_{\tilde{\chi}_1^-} \sim 100$GeV makes the fit better than the SM!

$$M_2/\mu = \begin{cases} 0.1 & : \text{gaugino like lighter chargino} \\ 1 & : \text{mixed state} \\ 10 & : \text{higgsino dominant lighter chargino} \end{cases}$$

# 5. Summary

We have studied the indirect constraints on the MSSM parameter space from the electroweak experiments

- oblique corrections: $S_Z, T_Z, m_W$

  $$S_Z = S + \Delta R - 0.064 x_\alpha, \qquad T_Z = T + 1.49\Delta R - \frac{\Delta \bar{\delta}_G}{\alpha}$$

  - running effect of the $Z$-boson propagator correction $\Delta R$ is important
  - light($\sim 100$GeV) chargino improves the fit through $\Delta R$
- non-oblique corrections:
  small if the left-handed sfermions are heavy enough ($\sim 1$TeV)

When all the sfermions and extra Higgs bosons ⇒ 1TeV:

$\chi^2_{\text{MSSM}} < \chi^2_{\text{SM}}$ : for light ($\sim 100$GeV) chargino
$\chi^2_{\text{MSSM}} \to \chi^2_{\text{SM}}$ : if chargino is heavier than $120$GeV

- a possible signal of SUSY which we expect:
  - $m_h \sim 120$GeV
  - $\tilde{\chi}_1^- \sim 100$GeV
  - $\tilde{\ell}_L, \tilde{q}_L$ cannot be $\sim 100$GeV
- target
  - Experiments
    Look for light Higgs, chargino at LEP-2, Tevatron and LHC
    Improve $m_t, \alpha(m_Z^2), m_W$
  - Theory
    hints from other experiments : $b \to s\gamma$ (in progress), muon $g-2$ (G.C.C., K. Hagiwara, M. Hayakawa PLB478(00)231)
    (constraints on the right-handed sfermions)

    SUSY breaking scenarios : supergravity, gauge interaction, anomaly mediation scenarios