ICHEP2000
OSAKA
7/27/00

YUKAWA TEXTURES AND HORAVA-WITTEN M-THEORY

R. ARNOWITT
B. DUTTA

[hep-th/0006172]

Texas A&M University

1. INTRODUCTION

Over the past year, considerable progress has been made in understanding Horava-Witten heterotic M-Theory with "non-standard" embeddings. In this picture space has (to lowest order) an 11 dimensional orbifold structure of the form

$$M_4 \times X \times S^1/\mathbb{Z}_2$$

where

M_4 = Minkowski space

X = 6D Calabi-Yau space

$-\pi\rho \leq x^{11} \leq \pi\rho$

The space has thus 2 orbifold 10D manifolds $M_4 \times X$:

$x^{11} = 0$; visable sector

$x^{11} = \pi\rho$; hidden sector

each with a priori E_8 gauge symmetry.

In addition, there will be a set of 5-branes in the bulk at points

$$0 < x_n < \pi\rho \; ; \quad n = 1 \dots N \; ; \quad \text{5-branes}$$

each spanning M_4 (for Lorentz invariance) and wrapped around a holomorphic curve in X (to preserve supersymmetry).

In general, physical matter lives on the $x^{11}=0$ orbifold plane and only gravity lives in the bulk. The existance of 5-branes allows one to satisfy the cohomological constraints with the E_8 on $x^{11}=0$ breaking:

$$E_8 \Rightarrow G \times H \; ; \quad \begin{aligned} G &= \text{structure grp} \\ H &= \text{unification grp} \end{aligned}$$

and we consider here the case

$$G = SU(5) \Rightarrow H = SU(5)$$

Within this framework, using torus fibered Calabi-Yau (with 2 sections) 3 generation models with a Wilson line breaking $SU(5) \to SU(3) \times SU(2) \times U(1)$ have been constructed [Donagi, Ovrut, Pantev, Waldram].

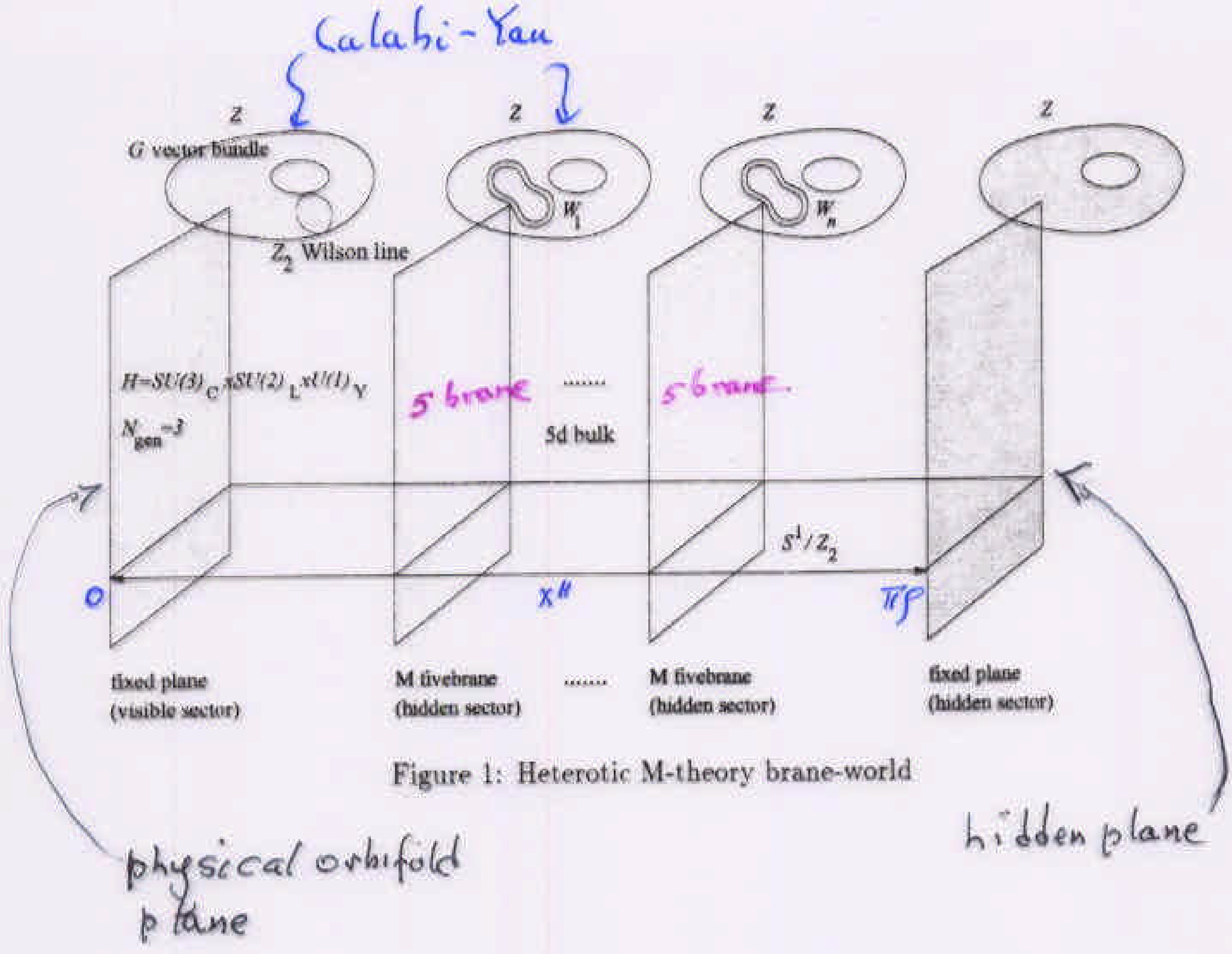

Figure 1: Heterotic M-theory brane-world

[Donagi, Ovrut, Pantev, Waldram, hep-th/9912208]

In following we consider: ④

* A brief summary of form of Kahler metric and Yukawa couplings (expressed as integrals over the Calabi-Yau manifold).

* A phenomenological choice of Yukawa textures arising from general form of M-Theory Kahler metric, and consistent with all CKM and quark mass data.

* Construction of 3 generation model with Wilson line (to break $SU(5) \to SM$) for Calabi-Yau with del Pezzo base dP_7 with properties needed for phenomenology (vanishing instanton charge: $\beta_i^{(0)} = 0$)

2. KAHLER METRIC AND YUKAWA COUPLINGS

In M-Theory, the gauge coupling constant α_G and Newtonian constant G_N obey

$$\alpha_G = \frac{(4\pi\kappa^2)^{2/3}}{2\mathcal{V}} \; ; \; G_N = \frac{\kappa^2}{16\pi^2\rho\mathcal{V}}$$

where

$1/\kappa^{2/9}$ = 11 D Planck mass ; $\mathcal{V}$ = Calabi-Yau volume

Grand unification at M_G is accounted for by assuming

$$\mathcal{V} = (1/M_G)^6 \; ; \; M_G \cong 3\times10^{16}\,\text{GeV}$$

and with $\alpha_G \cong 1/24$ one finds

$$1/\kappa^{2/9} = 2M_G \; ; \; 1/\pi\rho \cong 4.7\times10^{15}\,\text{GeV}$$

Thus the 11^{th} dimension is large and the fundamental mass scale, $\kappa^{-2/9}$, is $\approx M_G$, not M_{Pl}.

The Bose part of the gravity multiplet in the bulk are

$$g_{IJ} \; ; \; C_{IJK} \; ; \; G_{IJKL} \equiv 24\,\partial_{[I}C_{JKL]}$$

G_{IJKL} obeys field equations

$$D_I G^{IJKL} = 0$$

and Bianchi identities

$$(dG)_{11RSTU} = 4\sqrt{2}\pi\left(\frac{\kappa}{4\pi}\right)^{2/3}\Big[J^0\delta(x^{11}) + J^{N+1}\delta(x^{11}-\pi\rho) + \frac{1}{2}\sum_{n=1}^{N} J^n\left(\delta(x^{11}-x_n)+\delta(x^{11}+x_n)\right)\Big]_{RSTU}$$

where the J^n, $n=0,1,\dots N+1$ are sources from the orbifold planes and the 5-branes.

These equations can be solved in series of powers of $\kappa^{2/3}$ [Witten] and solutions to 1st order with 5-branes have been obtained [Lukas, Ovrut, Waldram]. The effective 4D theory is then characterized by a Kahler potential

$$K = Z_{IJ}\bar{C}^I C^J \; ; \; C^I = \text{matter fields}$$

Yukawa couplings Y_{IJK} and gauge function f on the physical orbifold plane $x^{11}=0$.

The general form for these solutions are:

$$Z_{IJ} = e^{-K_T/3}\left[G_{IJ} - \frac{\epsilon}{S+\bar{S}} \tilde{\Gamma}^i_{IJ} \left(\beta_i^{(0)} + \sum_{n=1}^{N} (1-Z_n)^2 \beta_i^{(n)} \right) \right]$$

where

$$\epsilon = \left(\frac{\kappa}{4\pi}\right)^{2/3} \frac{2\pi^2 \rho}{V^{2/3}}$$

$$K_T = -\ln\left[\frac{1}{6} d_{ijk} (T^i + \bar{T}^i)(T^j + \bar{T}^j)(T^k + \bar{T}^k)\right]$$

and

$Z_n = \frac{x_n}{\pi \rho}$; position of 5-branes

and S = dilaton and T^i = moduli

The G_{IJ}, $\tilde{\Gamma}^i_{IJ}$, Y_{IJK} can be expressed in terms of integrals over the Calabi-Yau manifold

The $\beta_i^{(n)}$ are instanton and magnetic charges:

$$\beta_i^{(n)} = \int_{C_{4i}} J^{(n)} \; ; \quad C_{4i} = 4\text{-cycles}$$

3. Phenomenological Yukawa Textures

Conventional treatments of Yukawa textures usually assumes symmetric matrices for the u and d quarks with high powers of the Wolfenstein parameter $\lambda = 0.2$ to account for the CKM and quark mass hierarchies.

Example [Ramond, Roberts, Ross]:

$$Y^u = \begin{pmatrix} 0 & \sqrt{2}\,\lambda^6 & 0 \\ \sqrt{2}\,\lambda^6 & \sqrt{5}\,\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \; ; \; Y^d = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \; \lambda = 0.2$$

The heterotic M-Theory offers an alternate possibility for generating the hierarchies. Here the Y_{IJK} are integrals over the Calabi-Yau and so cannot be explicitly evaluated; however, there is no a priori reason some elements are highly suppressed relative to rest. Further, there is the Kahler metric which consists of two parts: G_{IJ} which $\mathcal{O}(1)$ and the ϵ part. The ϵ part would be small provided:

$\beta_i^{(0)} = 0$ (instanton charges on physical plane)

$z_n \equiv 1 - d_n$; $d_n \simeq 0.1$ (5-branes cluster near hidden plane)

The $\beta_i^{(0)} = 0$ condition is non-trivial but it is possible to show that a 3 generation torus fibered Calabi-Yau using del Pezzo base dP_7 has this property.

The above two assumptions make the ϵ terms in Z_{IJ} to be $\mathcal{O}(10^{-2})$. We assume that these are the third generation contributions for Z^u. An example of a set of $Z^{u,d}_{IJ}$ and $Y^{u,d}$ that are qualitatively in accord with the above is the following:

<u>Table</u> Z^u, Z^d ; Y^u, Y^d

Note that no particularly small numbers (like λ^6) appears. The Yukawa matrices themselves are assumed diagonal, for simplicity.

One first diagonalized $Z^{u,d}$ with unitary $U^{(u,d)}$ and then rescales it to unit matrix:

$$C^{I}_{(u,d)} = \left(U^{(u,d)} S^{(u,d)}\right)_{IJ} C^{J\,\prime}_{(u,d)}$$

where

$$\mathrm{diag}\, S = (\lambda_1^{-\frac{1}{2}}, \lambda_2^{-\frac{1}{2}}, \lambda_3^{-\frac{1}{2}})\ ;\ \lambda_i = \text{eigenvalues of } Z^{(u,d)}$$

The Yukawa matrices are then transformed

$$\lambda^{(u)}_{IJ} = \left(S^{(u)} \tilde{U}^{(u)} Y^{(u)} U^{(u)} S^{(u)}\right)_{IJ}$$

$$\lambda^{(d)}_{IJ} = \left(S^{(u)} \tilde{U}^{(u)} Y^{(d)} U^{(d)} S^{(d)}\right)_{IJ}$$

It is the rescaling that generates the Yukawa hierarchies.

To obtain the CKM and quark masses at the electroweak scale:

* Use 2-loop gauge and 1-loop SUSY RGE down to m_t
* Below m_t assume Standard model holds and include QCD corrections (important)
* Diagonalize Yukawas at electroweak scale to get V_{CKM}, quark masses

Table V_{CKM}, m_q. Comparison with experiment.

Table 2. Kahler matrices Z_{IJ} and Yukawa matrices Y^u, Y^d for u and d quarks for tanβ=3. The parameter d is 0.1.

$$Z^u = f_T \begin{pmatrix} 1 & 0.345 & 0 \\ 0.345 & 0.132 & 0.639d^2 \\ 0 & 0.639d^2 & 0.333d^2 \end{pmatrix}; \; Z^d = f_T \begin{pmatrix} 1 & 0.821 & 0 \\ 0.821 & 0.887 & 0 \\ 0 & 0 & 0.276 \end{pmatrix}. \tag{27}$$

$$\mathrm{diag}Y^u = (0.0765, 0.536, 0.585Exp[\pi i/2]); \tag{28}$$

$$\mathrm{diag}Y^d = (0.849, 0.11, 1.3).$$

$$f_T \equiv \exp[-K_T/3]$$

Quantity	(GeV) Theoretical Value	(GeV) Experimental Value
m_t(pole)	170.5	175± 5
$m_c(m_c)$	1.36	1.1-1.4
m_u(1 GeV)	0.0032	0.002-0.008
$m_b(m_b)$	4.13	4.1-4.5
m_s(1 GeV)	0.110	0.093-0.125[35]
m_d(1 GeV)	0.0055	0.005-0.015
V_{us}	0.22	0.217-0.224
V_{cb}	0.036	0.0381±0.0021[36]
V_{ub}	0.0018	0.0018-0.0045
V_{td}	0.006	0.004-0.013
$\sin 2\beta$	0.31	
$\sin\gamma$	0.97	
$\frac{m_u}{m_d}$	0.582	0.553±0.043 [Leutwyler]
$\frac{m_s}{m_d}$	20.0	18.9±0.8 [Leutwyler]

The agreement with experiment is very and the Yukawa hierarchies achieved without introducing small parameters. Further, no undue fine tuning occurs in the choice of $Z^{(u,d)}$ to obtain the hierarchies. For example if one rounds off the entries:

Approximate $Z^{u,d}$ $(f_T = \exp(-K_T/3))$:

$$Z^u = f_T\begin{pmatrix} 1 & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{8} & \frac{2}{3}\delta^2 \\ 0 & \frac{2}{3}\delta^2 & \frac{1}{8}\delta^2 \end{pmatrix} ; \quad Z^d = f_T\begin{pmatrix} 1 & \frac{4}{5} & 0 \\ \frac{4}{5} & \frac{9}{10} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix} ; \quad \delta = 0.1$$

one still gets qualitative agreement with experiment.

4. CONCLUSIONS

Horava-Witten M-Theory has now progressed to the point where it offers a fundamental framework for building phenomenological models:

* Allows conventional GUT groups, $SU(5)$, $SO(10)$
* Accomodates grand unification at $M_G \cong 3\times10^{16}$ GeV
* With torus fibered Calabi-Yau there are 3 generation models with a Wilson line to break the GUT group to $SU(3)\times SU(2)\times U(1)$

M-Theory with non-standard embeddings offers an alternate possibility of encoding the Yukawa hierarchies in Kahler metric Z_{IJ}. This can happen if

* 5 branes cluster near hidden plane, $z_n = 1 - d_n$, $d_n \simeq 0.1$.
* Instanton charge of physical plane vanishes, $\beta_i^{(0)} = 0$.
* No exceptional fine tuning required
* Condition $\beta_i^{(0)} = 0$ can occur for torus fibered C-Y with base dP_7.