Status of theoretical $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s l^+ l^-$ calculations

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Plan:

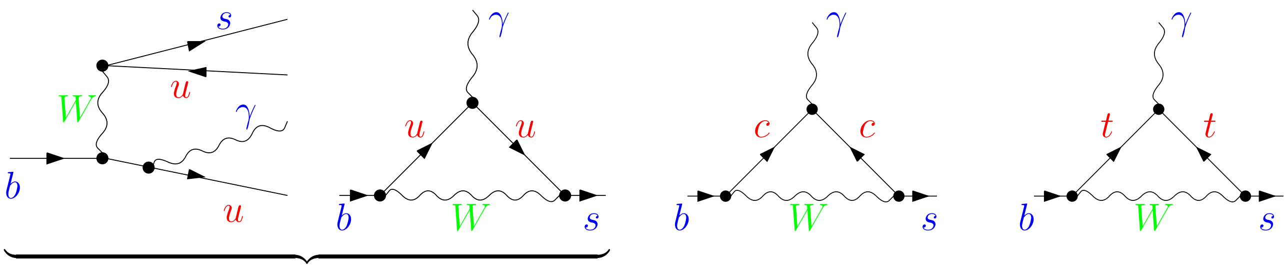
1. $\bar{B} \to X_s \gamma$

- **(i) Intermediate ψ contributions**
- **(ii) Perturbative calculations**
- **(iii) Non-perturbative effects**
- **(iv) Cuts on the photon spectrum**

2. $\bar{B} \to X_s l^+ l^-$

- **(i) Intermediate ψ contributions**
- **(ii) Perturbative calculations**
- **(iii) Non-perturbative effects**
- **(iv) Cuts on the dilepton invariant mass spectrum**

Electroweak transitions mediating $\bar{B} \to X_s \gamma$:

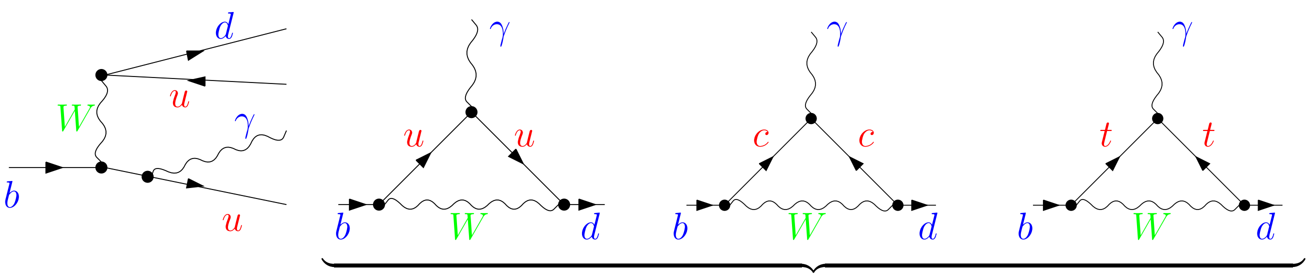


$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.

The decay $\bar{B} \to X_d\gamma$ is CKM suppressed with respect to $\bar{B} \to X_s\gamma$. Therefore, it does not make much difference whether it is excluded or included in $\bar{B} \to X_{\text{no charm}}\gamma$:



$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq 1\% \qquad \left|\frac{V_{td}}{V_{cb}}\right|^2 \in [2.5\%, 5\%]$$

If $\bar{B} \to X_d\gamma$ is included, one needs to remember that the perturbative results for this decay are subject to at least $\pm 30\%$ non-perturbative uncertainty ($\Rightarrow$ 2% uncertainty in $\bar{B} \to X_{\text{no charm}}\gamma$).

The $\bar{B} \to X_s\gamma$ branching ratio has been measured by CLEO [**Phys. Rev. Lett. 74 (1995) 2885, hep-ex/9908022**] **and ALEPH** [**Phys. Lett. B429 (1998) 169**]. **The (more precise) CLEO result can be written as follows:**

$$\mathrm{BR}[\bar{B} \to X_s\gamma] \simeq BR[\bar{B} \to X_{\text{no charm}}\gamma] \simeq (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$$

$$+ \sum_{P=\psi,\psi',\ldots} \mathbf{BR}[\bar{\mathbf{B}} \to \mathbf{X}^{(1)}_{\text{no charm}}\ \mathbf{P}] \times \mathbf{BR}[\mathbf{P} \to \mathbf{X}^{(2)}_{\text{no charm}}\ \gamma]$$

The intermediate ψ contribution in the latter term gives around 4×10^{-4}, even when $E'_\gamma > 0.3 m_\psi$ in the ψ rest-frame. The analogous contribution from ψ' is expected to be about 6 times smaller.

The effect of the photon energy cutoff $E_\gamma > E_0$ in the $\bar{B}$-meson rest frame can be easily estimated when $X^{(1)}_{\text{no charm}}$ is assumed to be massless and the spin of ψ is assumed to be irrelevant. For $E_0 > \frac{m_\psi^2}{2m_B} \simeq 0.91$ GeV one finds:

$$\left\{\mathbf{BR}[\bar{\mathbf{B}} \to \mathbf{X}^{(1)}_{\text{no charm}}\ \psi] \times \mathbf{BR}[\psi \to \mathbf{X}^{(2)}_{\text{no charm}}\ \gamma]\right\}_{E_\gamma > E_0}$$

$$= \mathbf{BR}[\bar{\mathbf{B}} \to \mathbf{X}^{(1)}_{\text{no charm}}\ \psi]\ \int_{2E_0/m_B}^{1} dx\, b(x) f(x),$$

where $x = \frac{2E'_\gamma}{m_\psi}$, $\quad b(x) = \frac{\partial}{\partial x} BR[\psi \to X^{(2)}_{\text{no charm}}\ \gamma]$ **and**

$$f(x) = \frac{1}{2} - \frac{1}{\pi} \arcsin\left[\frac{m_B^2}{m_B^2 - m_\psi^2}\left(\frac{4E_0}{x m_B} - \frac{m_B^2 + m_\psi^2}{m_B^2}\right)\right].$$

The function $b(x)$ for $x > 0.6$ can be found from the ancient MARK II data [Phys. Rev. D23 (1981) 43]. A naive fit to their fig. 9 reads:

$$b(x) = (4.1 \pm 0.8) \times 10^{-2}\; n(x),$$

where

$$n(x) = C \begin{cases} 0.2, & \text{for } 0.6 < x < 0.7, \\ \frac{20}{9}(1-x)^2, & \text{for } 0.7 < x < 1, \end{cases}$$

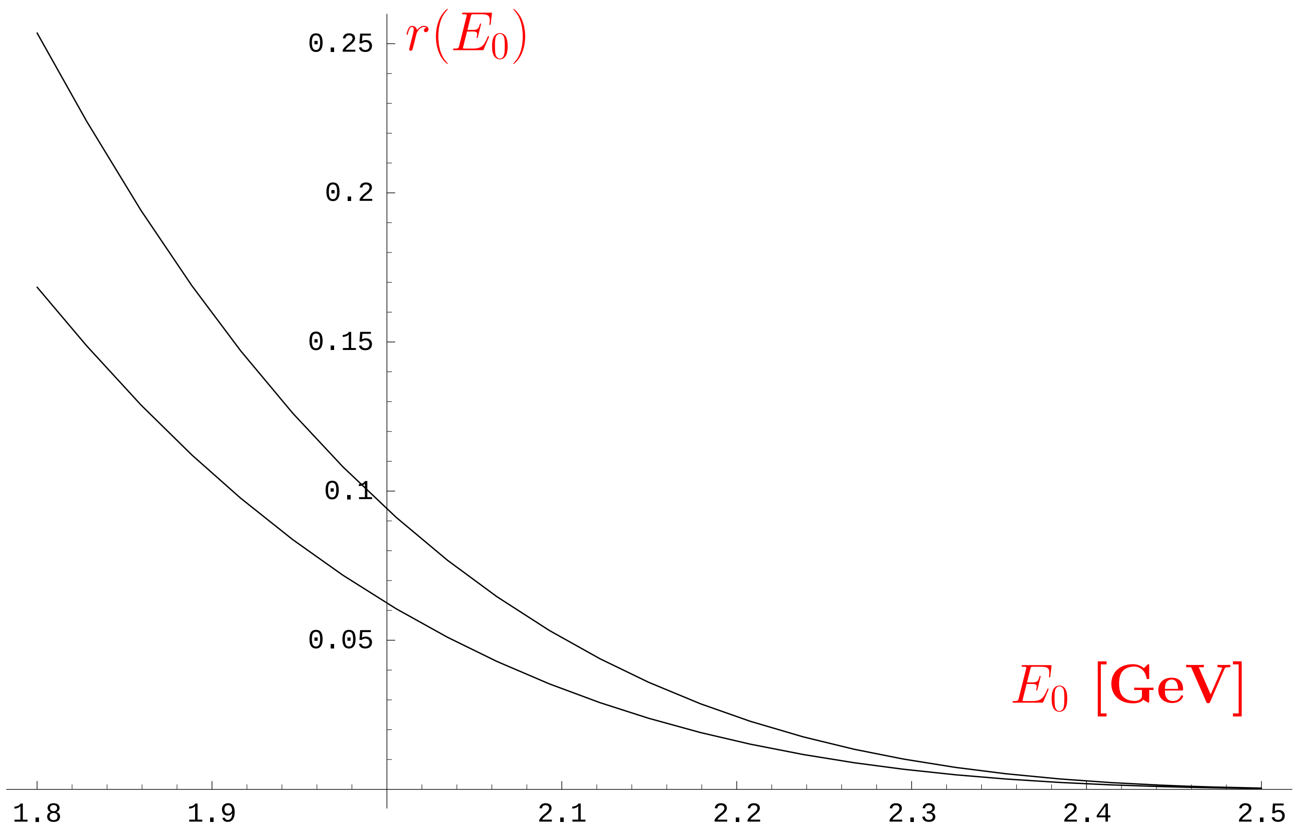
and the normalization constant C is fixed by the requirement

$$\int_{0.6}^{1} dx\; n(x) = 1.$$

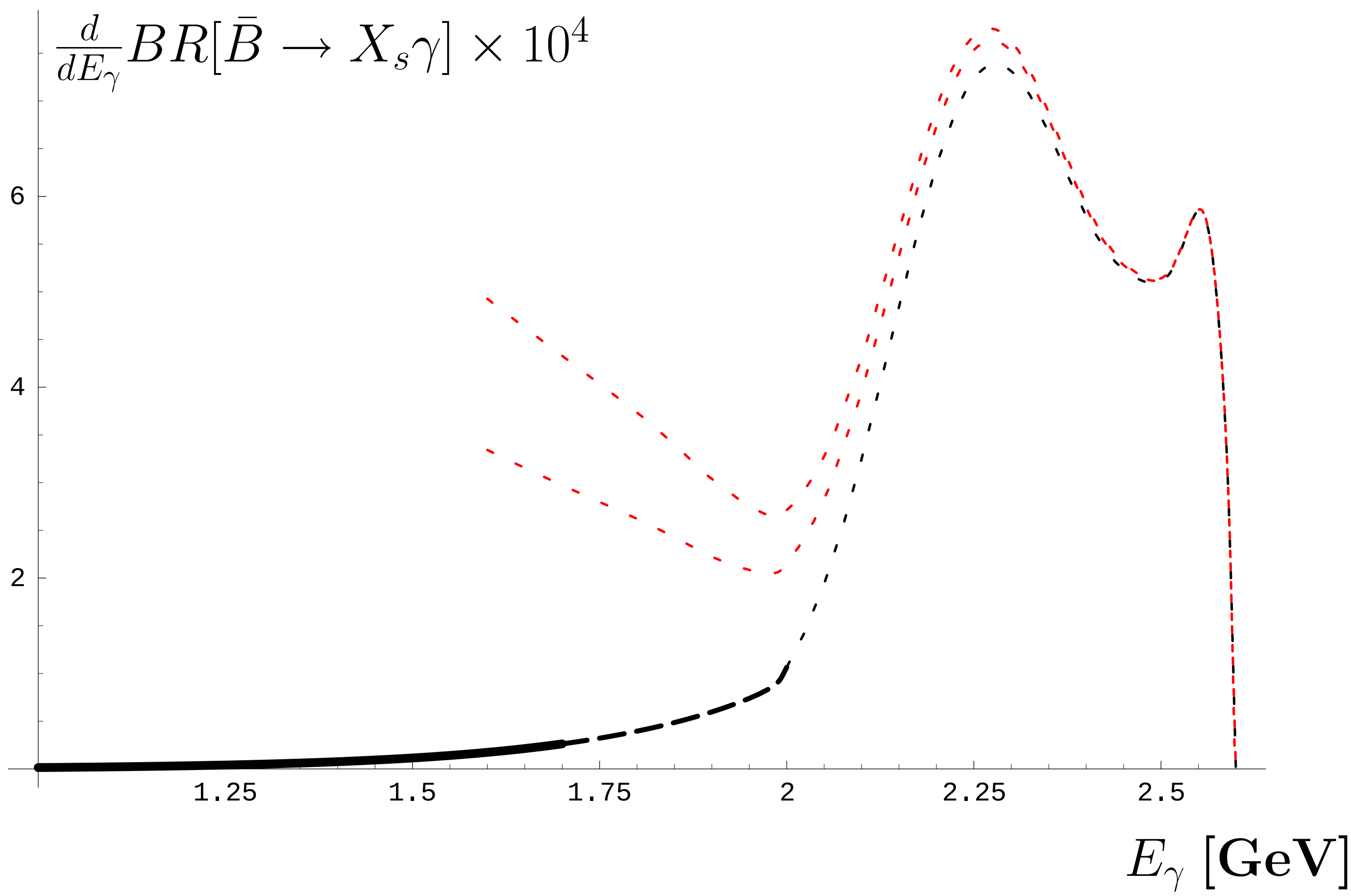
Knowing $b(x)$ for $x > 0.6$, one can calculate

$$r(E_0) = \frac{1}{3.15 \times 10^{-4}} \times$$
$$\times \left\{ BR[\bar{B} \to X^{(1)}_{\text{no charm}}\; \psi] \times BR[\psi \to X^{(2)}_{\text{no charm}}\; \gamma] \right\}_{E_\gamma > E_0}$$

for $E_0 > 0.3\, m_B \simeq 1.6$ GeV. The result is as follows:



The $\bar{B} \to X_s\gamma$ photon spectrum:

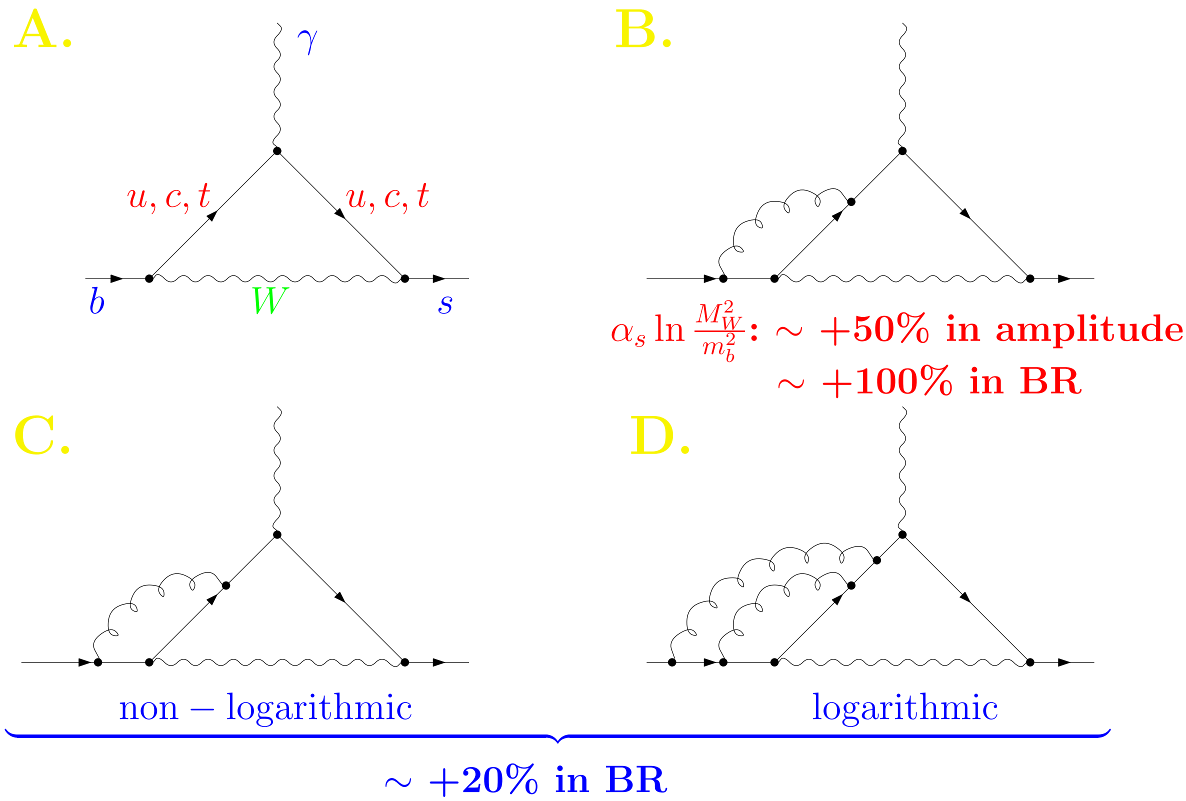


- - - with $\bar{B} \to X_s\psi$ followed by $\psi \to X'\gamma$.
- - - without " " " .

Present CLEO cut is $E_\gamma > 2.1GeV$. $\Rightarrow$ Strong sensitivity to unknown $\bar{B}$-meson shape function.

Lowering the cut to ∼1.6 GeV would practically remove the sensitivity to the shape function. However, a careful subtraction of the intermediate ψ contribution would become necessary.

Examples of Feynman diagrams contributing to $b \to s\gamma$ at various orders in the renormalization-group-improved perturbation theory:



A. [T. Inami and C.S. Lim, Prog. Theor. Phys. 65 (1981) 297],
B. [M. Ciuchini et al., Phys. Lett. B316 (1993) 127, Nucl. Phys. B421 (1994) 41],
[S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. 59 (1987) 180],
[N.G. Deshpande et al., Phys. Rev. Lett. 59 (1987) 183],
[B. Grinstein, R. Springer and M.B. Wise, Nucl. Phys. B339 (1990) 269],
[M.M., Phys. Lett. B269 (1991) 161, Nucl. Phys. B393 (1993) 23],
C. [C. Greub, T. Hurth and D. Wyler, Phys. Rev. D54 (1996) 3350],
[A. Ali and C. Greub, Z.Phys.C49 (1991) 431, Phys. Lett. B361 (1995) 146],
[N. Pott, Phys. Rev. D54 (1996) 938],
[P. Cho and B. Grinstein, Nucl. Phys. B365 (1991) 279],
[K. Adel and Y.P. Yao, Phys. Rev. D49 (1994) 4945],
[C. Greub and T. Hurth, Phys. Rev. D56 (1997) 2934],
[A.J. Buras, A. Kwiatkowski and N. Pott, Nucl. Phys. B517 (1998) 353],
D. [K. Chetyrkin, M.M. and M. Münz, Phys. Lett. B400 (1997) 206],
[M.M. and M. Münz, Phys. Lett. B344 (1995) 308].

The effective Lagrangian:

$$\mathcal{L} \;=\; \mathcal{L}_{QCD\times QED}(u,d,s,c,b) \;\; + \frac{4G_F}{\sqrt{2}} V_{ts}^{*} V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, & C_8(m_b) \sim -0.15 \end{cases}$$

Perturbative expansion of the Wilson coefficients:

$$C_i(\mu) \;=\; C_i^{(0)}(\mu) \;+ \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \frac{\alpha_{em}}{\alpha_s(\mu)} C_i^{em(0)}(\mu) \;+ \frac{\alpha_{em}}{4\pi \;\sin^2\theta_W} C_i^{ew(1)}(\mu) + \ldots$$

$C_7^{(1)eff}(m_b) \simeq +0.5 \;\Rightarrow\; \mathbf{-4.9}\%$ in BR [**C**, **D**]

$C_7^{em(0)eff}(m_b) \simeq +0.03 \;\Rightarrow\; \mathbf{-0.8}\%$ in BR [**E**]

$C_7^{ew(0)eff}(m_b) \simeq +1.9 \;\Rightarrow\; \mathbf{-1.6}\%$ in BR [**F**]

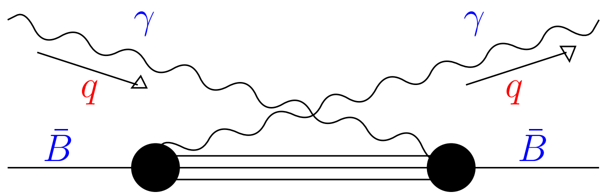
E. [K. Baranowski and M.M., Phys. Lett. B483 (2000) 410].
[A. Kagan and M. Neubert, Eur. Phys. J. C7 (1999) 5],

F. [P. Gambino and U. Haisch, CERN-TH-2000/211],
[A. Czarnecki and W. Marciano, Phys. Rev. Lett. 81 (1998) 277],
[A. Strumia, Nucl. Phys. B532 (1998) 28].

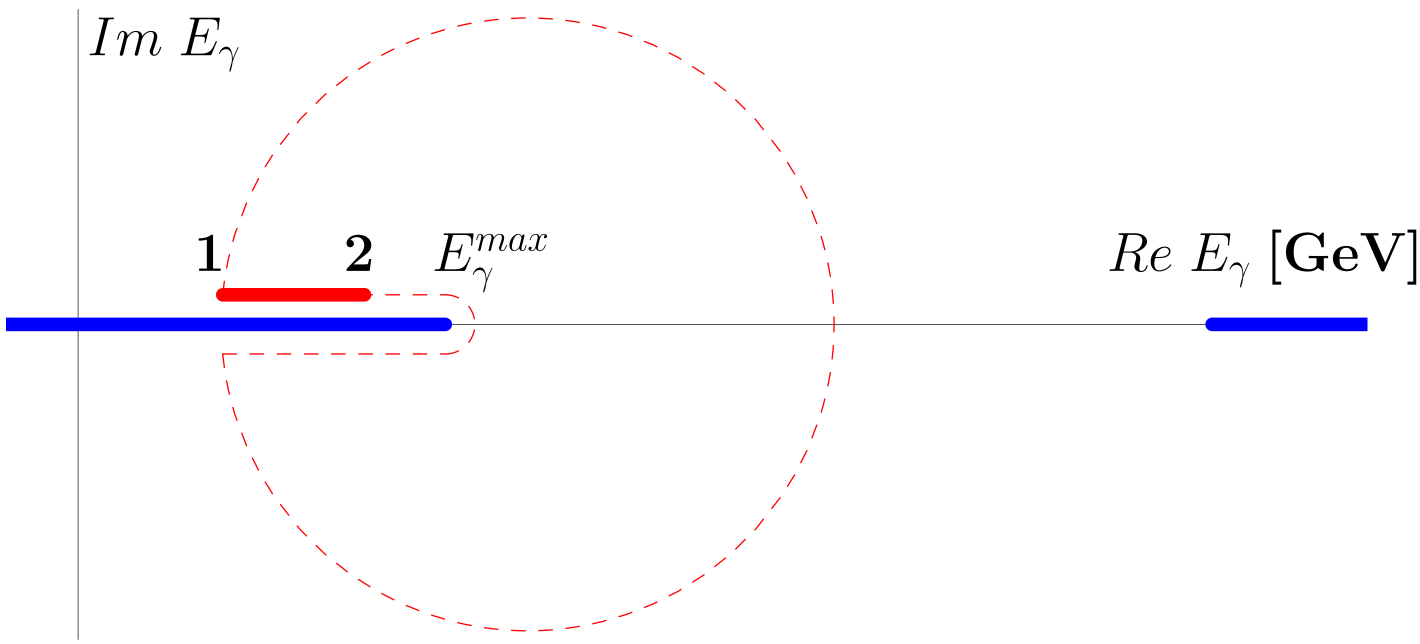
Once the Wilson coefficient are known, we need to find the $\bar{B} \to X_s\gamma$ amplitude, i.e. the matrix elements of the effective Hamiltonian. This can be done with help of OPE and HQET. We need to calculate:

$$\Sigma_{X_s} \left| C_7 \langle X_s\gamma | O_7 | \bar{B} \rangle + C_2 \langle X_s\gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$$

The "77" interference term can be related via optical theorem to the imaginary part of the elastic forward scattering amplitude:

$Im\{$  $\} \equiv Im\ A$

In this amplitude, we can perform OPE when the photons are soft enough, i.e. when $|m_B - 2E_\gamma| >> \Lambda_{QCD}$.



$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{max}} dE_\gamma\ E_\gamma^n\ Im\ A(E_\gamma) \sim \oint_{\mathrm{big\ circle}} dE_\gamma\ E_\gamma^n\ A(E_\gamma)$$

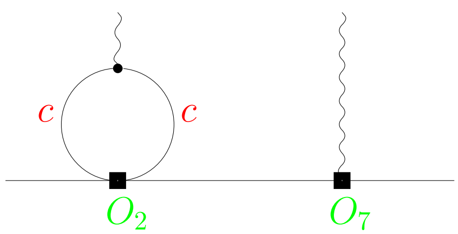
[J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B247 (1990) 399],
[A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D49 (1994) 3367].

We have a double expansion:

$$\Sigma_{X_s} BR[\bar{B} \to X_s \gamma]_{E_\gamma > 1 \ \text{GeV}} = \left[a_{00} + a_{02} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right] + \frac{\alpha_s(m_b)}{\pi} \left[a_{10} + a_{12} \left(\frac{\Lambda}{m_B} \right)^2 + \ldots \right] + \mathcal{O}\left[\left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right]$$

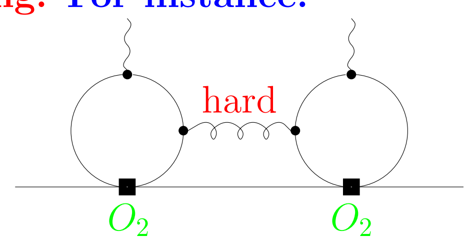
+ [Contributions other than the "77" interference term].

There is no OPE for the latter term. However, operators containing no charm quark are suppressed by their small Wilson coefficients. As far as the operators containing the charm quark are concerned, we know that their contribution at the leading order in α_s can be expressed as a power series:

$\langle \bar{B} |$ 

$| \bar{B} \rangle = \frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda}{m_c^2} \right)^n ,$

which can be truncated to the leading $n = 0$ term, because the coefficients b_n decrease fast with n. The calculable $n = 0$ term makes $BR[\bar{B} \to X_s \gamma]$ increase by around 3%. However, an analysis of non-perturbative effects in the matrix elements of O_1 and O_2 at $\mathcal{O}(\alpha_s)$ is missing. For instance:

$\langle \bar{B} |$ 

$| \bar{B} \rangle = A_{1-\text{loop}} + B_\psi + C_?,$

where $A_{1-\text{loop}}$ stands for the very small ($< 1\%$ in BR) one-loop perturbative contribution, B_ψ is a part of the intermediate ψ contribution, and $C_?$ denotes the remaining non-perturbative terms. $C_?$ would not be numerically important if it was either suppressed by $\Lambda/m_{c,b}$, or small for other reasons, or could be absorbed into the intermediate ψ contribution. Is any of those three possibilities realized?

Neglecting all the non-perturbative effects that arise at order $\mathcal{O}(\alpha_s(m_b))$, we can write:

$$\frac{\Gamma[\bar{B} \to X_s\gamma]^{\text{subtracted } \psi}_{E_\gamma > E_{\text{cut}}}}{\Gamma[\bar{B} \to X_c e\bar{\nu}_e]} \simeq \frac{\Gamma[b \to X_s\gamma]^{\text{perturbative NLO}}_{E_\gamma > E_{\text{cut}}}}{\Gamma[b \to X_c e\bar{\nu}_e]^{\text{perturbative NLO}}} \times$$

$$\times \left[1 + (\mathcal{O}(\Lambda^2/m_b^2) \simeq 1\%) + (\mathcal{O}(\Lambda^2/m_c^2) \simeq 3\%)\right].$$

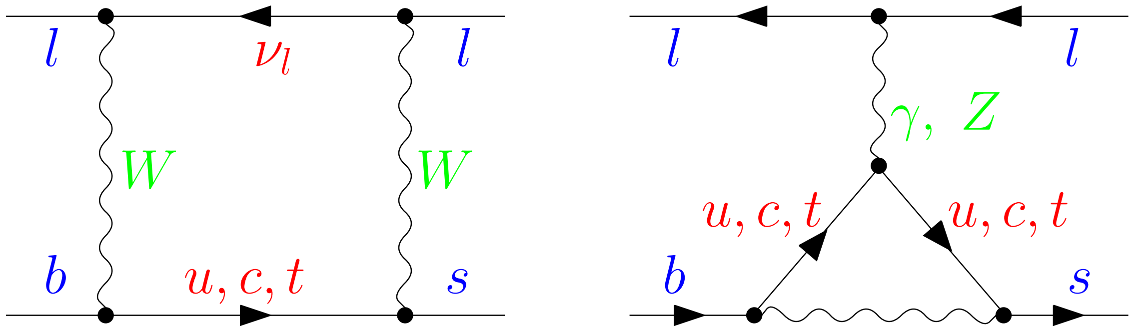
For $E_{\text{cut}} = 1$ GeV , one obtains:

$$BR[\bar{B} \to X_s\gamma]^{\text{subtracted } \psi}_{E_\gamma > E_{\text{cut}}} = (3.29 \pm 0.33) \times 10^{-4}.$$

The dominant errors originate from $\mathcal{O}(\alpha_s^2)$ effects, and from m_c/m_b in the semileptonic decay (around 7% each).

$E_{cut} = 1$ GeV is not accessible experimentally. We need the photon spectrum.

Electroweak transitions mediating $\bar{B} \to X_s l^+ l^-$:



In the effective Lagrangian, two operators need to be included, in addition to those already present in the $\bar{B} \to X_s \gamma$ analysis:

$$O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b) (\bar{l} \gamma^\mu l)$$

$$O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b) (\bar{l} \gamma^\mu \gamma_5 l)$$

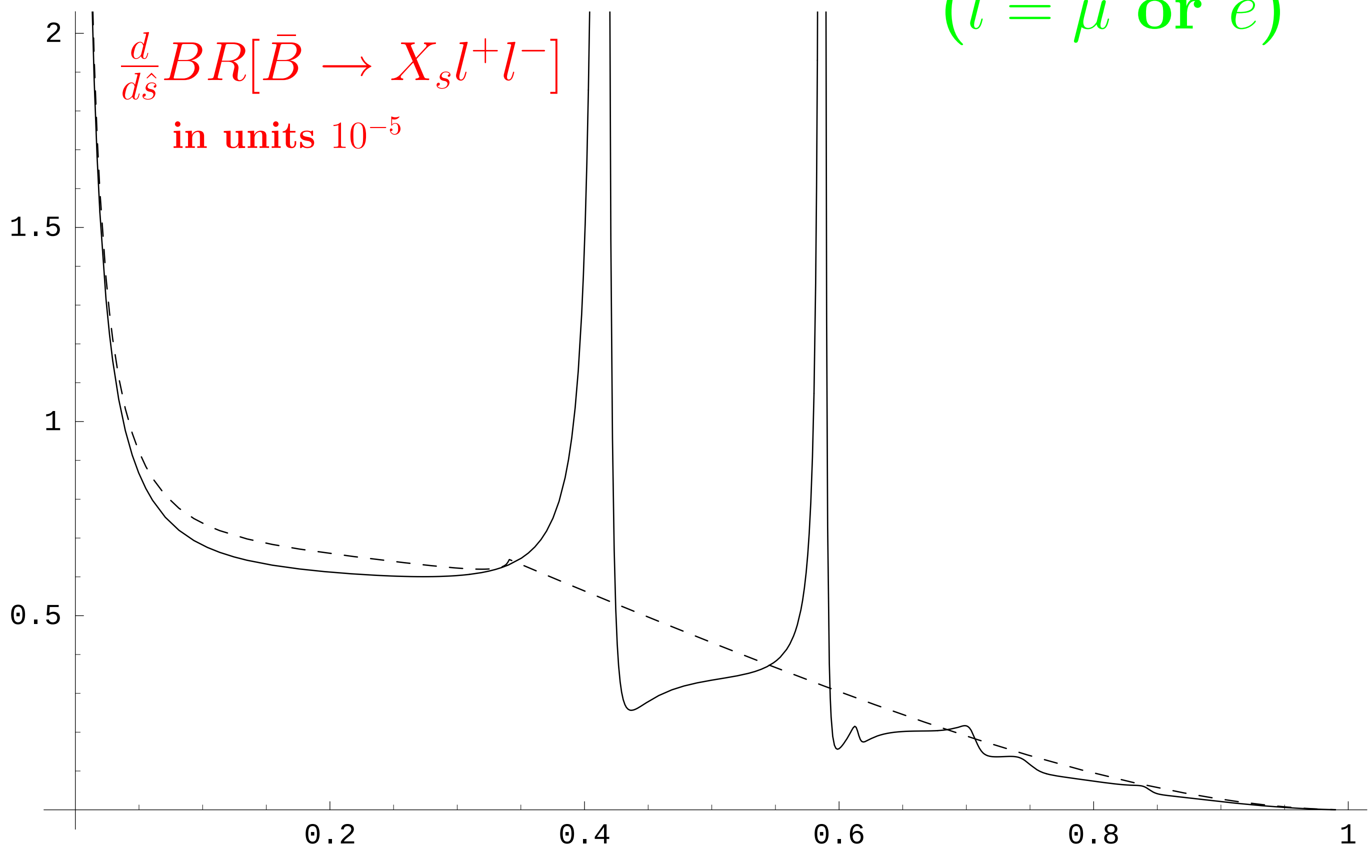
Their Wilson coefficients are relatively large:

$$C_9(m_b) \simeq +4.1$$

$$C_{10}(m_b) \simeq -4.2$$

Dilepton mass spectrum in $\bar{B} \to X_s l^+ l^-$.

($l = \mu$ or e)



$\hat{s} = \left(\frac{m_{l^+l^-}}{m_b}\right)^2$

- - - - - - - - - perturbative,

——— with inclusion of non-perturbative $c\bar{c}$ in the factorization approximation.

Region $\hat{s} \in [0.05, 0.25]$:

(i) Quite clean theoretically. However, the effect of $\bar{B} \to \psi X^{(1)}$ followed by $\psi \to X^{(2)} l^+ l^-$ should be studied.

(ii) Sensitive to new physics in a different way than $\bar{B} \to X_s \gamma$. For instance, when the coefficient $C_7^{eff}(m_b)$ changes sign, the integrated BR in this region changes from 1.5×10^{-6} to 3×10^{-6}. Thus, its sensitivity to the sign of C_7^{eff} is the same as that of leptonic forward-backward or energy asymmetries.

Perturbative expansion of C_9 and C_{10}:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

After a formal expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electroweak logarithm originating from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$

Numerically $C_9^{(-1)}(m_b) \simeq 0.033 << 1 \quad \Rightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$**.**

On the other hand, $C_9^{(0)}(m_b) \simeq 2.2$**. Consequently, the accuracy of** $\sim 10\%$ **in the Wilson coefficients can be achieved only after including the formally NNLO** $\mathcal{O}(\alpha_s)$ **terms** $\Rightarrow$ **2-loop matching and 3-loop RGE evolution, as in** $\bar{B} \to X_s \gamma$**.**

Present status:

$C_9^{(-1)}(m_b)$ **and** $C_{10}^{(0)}$ **are known.**

[B. Grinstein, M Savage and M.B. Wise, Nucl. Phys. B319 (1989) 271]

$C_9^{(0)}(m_b)$ **is known.**

[M.M, Nucl. Phys. B393 (1993) 23, B439 (1995) 461 (E)]
[A.J. Buras and M. Münz, Phys. Rev. D52 (1995) 186]

$C_{10}^{(1)}$ **is known.**

[G. Buchalla amd A.J. Buras, Nucl. Phys. B398 (1993) 285, B400 (1993) 225]
[M.M., J. Urban, Phys. Lett. B451 (1999) 161]

$C_9^{(1)}(m_b)$ **is known up to (small) 3-loop RGE effects.**

[C. Bobeth, M.M. and J. Urban, Nucl. Phys. B574 (2000) 291]

Unfortunately, 2-loop matrix elements of the 4-quark operators are not known...

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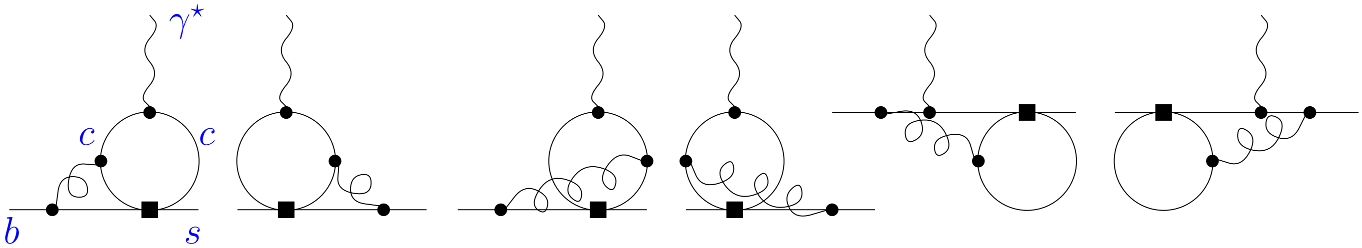


The $b \to s\gamma$ calculation of Greub, Hurth and Wyler has to be generalized to off-shell photons.

Present prediction:

$$BR[\bar{B} \to X_s l^+ l^-]_{\hat{s}\in[0.05,\, 0.25]}$$

$$= [\underbrace{(1.42 \pm 0.19)}_{\text{perturbative}} - \underbrace{0.02}_{\Lambda^2/m_c^2} + \underbrace{0.06}_{\Lambda^2/m_b^2}] \times 10^{-6}$$

$$= (1.46 \pm 0.19) \times 10^{-6},$$

where only the perturbative uncertainty from μ_b-dependence has been taken into account.

HQET corrections:

Λ^2/m_c^2:

[G. Buchalla, G. Isidori and S.-J. Rey, Nucl. Phys. B511 (1998) 594],
[J.-W. Chen, G. Rupak and M.J. Savage, Phys. Lett. B410 (1997) 285],

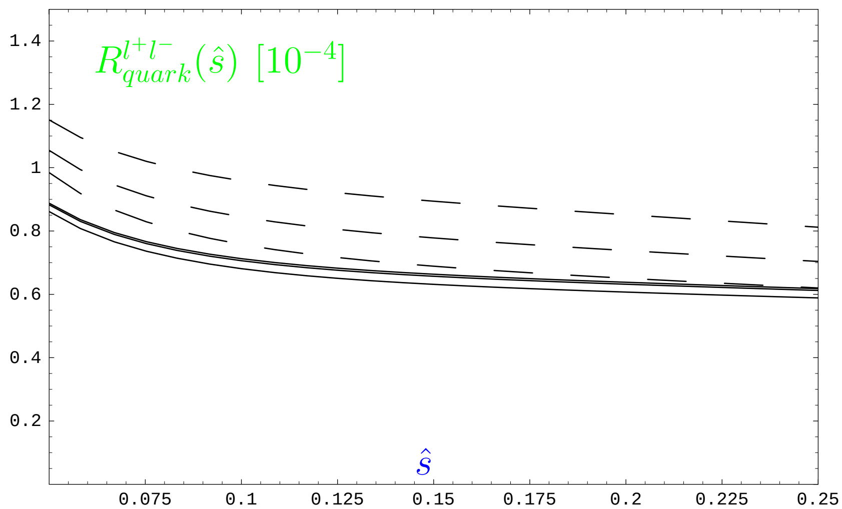
Λ^2/m_b^2:

[A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D49 (1994) 3367],
[G. Buchalla and G. Isidori, Nucl. Phys. B525 (1998) 333].

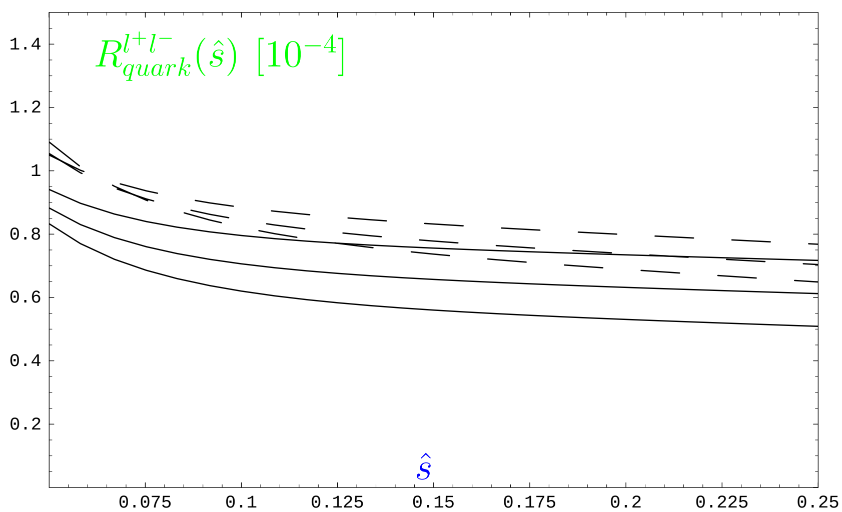
In those calculations, the quantity C_9^{eff} was treated as a Wilson coefficient of a local operator, which it is not (contrary to C_7^{eff} that is relevant for $B \to X_s\gamma$). An estimate of the accuracy of such an approximation is necessary.

Dilepton mass spectrum in $b \to X_s l^+ l^-$ for $\hat{s} = (m_{l^+l^-}/m_b)^2 \in [0.05, 0.25]$.

$$R^{l^+l^-}_{quark}(\hat{s}) = \frac{1}{\Gamma[b \to X_c e \bar{\nu}_e]} \frac{d}{d\hat{s}} \Gamma(b \to X_s l^+ l^-).$$



Reduction of μ_0-dependence of $R^{l^+l^-}_{quark}(\hat{s})$



Remaining μ_b-dependence of $R^{l^+l^-}_{quark}(\hat{s})$

[C. Bobeth, M.M. and J. Urban, Nucl. Phys. B574 (2000) 291]

Conclusions:

1. $\bar{B} \to X_s \gamma$

(i) Since the completion of NLO QCD calculation 4 years ago, many new calculations have been made. They include evaluation of non-perturbative Λ^2/m_c^2 corrections and the leading electroweak corrections. None of these results exceeds half of the overall $\sim 10\%$ uncertainty. They tend to cancel among each other. Therefore, the prediction for $BR[\bar{B} \to X_s \gamma]$ remains almost the same: $(3.29 \pm 0.33) \times 10^{-4}$.

(ii) This prediction agrees very well with the measurements of CLEO and ALEPH, whose combined result is $(3.14 \pm 0.48) \times 10^{-4}$.

(iii) Future measurements of $\bar{B} \to X_s \gamma$ should rely as little as possible on theoretical predictions for the precise shape of the photon spectrum above $E_\gamma \sim 2$ GeV. On the other hand, the intermediate ψ background should be carefully subtracted for E_γ below ~ 2 GeV.

(iv) A systematic analysis of non-perturbative effects at order $\mathcal{O}(\alpha_s)$ is missing. Most probably, they do not exceed the overall $\sim 10\%$ uncertainty when the energy cutoff is between 1 and 2 GeV, and when the intermediate $\psi^{(\prime)}$ contribution is subtracted.

2. $\bar{B} \to X_s l^+ l^-$

(i) A calculation of $\mathcal{O}(\alpha_s)$ terms in all the Wilson coefficients has been recently completed (up to small effects originating from 3-loop RGE evolution of C_9). However, the perturbative uncertainty remains close to $\sim 13\%$, because the 2-loop matrix elements of the 4-quark operators are unknown.

(ii) Good control over non-perturbative effects can be achieved in the region of low dilepton invariant mass ($\hat{s} \in [0.05,\ 0.25]$). The present prediction for the branching ratio integrated over this region is $(1.46 \pm 0.19) \times 10^{-6}$. However, a careful analysis of non-perturbative effects from intermediate $c\bar{c}$ states (including ψ) is necessary.