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ε'/ε in the Chiral Limit

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2. Technique
   * Matching and Scheme Dependence
3. Results and Conclusions
   * ε'/ε in χ-limit
   * Main corrections

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1. Introduction

CP - violation in Kaon Physics

$K^0 - \bar{K}^0$ mixing : Indirect

$$\varepsilon \equiv \frac{A[K_L \to (\pi\pi)_0]}{A[K_S \to (\pi\pi)_0]}$$

$K \to \pi\pi$: Direct

$$\varepsilon'/\varepsilon \equiv \frac{1}{\sqrt{2}}\left[\frac{A[K_L \to (\pi\pi)_2]}{A[K_L \to (\pi\pi)_0]} - \frac{A[K_S \to (\pi\pi)_2]}{A[K_S \to (\pi\pi)_0]}\right]$$

We need $A[K \to \pi\pi]$.

In isospin symmetry limit,

$$A[K^0 \to \pi^0\pi^0] = \frac{1}{\sqrt{3}} A_0 - \sqrt{\frac{2}{3}} A_2$$

$$A[K^0 \to \pi^+\pi^-] = \frac{1}{\sqrt{3}} A_0 + \frac{1}{\sqrt{6}} A_2$$

$$A[K^+ \to \pi^+\pi^0] = \frac{\sqrt{3}}{2} A_2 .$$

$$A_I \equiv -i\, a_I\, e^{i\delta_I}$$

δ_I : strong phases from Final State Interactions

$\Delta S = 1$ Lowest Order CHPT

$$\mathcal{L}^{(2)}_{\Delta S=1} = \frac{-3}{5}\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \; F_0^4 \; *$$

$$\Big\{ e^2 G_E F_0^2 \langle \Delta_{32} u^\dagger Q u \rangle$$

$$+ G_8 \langle \Delta_{32} u^\mu u_\mu \rangle + G_8' \langle \Delta_{32} \chi_+ \rangle$$

$$+ G_{27} t^{ij,kl} \langle u_\mu \Delta_{ij} \rangle \langle u^\mu \Delta_{kl} \rangle \Big\}$$

$$U \equiv e^{\frac{i\sqrt{2}\Phi}{F_0}} \equiv u u \; ; \quad u^\mu \equiv i u^\dagger (D^\mu U) u$$

$\Delta_{ij} \sim \delta_{ij}$; Q : e.m. quark charges

$$\chi_+ \equiv 2 B_0 \{ u^\dagger \mathcal{M} u^\dagger + u \mathcal{M}^\dagger u \}$$

$$a_0 \equiv \frac{\sqrt{6}}{9} C F_0 \left[(9 G_8 + G_{27})(m_K^2 - m_\pi^2) - 6 e^2 G_E F_0^2 \right]$$

$$a_2 \equiv \frac{\sqrt{3}}{9} C F_0 \left[10\, G_{27} (m_K^2 - m_\pi^2) - 6 e^2 G_E F_0^2 \right]$$

$$\delta_0 = \delta_2 = 0$$

Large N_c : $G_8 = G_{27} = 1$

$$G_E F_0^2 = 0$$

➔ Calculate G_E, G_8, and G_E at next-to-leading order in $1/N_c$.

2. Technique

Matching Short-Distances

$\Gamma_{\Delta S=1}$ is generated by virtual W- exchange.

$[0,\infty[$

R.G. + Perturbative Matching at thresholds

$$\sum_{i=1}^{10} C_i(\nu)\, Q_i(x)$$

NDR, HV, ...
Choice of operators, ..

Short-Distance Matching

$\Gamma_{LD}(g_i, \mu_c, \dots)$

* Different degrees of freedom
* Different schemes

...

D=4

Long-Distance

$$\langle 2 | \Gamma_{LD}(g_i, \mu_c, \dots) | 1 \rangle = \langle 2 | \sum_{i=1}^{10} C_i(\nu)\, Q_i(x) | 1 \rangle$$

asympotic states

Fixes analytically short-distance behaviour of Γ_{LD} couplings g_i .

$$g_i(\mu_c, \cdots) = \mathcal{F}(G_i(\nu), \alpha_s(\nu), \cdots)$$

* Large $\alpha_s^n \log^n\left(\frac{M_W}{\nu}\right)$ summed at NLO.

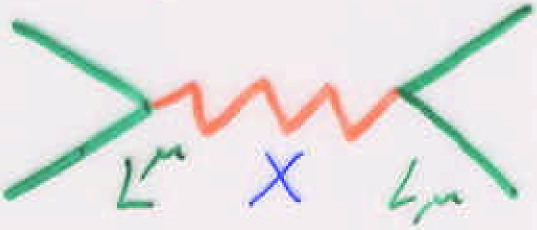
{ A. Buras et al.
M. Ciuchini et al.
HV, NDR, ...

* Scale ν and Scheme independence to $\mathcal{O}(\alpha_s^2)$

Our approach

Effective field theory of heavy $\sim \frac{1}{p^2 - M_X^2}$ color-singlet X-bosons coupled to QCD currents and densities.

& $D=4$ Euclidean cut-off μ_c.



$$2L_\mu^{sd} \equiv [\bar{s}\gamma_\mu(1-\gamma_5)d]$$

$$\Gamma_{LD} \equiv 2g_1(\mu_c, \ldots) \int d^4y\, X_1^\mu (L_\mu^{sd} + L_\mu^{uu})$$

$$+ \cdots$$

Reproduces the physics of $\Gamma_{\Delta S=1}$ below some scale μ_c to $\mathcal{O}(1/M_X^2)$

Calculate

$$\Pi_{ij}(q^2) \equiv i \int d^4x \, e^{iqx} \langle 0 | T(P_i^{\dagger}(0) P_j(x) e^{i\Gamma_{LD}}) | 0 \rangle$$

in the presence of strong and e.m. interactions.

$$P_{K^+}(x) \equiv [\bar{s} i \gamma_5 u] ; \ldots$$

Taylor expanding in q^2 one can obtain the CHPT couplings

$G_8, G_{27}, G_8', G_E, \ldots$

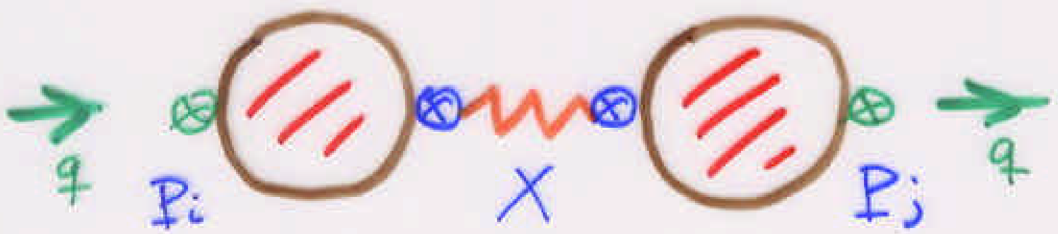
$\hookrightarrow$ Prediction for $\Delta I = 1/2$ and ε'/ε

C. Bernard et al $\mathcal{O}(p^2)$

J. Bijnens et al $\mathcal{O}(p^4)$

$1/N_c$ Expansion Analysis

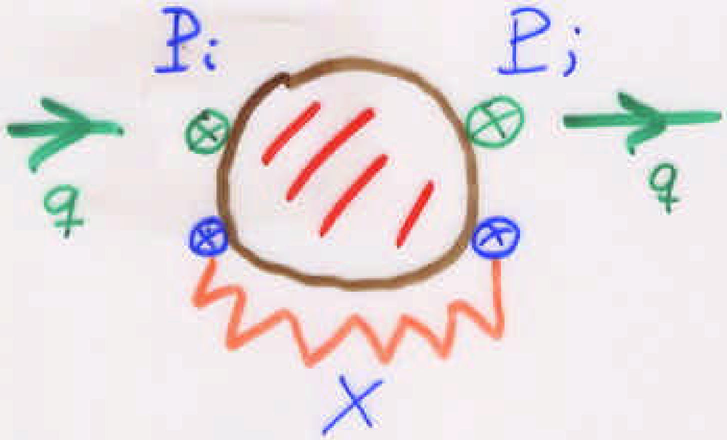
L.O.



$\mathcal{O}(N_c^2)$: Factorizable

Model independent!

N.L.O.



$\mathcal{O}(N_c)$: Non-Factorizable

$$\Pi(q^2) \sim i \int \frac{d^4 r_E}{(2\pi)^4}\, g_{\mu\nu}\, \Pi^{\mu\nu}_{PPLL}(q_E, r_E)$$

$$\int_0^\infty d|r_E| \equiv \underbrace{\int_0^\mu d|r_E|}_{\text{Long-Distance}} + \underbrace{\int_\mu^\infty d|r_E|}_{\text{Short-Distance}}$$

Short-Distance part is under control at NLO in $1/N_c$!

[μ high enough to use OPE QCD]

Remains the Long-Distance part

$$\int_0^\mu d|r_E|\, r_E^3\, \Pi^{\mu\nu}_{PPLL}(q_E, r_E)$$

At very low μ we use CHPT,
To go beyond we use a good hadronic model as 1st step

ENJL Model

Good Features
* Reproduces CHPT
* Correct χ-symmetry
* Good Phenomenology

Drawbacks
* Doesn't confine
* Does not have all QCD short-distance constraints

* S. Peris, M. Perrottet, E. de Rafael

Large N_c QCD: M. Knecht, S. Peris, M. Perrottet E. de Rafael

3. Results

$$\left|\varepsilon'/\varepsilon\right| \simeq \frac{1}{\sqrt{2}\,|\varepsilon|} \frac{\mathrm{Re}\, a_2}{\mathrm{Re}\, a_0} \left\{ \frac{-\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0} + \frac{\mathrm{Im}\, a_2}{\mathrm{Re}\, a_2} \right\}$$

(8)

$|\varepsilon|_{exp}$ or $|\varepsilon|_{th}$ ✓

For Re a_0 and Re a_2 :
We take the L.O. CHPT values from a fit to experimental amplitudes to $\mathcal{O}(p^4)$

Kambor, Missimer, Wyler

→ Predict imaginary parts of CHPT couplings.

In SM, they are all proportional to $\mathrm{Im}\,\tau \equiv -\mathrm{Im}\left[\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\right]$

Im G_8 almost all from Q_6

L.O.		NLO
$(2.0 \pm ?)\,\mathrm{Im}\,\tau$	→	$(5.1 \pm 1.5)\,\mathrm{Im}\,\tau$

Im $e^2 G_E$ almost all from Q_8

L.O.		NLO
$-(3.9 \pm ?)\,\mathrm{Im}\,\tau$	→	$-(5.2 \pm 1.0)\,\mathrm{Im}\,\tau$

* Leading in $1/N_c$ contributions from Q_6 and Q_8 are <u>scale</u> independent.

{E. de Rafael
{A. Buras, J.-M. Gérard

→ <u>Next-to-Leading</u> in $1/N_c$ Long-Distance contribution to G_E from Q_8 <u>vanishes</u> in χ-limit

<u>Model independent result</u>!

$B_8 \simeq 1$ at NLO

At <u>Lowest Order</u> <u>CHPT</u> and <u>next-to-leading</u> in <u>$1/N_c$</u>!

$$\left|\varepsilon'/\varepsilon\right|_{\chi} = [\underset{a_0}{8.3} - \underset{a_2}{2.2}] \cdot 10^{-3} = [6 \pm 3] \cdot 10^{-3}$$

→ <u>Chiral Corrections</u>

1. Final State Interactions

$$\frac{\mathrm{Re}\, a_0}{\mathrm{Re}\, a_2} = 22.2 \pm 0.1 \qquad (29 \pm 5)$$

Gardner, Valencia

$\frac{\mathrm{Im}\, a_I}{\mathrm{Re}\, a_I}$ has no FSI to all orders

2. Isospin Breaking

Only π^0-η mixing under control ✓

$$\frac{[\mathrm{Im}\, a_2]_{\pi^0\text{-}\eta}}{\mathrm{Re}\, a_2} = [0.16 \pm 0.03] \, \frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0}$$

G. Ecker et al

$O(p^4)$ Constants Unknown !

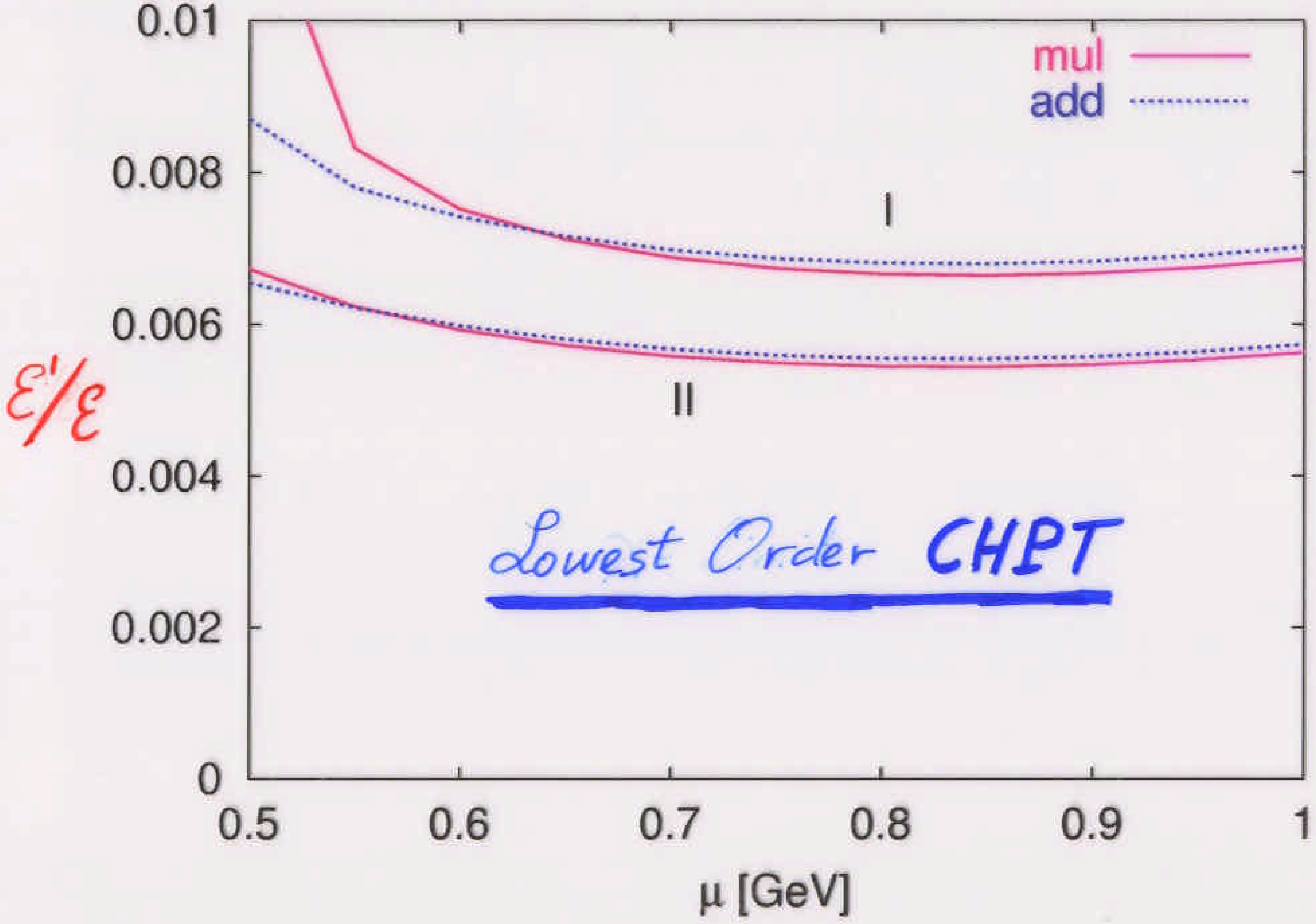
3. Purely real CHPT corrections

Only real part of loop diagrams known.

$O(p^4)$ Constants Unknown !

$$\left| \frac{\varepsilon'}{\varepsilon} \right|_{th} = [3.4 \pm 1.8] \cdot 10^{-3}$$

Exp. $[1.92 \pm 0.46] \cdot 10^{-3}$



mul
add
I
II
Lowest Order CHPT
0.01
0.008
0.006
0.004
0.002
0
0.5
0.6
0.7
0.8
0.9
1
μ [GeV]
ε'/ε