

**Coherent exclusive exponentiation  
for precision Monte Carlo  
calculations of fermion pair  
production / Precision predictions  
for (un)stable  $W^+W^-$  pairs**

**July 27, 2000**

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**Outline:**

- **Introduction**
- **KKMC**
- **Results: CEEX**
- **YFSWW3-1.14**
- **Results: YFSWW3-1.14**
- **Conclusions**

*with S. Jadach, W. Placzek, M. Skrzypek and Z. Was*

## The CEEEX solution: KKMC

Documented in CERN-TH-99-235, DESY-99-106, UTHEP-99-0901, CERN-TH-2000-087, UTHEP-99-0901

- ISR/FSR interferences are included in a natural way.  
Spin amplitudes for ISR and FSR summed squared numerically.  
(The BHWIDE approach gets cumbersome beyond first order)
- Exact treatment of fermion spin polarizations (transv.&longitudinal).  
Numerical Wigner rotations of spin density matrices available.  
(Necessary for interfacing with decay M.C. simulating fermion decays).
- Exact matrix element for 2 and 3 and more photons using  
Kleiss-Stirling (KS) spinor technique.

## The EEX solution: YFSWW3-1.14

Application of YFS3(PLB274,470(1992)) to the WW pair production and decay, YFSWW3, PLB417,326(1998), CERN-TH-99-222, PRD61 (2000) 113010, UTHEP-00-0101, hep-ph/0007012: **Exclusive Exponentiation** (EEX) technique, as implemented in KORALZ/YFS3, BHLUMI, BHWIDE, KORALW:

- Exact  $\mathcal{O}(\alpha)$  YFS exponentiation of production process
- FSR treated via PHOTOS to  $\mathcal{O}(\alpha^2)$  LL  
Finite  $P_T$  in FSR correct for soft limit in  $\mathcal{O}(\alpha)$
- Ratio of BR's used to correct decay rate through  $\mathcal{O}(\alpha)$

### Comparison with Other Calculations

KKMC

- **EEEX**:  $\mathcal{O}(\alpha^3)$  LL (KORALZ4.X-type)
- **IEEX**:  $\mathcal{O}(\alpha^2)_{prag}$
- **ZFITTER 6.21**:  $\mathcal{O}(\alpha^3)$  LL, Bardin *et al.*, hep-ph/9908433

YFSWW3-1.14

- RacoonWW, Dittmaier *et al.*

Exact  $\mathcal{O}(\alpha)$  LPA, complete  $\mathcal{O}(\alpha) e^+e^- \rightarrow 4f + \gamma$

Soft Photon KF exponentiation for  $\mathcal{O}(\alpha^3)$  LL ISR via structure fns.

hep-ph/9912261, 9912290, 9912447; Phys. Lett. **B475** (2000) 127; BI-TP  
2000/06, hep-ph/0006307

- Beenakker *et al.* **SEMIANALYTICAL APPROACH**

Exact  $\mathcal{O}(\alpha)$  LPA, **NO HIGHER ORDER RESUMMATION**

hep-ph/9902333, 9811481

KKMC
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**Main Difference: Old EEX/YFS vs CEEEX for**

$$e^-(p_1, \lambda_1) + e^+(p_2, \lambda_2) \rightarrow$$

$$f(q_1, \lambda'_1) + \bar{f}(q_2, \lambda'_2) + \gamma(k_1, \sigma_1) + \dots + \gamma(k_n, \sigma_n).$$

(1)

**EEX total cross section**

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} e^{Y(m_\gamma)} D_n(q_1, q_2, k_1, \dots, k_n)$$

(2)

$\mathcal{O}(\alpha^1)$  distributions for  $n_\gamma = 0, 1, 2$

$$D_0 = \bar{\beta}_0$$

$$D_1(k_1) = \bar{\beta}_0 \tilde{S}(k_1) + \bar{\beta}_1(k_1)$$

(3)

$$D_2(k_1, k_2) = \bar{\beta}_0 \tilde{S}(k_1) \tilde{S}(k_2)$$

$$+ \bar{\beta}_1(k_1) \tilde{S}(k_2) + \bar{\beta}_1(k_2) \tilde{S}(k_1)$$

Real Soft Factors as usual

$$\begin{aligned}
 4\pi \tilde{S}(k) &= \sum_{\sigma} |5_{\sigma}(k)|^2 = |5_{+}(k)|^2 + |5_{-}(k)|^2 \\
 &= -\frac{\alpha}{\pi} \left( \frac{q_1}{kq_1} - \frac{q_2}{kq_2} \right)^2.
 \end{aligned} \tag{4}$$

**Important:** the IR-finite building blocks

$$\begin{aligned}
 \tilde{\beta}_0 &= \sum_{\lambda} |\mathcal{M}_{\lambda}|^2, \\
 \tilde{\beta}_1(k) &= \sum_{\lambda\sigma} |\mathcal{M}_{\lambda\sigma}^{1\text{-phot}}|^2 - \sum_{\sigma} |5_{\sigma}(k)|^2 \sum_{\lambda} |\mathcal{M}_{\lambda}^{\text{Born}}|^2
 \end{aligned} \tag{5}$$

in the multiphoton distributions are all in terms of  $\sum_{spin} |\dots|^2$ !! Here,  $\lambda =$  fermion helicities and  $\sigma =$  photon helicity.

Contrast: the analogous  $\mathcal{O}(\alpha^1)$  case of CEEK

$$\sigma = \sum_{n=0}^{\infty} \sum_{m_\gamma} \int d\Phi_{n+2}$$

$$\sum_{\lambda, \sigma_1, \dots, \sigma_n} |e^{B(m_\gamma)} \mathcal{M}_{n, \sigma_1, \dots, \sigma_n}^\lambda(k_1, \dots, k_n)|^2 \quad (6)$$

Differential Distributions for  $n_\gamma = 0, 1, 2$  photons:

$$\mathcal{M}_0^\lambda = \hat{\beta}_0^\lambda, \quad \lambda = \text{fermion helicities,}$$

$$\mathcal{M}_{1, \sigma_1}^\lambda(k_1) = \hat{\beta}_{0\mathbf{5}\sigma_1}^\lambda(k_1) + \hat{\beta}_{1, \sigma_1}^\lambda(k_1),$$

$$\mathcal{M}_{2, \sigma_1, \sigma_2}^\lambda(k_1, k_2) = \hat{\beta}_{0\mathbf{5}\sigma_1}^\lambda(k_1) \mathbf{5}_{\sigma_2}(k_2)$$

$$+ \hat{\beta}_{1, \sigma_1}^\lambda(k_1) \mathbf{5}_{\sigma_2}(k_2) + \hat{\beta}_{1, \sigma_2}^\lambda(k_2) \mathbf{5}_{\sigma_1}(k_1) \quad (7)$$

## IR-finite building blocks

$$\hat{\beta}_{1,0}^\lambda = (e^{-B} \mathcal{M}_\lambda^{\text{Born+Virt.}}) \Big|_{\mathcal{O}(\alpha^1)}, \quad (8)$$

$$\hat{\beta}_{1,\sigma}^\lambda(k) = \mathcal{M}_{1,\sigma}^\lambda(k) - \hat{\beta}_{1,0}^\lambda \mathcal{M}_\sigma(k).$$

Explicitly, this time everything is in terms of  $\mathcal{M}$ -spin-amplitudes! THE BASIC DIFFERENCE BETWEEN EEXYFS AND CEEEX.

Complete expressions for spin amplitudes with CEEEX exponentiation,  $n_\gamma$  arbitrary, in *Phys. Lett. B449, 97 (1999)* for the  $\mathcal{O}(\alpha^1)$  case and in *CERN-TH/2000-087, UTHEP-99-09-01*, for the  $\mathcal{O}(\alpha^2)$  case, all based on GPS spinor conventions in *CERN-TH-98-235, hep-ph/9905452*.

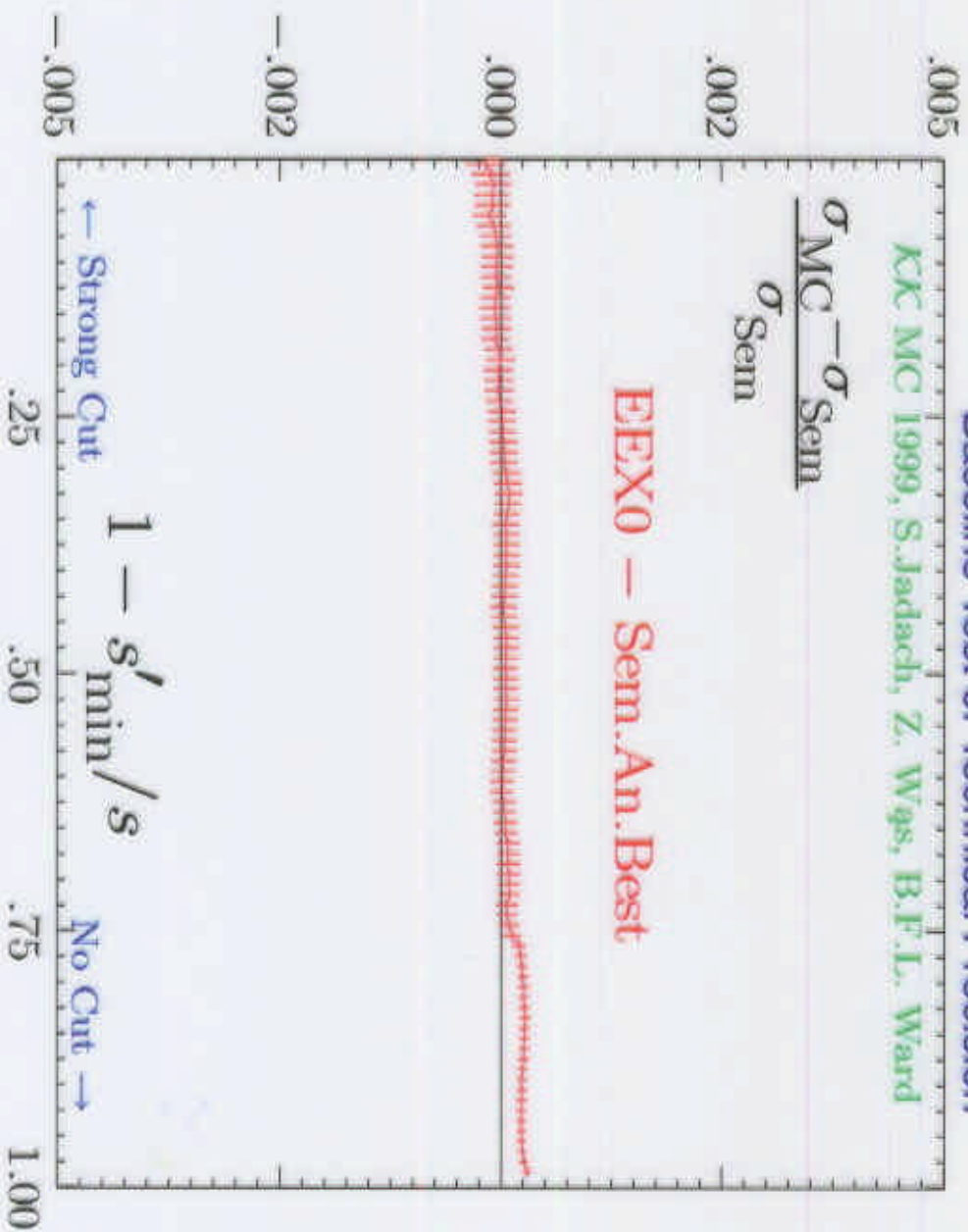
## RESULTS: CEEEX

### Baseline Test of Technical Precision

KK MC 1999, S.Jadach, Z. Wgs, B.F.I. Ward

$$\frac{\sigma_{MC} - \sigma_{Sem}}{\sigma_{Sem}}$$

**EEX0 - Sem.An.Best**



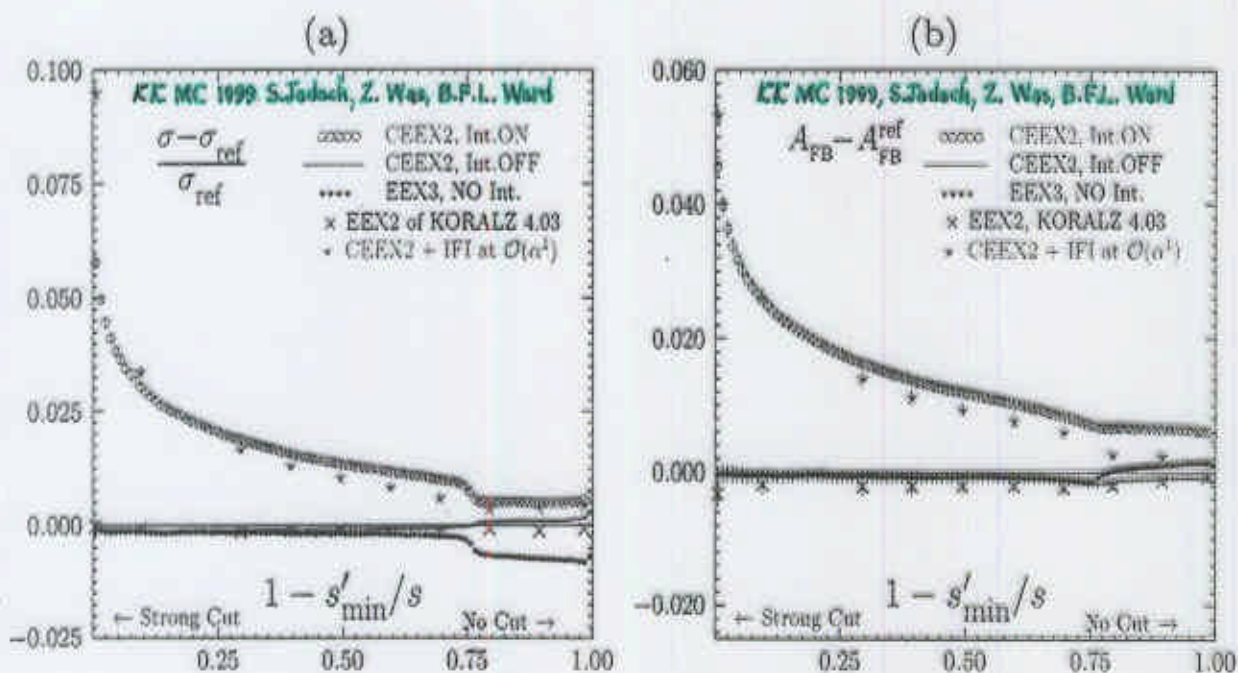
$\Rightarrow$  0.02% TTU,  $\psi < 0.999$ , etc.



## ICHEP 2000

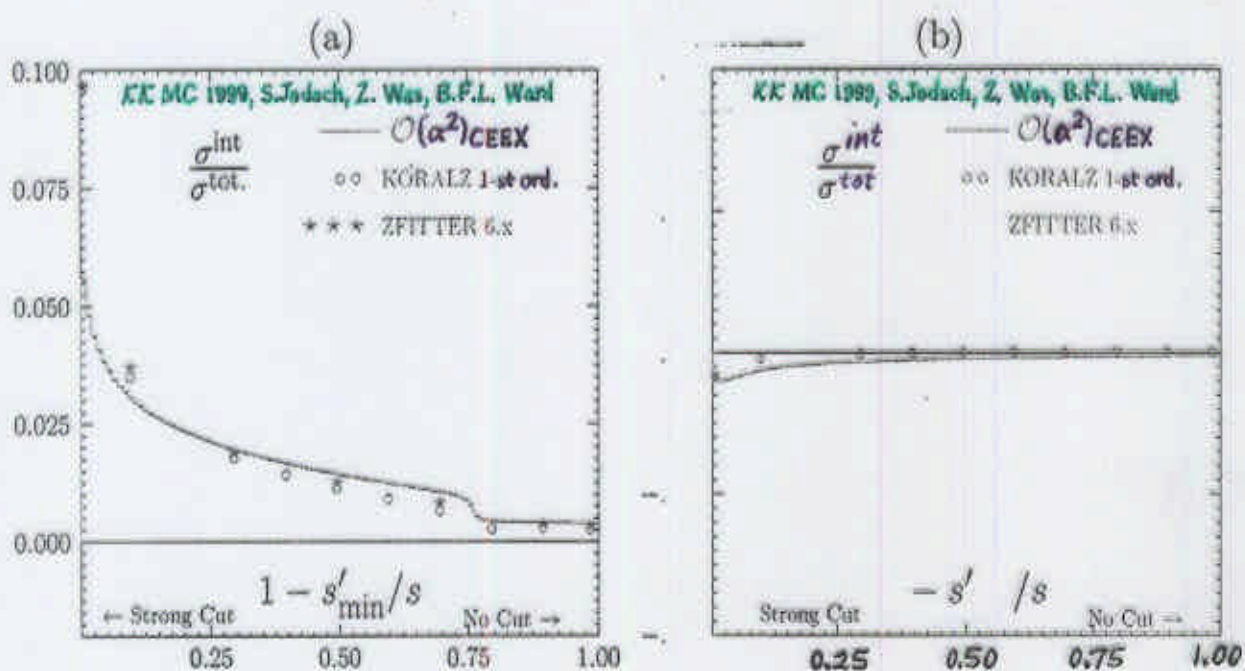
## III-2

## Physical Precision Tests



Absolute predictions for  $\sigma_{\text{tot}}$ ,  $A_{\text{FB}}$ :  $\mu, \bar{\mu}$ , 189 GeV  
 $\Rightarrow$  0.2% (0.2 - 0.4%) TU for  $\sigma_{\text{tot}}$  ( $A_{\text{FB}}$ ) at LEP2, etc

## Physical Precision Tests



$s'$ -cut dependence of  $\delta\sigma$ , No  $\theta$  cut: (a), 189 GeV  
 (b),  $M_Z \Rightarrow$  IFI  $\approx 1.5\%$  for energy cut 0.3,  
 $|\cos\theta| < 0.9$  reduces IFI by 25%,  
 IFI very small at  $Z$  return, etc.

**YFSWW3-1.14**

**Process of interest**

$$e^-(p_1) + e^+(p_2) \rightarrow f_1(r_1) + \bar{f}_2(r_2) + f'_1(r'_1) + \bar{f}'_2(r'_2) + \gamma(k_1), \dots, \gamma(k_n)$$

$$\sigma_n = \frac{1}{flux} \int d\tau_{n+4}(p_1 + p_2; r_1, r_2, r'_1, r'_2, k_1, \dots, k_n) \sum_{ferm. spin} \sum_{phot. spin} |M_{4f}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n)|^2$$

(9)

**$W^+W^-$  production and decay**

$$e^-(p_1) + e^+(p_2) \rightarrow W^-(q_1) + W^+(q_2),$$

$$W^-(q_1) \rightarrow f_1(r_1) + \bar{f}_2(r_2), \quad W^+(q_2) \rightarrow f'_1(r'_1) + \bar{f}'_2(r'_2),$$

$$\sigma_n = \frac{1}{\text{flux}} \int d\tau_{n+4}(p_1 + p_2; r_1, r_2, r'_1, r'_2, k_1, \dots, k_n)$$

$$\sum_{\text{ferm. spin}} \sum_{\text{phol. spin}} |\mathcal{M}_{LPA}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n)|^2$$

(10)

**LPA<sub>a,b</sub>:**

$$\begin{aligned}
 & M_{4f}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n) \Rightarrow LPA \ M_{LPA}^{(n)}(p_1, p_2, r_1, r_2, r'_1, r'_2, k_1, \dots, k_n) \\
 & = \sum_{\text{Phot. Partitions}} M_{\text{Prod}}^{(n), \lambda_1 \lambda_2}(p_1, p_2, q_1, q_2, k_1, \dots, k_a) \\
 & \quad \times \frac{1}{D(q_1)} M_{\text{Dec1}, \lambda_1}^{(n)}(q_1, r_1, r_2, k_{a+1}, \dots, k_b) \\
 & \quad \times \frac{1}{D(q_2)} M_{\text{Dec2}, \lambda_2}^{(n)}(q_2, r'_1, r'_2, k_{b+1}, \dots, k_n), \\
 & D(q_i) = q_i^2 - M^2, \quad M^2 = (M_W^2 - i\Gamma_W M_W)(1 - \Gamma_W^2/M_W^2 + \mathcal{O}(\alpha^3)), \\
 & q_1 = r_1 + r_2 + k_{a+1} + \dots + k_b; \quad q_2 = r'_1 + r'_2 + k_{b+1} + \dots + k_n,
 \end{aligned} \tag{11}$$

$\Rightarrow$  **LPA<sub>a,b</sub>:** Edén et al., Stuart, hep-ph/9706431, etc.

$$M = \sum_j \ell_j A_j(\{q_k q_l\}), \tag{12}$$

We do both.

Standard YFS Methods (EEX-Type)  $\Rightarrow$

$$d\sigma = e^{2\Re\alpha B' + 2\alpha\tilde{B}} \frac{1}{(2\pi)^4}$$

$$\int d^4y e^{iy(p_1+p_2-q_1-q_2)+D} [\bar{\beta}_0 + \sum_{n=1}^{\infty} \frac{d^3k_j}{k_j^0} e^{-iyk_j} \bar{\beta}_n(k_1, \dots, k_n)] \quad (13)$$

$$\times \frac{d^3r_1}{E_1} \frac{d^3r_2}{E_2} \frac{d^3r'_1}{E'_1} \frac{d^3r'_2}{E'_2},$$

where

$$D = \int \frac{d^3k}{k_0} \tilde{S} [e^{-iy \cdot k} - \theta(K_{max} - |\vec{k}|)] \quad (14)$$

$$2\alpha\tilde{B} = \int \frac{d^3k}{k_0} \theta(K_{max} - |\vec{k}|) \tilde{S}(k).$$

$K_{max} \Leftrightarrow$  Dummy

**SCHEMES: RELATED BY RENORMALIZATION GROUP**

- **Version 1.13:**  $G_\mu$ -Scheme of Fleischer *et al.*, *Z. Phys. C42* (1989) 409, etc.
- **Version 1.14:** **Scheme A** – ONLY HARD EW CORR. HAS  $\alpha_{G_\mu}$ ;  
**Scheme B** – ENTIRE  $\mathcal{O}(\alpha)$  HAS  $\alpha(0)$

$\Rightarrow -0.3 \div -0.4\%$  **SHIFT OF NORMALIZATION** of 1.14 RELATIVE TO 1.13

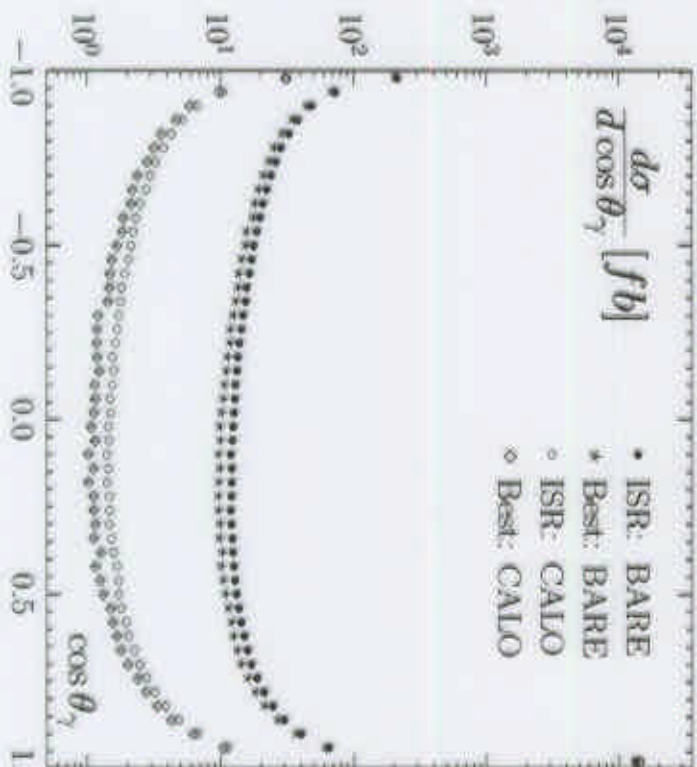
See Passarino's Talk for Details and References

### RESULTS: YFSWW3-1.14

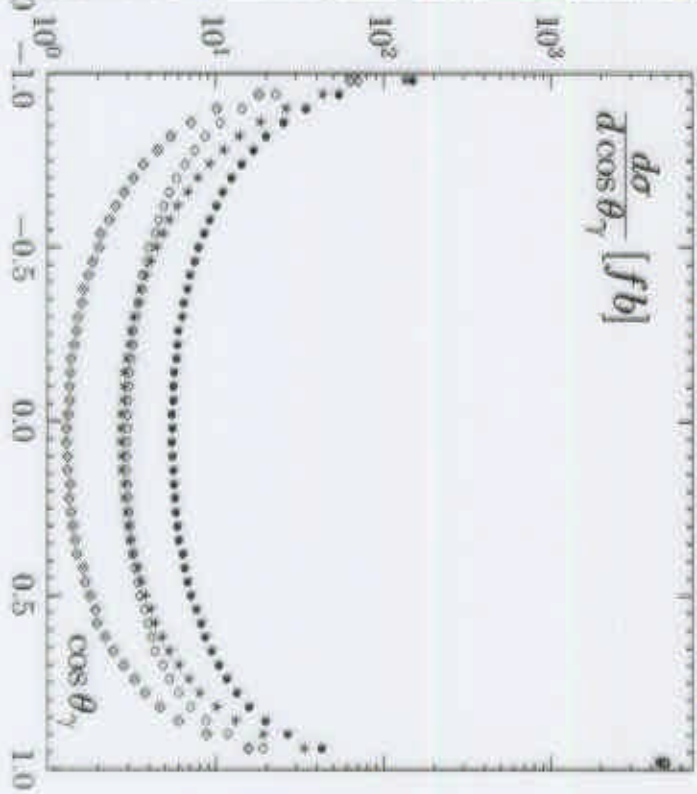
Hardest Photon Angular Distribution

$$e^+e^- \longrightarrow W^+W^- \longrightarrow u\bar{d}\mu^-\bar{\nu}_\mu$$

$E_{CM} = 200 \text{ GeV}$



$E_{CM} = 500 \text{ GeV}$



$\cos\theta_\gamma$  w.r.t.  $e^+$  beam  $\Rightarrow$  NL away from beams, etc.



## Comparison with RacoonWW

final state	no cuts		$\sigma_{\text{tot}}$ [fb]	
	program	Born	best	
$\nu_{\mu}\mu^{+}\tau^{-}\bar{\nu}_{\tau}$	YFSWW3	219.770(23)	199.995(62)	
	RacoonWW	219.836(40)	199.551(46)	
	(Y-R)/Y	-0.03(2)%	0.22(4)%	
$u\bar{d}\mu^{-}\bar{\nu}_{\mu}$	YFSWW3	659.64(07)	622.71(19)	
	RacoonWW	659.51(12)	621.06(14)	
	(Y-R)/Y	0.02(2)%	0.27(4)%	
$u\bar{d}\bar{s}c$	YFSWW3	1978.18(21)	1937.40(61)	
	RacoonWW	1978.53(36)	1932.20(44)	
	(Y-R)/Y	-0.02(2)%	0.27(4)%	

Total cross sections, CC03 from RacoonWW, YFSWW3,  $\sqrt{s} = 200$  GeV  
without cuts. Statistical errors – last digits in ( ), etc.  $\Rightarrow$  0.4% TU.

## Conclusions: KKMC(CEEX)

CEEX: clear upgrade path for EEX in a spin amplitude level MC.

- IFI (ISR $\otimes$ FSR) included and under firm control -- (see below).
- All coherence effects, including narrow resonances.
- All spin-spin effects ( $||\oplus\perp$ ) for beams and final (unstable) fermions.
- Exact M.E. for 2 high hard photons (3 photons pending).
- Extension to  $\nu\bar{\nu}$  in progress.
- Future extension to all-angle second order Bhabha is possible.
- For LEP2, total TU is 0.2%(0.2-0.4%) for  $\sigma_{tot}(AFB)$ , for typical cuts. For the LC at 0.5 TeV, these are a factor of 2 worse.
- For  $\gamma\gamma^*$ , the TU is 0.3% for LEP2 (no firm result for LC)

*KKMC*: first MC implementation for 2f production at LEP, LC's,  $\mu$ -colliders,  $\tau$  and  $b$  factories.

## Conclusions: KKMC(IFI)

### CEEX: firm control of IFI

- For a typical **energy cut of 0.3 IFI** is about **1.5%** in  $\sigma_{tot}$  and  $A_{FB}$ .
- For the **energy cut of 0.1** it is a factor **2** bigger.
- The cut  $|\cos\theta| < 0.9$  makes it **25%** smaller.
- $\mathcal{O}(\alpha^1)$  IFI controlled using **KORALZ** and **KKMC** for arbitrary cuts.
- Effects beyond  $\mathcal{O}(\alpha^1)$  are **negligible**, ( $< 20\%$  of  $\mathcal{O}(\alpha^1)$ ), except for energy cuts stronger than **0.1**.
- IFI at the **Z radiative** return is **very small**, as expected.
- Changing from  $s'$  to  $Q^2$ -propagator in the energy cut has **no effect**.

**Conclusions: YFSWW3-1.14**

We are currently at an exciting point in the tests of the **EW Theory in gauge boson physics**. The **WW** pair production is an important aspect of these tests:

- **Mass Distributions: FSR – Peak Position and Height Shifts**
- **W Angular Distributions: LL AND NL**
- **$\ell$  Angular Distributions: LL AND NL**
- **Photon Angular Distributions: LL AND NL**
- **Photon Energy Distributions: LL**
- **Normalization: LL AND NL**  
Current **200 GeV TU**: 0.4% {YFSWW3/RaccoonWW}