Anisotropic color superconductor

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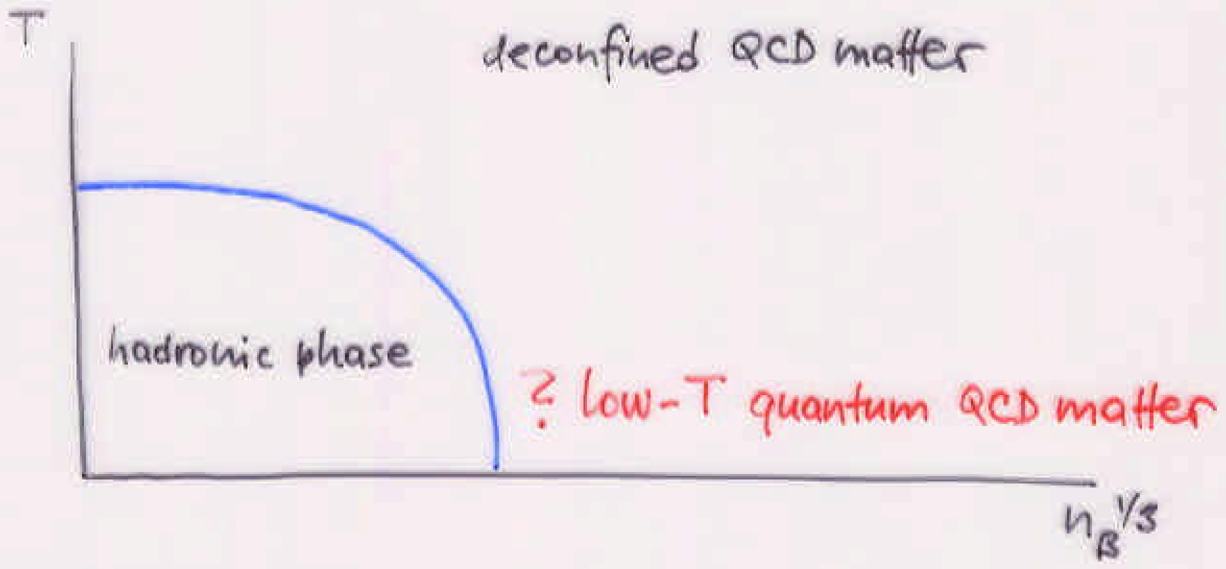
Outline :

- formulation of the problem and of the tasks
- playing with condensates
- quasiquark dispersion laws
- Nambu-Goldstone excitations
- fate of the gauge fields
- outlook

1. Problem and tasks



- (assumption) relativistic Landau Fermi liquid
  m_* (?) ; $\mathcal{L}_{int}$ (?) ; gauge int. perturbative

$$\mathcal{L}_{eff} = \overline{\psi}(i\not{D} - m_* + \mu\gamma_0)\psi + \mathcal{L}_{int} - \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu}$$

$$SU(3)_c \times SU(2)_I \times U(1)_V \times O(3)$$

$\mathcal{L}_{int}$: instantons, Debye screening, ...
local 4-f interaction

- Cooper phenomenon (Collins, Perry, ..., Wilczek, ...)
- determine the physical excitation spectrum
- draw physical conclusions from it (as if what we calculate can be measured)

2. Playing with condensates

$$\text{I.}\quad \langle \bar{\Psi}_{\alpha a A}(x)\, \epsilon^{ab3} (\tau_2)_{AB} (\gamma_5 C)_{\alpha\beta}\, \bar{\Psi}_{\beta b B}(x) \rangle = \Delta$$

- isotropic superconductor; Ginzburg-Landau-Higgs field $\phi^c(x)$
- $SU(3)_c \to SU(2)$:
  quarks of 2 colors are gapped BV quasiparticles

$$E_k = \sqrt{(\epsilon_k+\mu)^2 + |\Delta|^2} \qquad \epsilon_k = \sqrt{\vec{k}^2 + m_*^2}$$

$$E_k = \sqrt{(\epsilon_k-\mu)^2 + |\Delta|^2}$$

  Δ fixed by the gap equation
  5 gluons massive, $m \sim g\Delta$
  3 -"- massless
- plenty of work done

$$\text{II.}\quad \langle \bar{\Psi}_{\alpha a A}(x) (\tau_3\tau_2)_{AB} (\gamma_5 C)_{\alpha\beta}\, \bar{\Psi}_{\beta b B}(x) \rangle = \Delta_a \delta_{ab}$$

- isotropic superconductor, GLH field $\phi_{I;ab}(x)$
- not elaborated but straightforward
- spontaneous breakdown of isospin symmetry $\Rightarrow$ 2 physical NG excitations
- generic form of quasiparticles as in I.

III. $\langle \bar{\Psi}_{\alpha a A}(x)(\tau_2)_{AB}(\gamma_0\gamma_3 C)_{\alpha\beta}\bar{\Psi}_{\beta b B}(x)\rangle = \Delta_a \delta_{ab}$

anisotropic superconductor ; $\phi_{ab;\mu\nu}(x)$

IV. $\langle \bar{\Psi}_{\alpha a A}(x)(\tau_3\tau_2)_{AB}\,\epsilon^{ab3}(\gamma_0\gamma_3 C)_{\alpha\beta}\bar{\Psi}_{\beta b B}(x)\rangle = \Delta$

anisotropic superconductor ; $\phi^{a}_{I\,\mu\nu}(x)$

3. Quasiquark dispersion laws

Nambu-Gorkov-like self-consistent approximation

J. H., hep-ph/9812515
hep-ph/9812516
hep-ph – to appear

$$\mathcal{L}_{eff} = \bar{\Psi}(i\partial\!\!\!/ - m_* + \mu\gamma_0)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{int}$$

$U(1)\times SU(2)\times O(3)$

$$\mathcal{L}_{int} = G\left[(\bar{\Psi}\gamma_0\Psi)^2 - (\bar{\Psi}\gamma_0\vec{\tau}\Psi)^2\right]$$

isotropic condensate

$$\langle \bar{\Psi}_{\alpha A}(x)(\tau_3\tau_2)_{AB}(\gamma_5 C)_{\alpha\beta}\bar{\Psi}_{\beta B}(x)\rangle \equiv 0$$

anisotropic condensate

$$\langle \bar{\psi}_{\alpha A}(x)(\tau_2)_{AB}(\gamma_0\gamma_3 C)_{\alpha\beta}\bar{\psi}_{\beta B}(x)\rangle = \Delta$$

formalism : $q = \frac{1}{\sqrt{2}}\left|\begin{matrix}\psi \\ \tau_2\psi^c\end{matrix}\right) \Rightarrow \Delta = \langle \bar{q}\,\gamma_0\gamma_3(\Gamma_1 + i\Gamma_2)q\rangle$

$$E^2_{(1)}(\vec{p}) = \epsilon_p^2 + |\Delta|^2 + \mu^2 + 2\sqrt{\epsilon_p^2\mu^2 + (p_1^2 + p_2^2 + m_*^2)|\Delta|^2}$$

$$E^2_{(2)}(\vec{p}) = \epsilon_p^2 + |\Delta|^2 + \mu^2 - 2\sqrt{\epsilon_p^2\mu^2 + (p_1^2 + p_2^2 + m_*^2)|\Delta|^2}$$

spontaneous breakdown of rotational symmetry in action

gap equation :

$$\Delta + 2G\Delta\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{E_{(1)}(\vec{p})} + \frac{1}{E_{(2)}(\vec{p})}\right]\left(1 - \frac{4(p_1^2+p_2^2)}{E_{(1)}(\vec{p})E_{(2)}(\vec{p})}\right) = 0$$

lowest app. : noninteracting quasiquarks

4. Nambu-Goldstone excitations

$U(1) \times SU(2) \times O(3) \rightarrow SU(2) \times O(2) \Rightarrow$

$1+2$ NG excitations (gauge int. switched off)

(i) $Q = \int d^3x\, \bar{\psi}\gamma_0\psi = \frac{1}{2}\int d^3x \left[\bar{\psi}\gamma_0\psi - \bar{\psi}^c\gamma_0\psi^c\right] =$

$$= \int d^3x\, \bar{q}\gamma_0\Gamma_3 q$$

$$[Q, \underline{\bar{q}\gamma_0\gamma_3(\Gamma_1 - i\Gamma_2)q}] = -2\bar{q}\gamma_0\gamma_3(\Gamma_1 + i\Gamma_2)q$$

take ground-state exp. value ; $\Delta \neq 0$ by ass. $\Rightarrow$

(ii) $O(3)$ generators (spin angular momentum)

$$Q_1 = \frac{1}{2}i\int d^3x\, \bar{q}\gamma^0\gamma^2\gamma^3 q$$

$$Q_2 = \frac{1}{2}i\int d^3x\, \bar{q}\gamma^0\gamma^3\gamma^1 q$$

$$Q_3 = \frac{1}{2}i\int d^3x\, \bar{q}\gamma^0\gamma^1\gamma^2 q$$

$$[Q_1, \underline{\bar{q}\gamma_0\gamma_2(\Gamma_1 + i\Gamma_2)q}] = i\bar{q}\gamma_0\gamma_3(\Gamma_1 + i\Gamma_2)q$$

$$[Q_2, \underline{\bar{q}\gamma_0\gamma_1(\Gamma_1 + i\Gamma_2)q}] = i\bar{q}\gamma_0\gamma_3(\Gamma_1 + i\Gamma_2)q$$

For the gauge int. switched off 3 NG excitations are in the spectrum.

5. Fate of the gauge field in anisotropic supercond.

- microscopic calculation remains to be done
- GLH approach : $\phi^{\mu\nu}(x)$ order parameter
- reminder

$$\mathcal{L}_0 = -\frac{1}{2}\partial^\lambda \phi_{\lambda\mu}\, \partial_\nu \phi^{\nu\mu} + \frac{1}{4}M^2 \phi_{\mu\nu}\phi^{\mu\nu}$$

time - space components propagate

$$\mathcal{L}_0 = -\frac{1}{8}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \phi_{\alpha\beta}\, \epsilon_{\mu\eta\sigma\tau}\partial^\eta \phi^{\sigma\tau} - \frac{1}{4}M^2 \phi_{\mu\nu}\phi^{\mu\nu}$$

space - space components propagate

$$\mathcal{L}_{GLH} = -\left(D^\lambda \phi_{\lambda\mu}\right)^+ D_\nu \phi^{\nu\mu} - V(\phi)$$

$$D^\lambda \equiv \partial^\lambda - ie^* A^\lambda$$

$V(\phi)$ minimal at $\phi_{03} = \Delta$

$$\mathcal{L}_{mass} = -e^{*2} A^\lambda \phi^+_{\lambda 0} A_\nu \phi^{\nu 0} + e^{*2} A^\lambda \phi^+_{\lambda m} A_\nu \phi^{\nu m}$$

$$= e^{*2}|\Delta|^2\left(A_0^2 - A_3^2\right)$$

$$= e^{*2}|\Delta|^2 A_\mu A^\mu + e^{*2}|\Delta|^2\left(A_1^2 + A_2^2\right)$$

peculiar, but not unexpected

6. Outlook

- our beloved QCD can in the deconfined regime at low T exist in a plenty of physically distinct macroscopic quantum phases (add orbital angular momentum!)
- changes in the form of E_k => changes in the specific heat
- analogies falter: no external chromoelectric and chromomagnetic fields
- fortunately, ordinary external fields (electric, magnetic, velocity) available
- what if not the Landau Fermi liquid?

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