Photon/dilepton production in thermal field theory.

P. Aurenche

- γ , $\gamma^* \to \ell^+ \ell^-$ in the quark-gluon plasma
- a possible signal for QGP formation ?

$\frac{dN}{dq_0} \Rightarrow T$, temperature of QGP

if signal above backgrounds!

We assume ; equilibrium : T , $\mu = 0$

Production rate / unit time, volume :

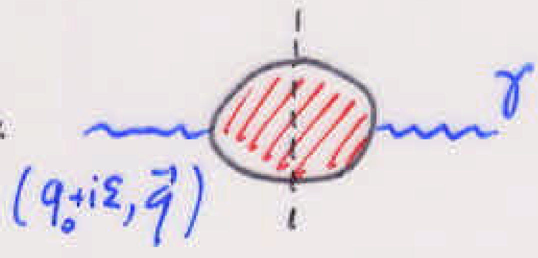
- $$\frac{q_0\, dN^\gamma}{d^3q\, d^4x} = -\frac{1}{8\pi^3}\, n_B(q_0)\, \mathrm{Im}\, \Pi_R(q_0, \vec{q})$$

- $$\frac{dN^{\ell^+\ell^-}}{d^4q\, d^4x} = -\frac{1}{12\pi^4}\, \frac{\alpha}{Q^2}\, n_B(q_0)\, \mathrm{Im}\, \Pi_R(q_0, \vec{q}) \, , \quad Q^2 = (\ell^+ + \ell^-)^2$$

- $$n_B(q_0) = \frac{1}{e^{q_0/T} - 1} \sim e^{-\frac{q_0}{T}} \, , \quad q_0 \gg T$$ [pheno. interest]

the Bose factor

$\mathrm{Im}\, \Pi_R(q_0, \vec{q}) =$



retarded 2-point function.

Calculate $\mathrm{Im}\,\Pi_R$ in HTL (hard thermal loop resummed) effective theory of Braaten-Pisarski, Frenkel-Taylor. Assume strong coupling $g \ll 1$. ②

I HTL effective theory

II Application to γ, γ^* up to 2-loop; unexpected result: $dN|_{2loop} \gtrsim dN|_{1loop}$;

very prelim. phenomenology.

III Higher loops: Is dN^γ calculable in pert. theory?

I HTL effective theory:

Basic point: interactions in QGP modify behavior of QCD in soft region.

2 scales:

- hard scale $T \gg \Lambda_{QCD}$: $E_q \sim E_G \sim T$ in QGP
- soft scale gT, $g \ll 1$

Collective effects modify physics at scale gT.

Example:

Quark propagator:

 $= \frac{1}{\not p} \sim \frac{1}{p}$

 $\sim g^2 \frac{T^2}{p}$ momentum of particles in loop $\sim T$ **(HTL)**

When $p \sim gT \Rightarrow$  $\sim \frac{g^2T^2}{gT} \sim gT \sim p$

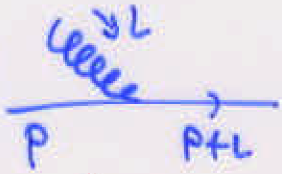
1-loop correction $\sim$ lowest order contrib $\Rightarrow$ should consider resummed propagator

 $= \frac{1}{\not p - }$  $\Rightarrow$ 2 consequences:

1) $\underline{p^2 > 0}$  $\sim \frac{\not p}{p^2 - m_q^2(p_0, p)}$ $\boxed{m_q^2 \sim g^2T^2}$

p large, close to mass shell

q effective mass (thermal)

2) $\underline{p^2 < 0}$ $\mathrm{Im}($  $) \sim \Big|$  $\Big|^2 \neq 0$

Im part nonvanishing due to Landau damping

Both effects are specific to thermal f.t.

4

gluon propagator

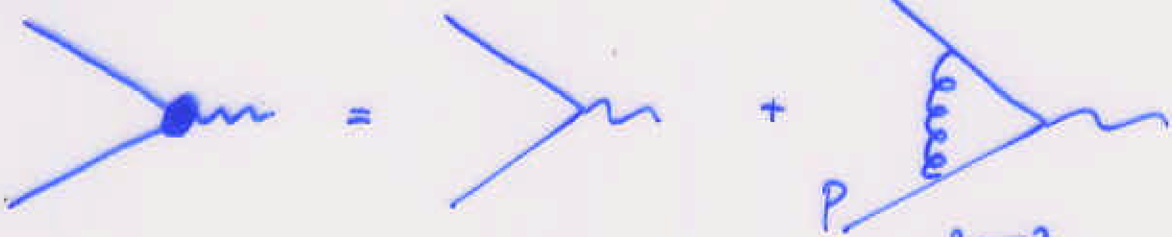
$$\sim \frac{1}{L^2 - \ldots - \ldots}$$

1) $\underline{L^2 > 0}$ $\quad \sim \dfrac{P_{T,L}}{L^2 - m^2_{T,L}}$ $\qquad \boxed{m^2_{T,L} \sim g^2 T^2}$

2) $\underline{L^2 < 0}$ $(\ell_0 = 0)$ $\quad \mathrm{Im}\Big($  $\Big) = \dfrac{1}{\ell^2 + m_D^2}$ $\quad$ longitudinal $m_D^2 \sim g^2 T^2$

$$= \frac{1}{\ell^2 + m^2_{mag}}$$ transverse $m^2_{mag} \sim g^4 T^2$

Vertices



$$= \quad e \quad + \; e\,\frac{g^2 T^2}{p\,r}$$

One can construct $\mathcal{L}_{eff}$ (Taylor. Wong) with effective propagators and vertices (gauge invariance)

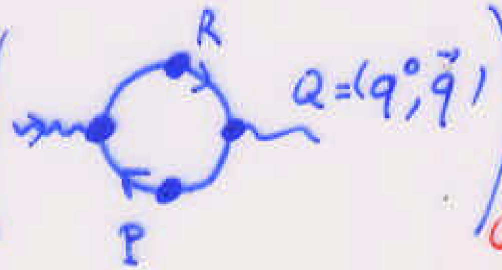
In following we start from $\mathcal{L}_{eff}$ and apply pert. theory.

II Application to γ, γ^* production

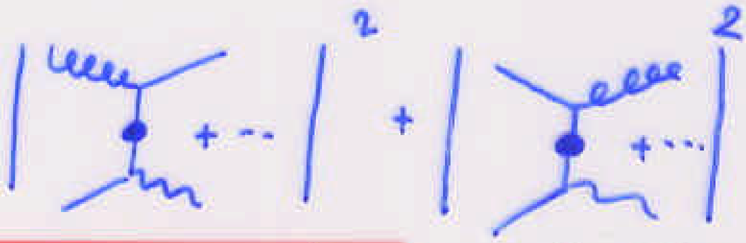
Consider hard γ, γ^* : $q^0 \gtrsim T$

small virtuality : $\sqrt{\frac{Q^2}{q_0}} << 1$ or $Q^2 = 0$

a) 1. loop result

$\Im m \Big($  $\Big)$ $\sim \Im m \Big($  $\Big)$

Q hard

$\sim$ 

$$\boxed{\gamma : \Im m\, \Pi(Q) \sim \alpha g^2 T^2 \left(\ln \frac{q_0 T}{m_q^2} + C \right)}$$

Kapusta, Lichard, Seibert (Baier, Nakkagawa, Niegawa, Redlich)

$$\boxed{\gamma^* : \Im m\, \Pi(Q) \sim \alpha g^2 T^2 \left(\ln \frac{q_0 T}{m_q^2} + C\left(\frac{m_q^2}{Q^2}, \frac{Q^2}{q_0^2}\right) \right)}$$

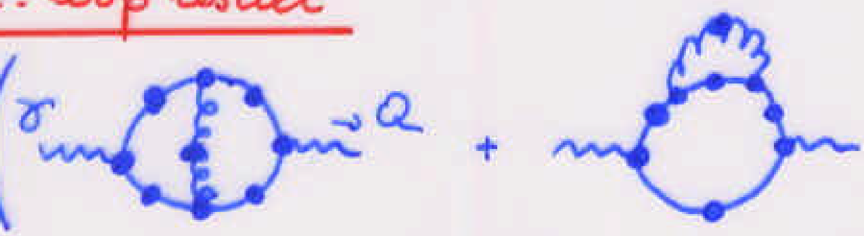
Altherr, Ruuskanen, Thoma, Traxler

$$\Im m\, \Pi(Q) \sim \alpha g^2 T^2 \ln\left(\frac{q_0}{T}\frac{1}{g^2}\right)$$

(6)

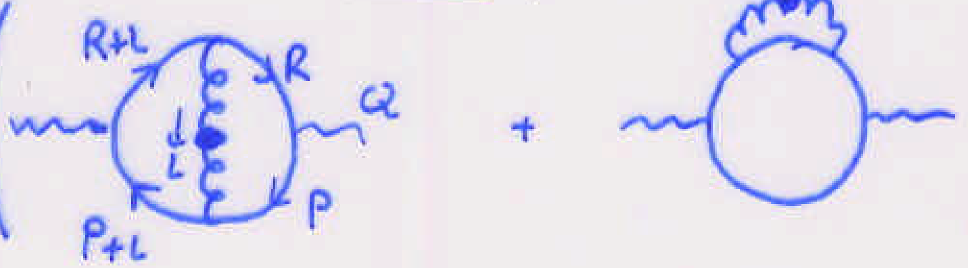
b) 2. loop result

[with Gelis, Kobes, Zaraket]

Im(

+ ...)

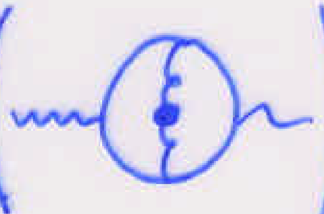
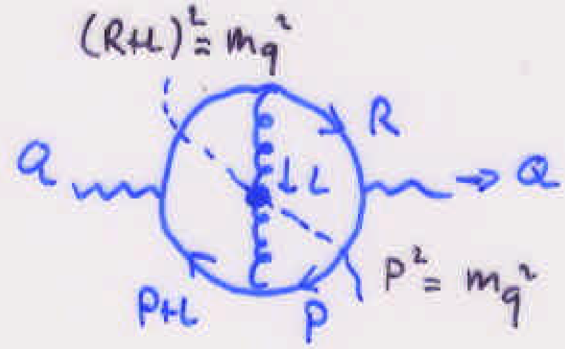
but dominance of hard quarks in the loop ⇒

Im(

) { $P, R \sim Q \gtrsim T$; L soft

keep only effective gluon which can be soft.

But:

Im() = 

[Physical processes]

When calculating Im part, put quarks P, $R+L$ on (effective) mass-shell:

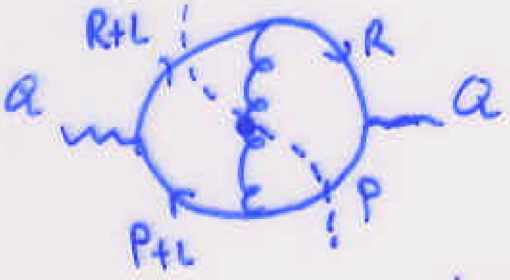
$$\boxed{\operatorname{Im} \Pi(Q) \sim \alpha\, g^2 \int d^4P\, d^4L\ \frac{\delta(P^2-m_q^2)}{R^2-m_q^2}\,\frac{\delta((R+L)^2-m_q^2)}{(P+L)^2-m_q^2} \cdots}$$

but $R^2 - m_q^2 = (P+Q)^2 - m_q^2$ can be very small as $P /\!/ Q$:

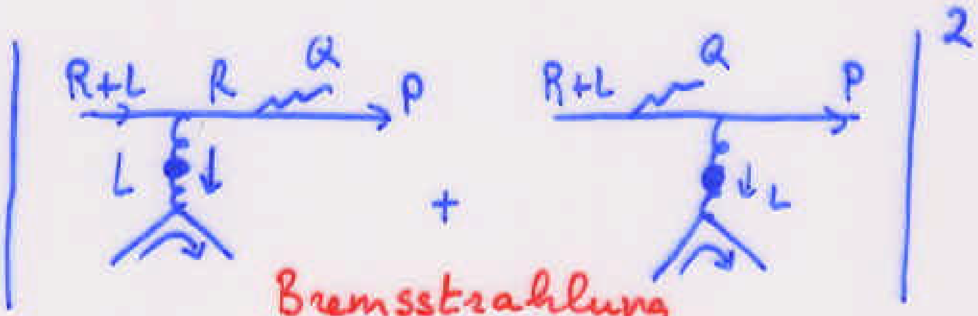
$$(R^2-m_q^2)\Big|_{P^2=m_q^2} \simeq 2pq\left(1-\cos\theta + \mathcal{O}\left(\frac{m_q^2}{p^2}\right)\right), \quad \theta = (\vec{p}, \vec{q})$$

$$\sim m_q^2 \quad \text{when } \cos\theta \simeq 1$$

Physical processes in 2-loop diagrams (some ...)



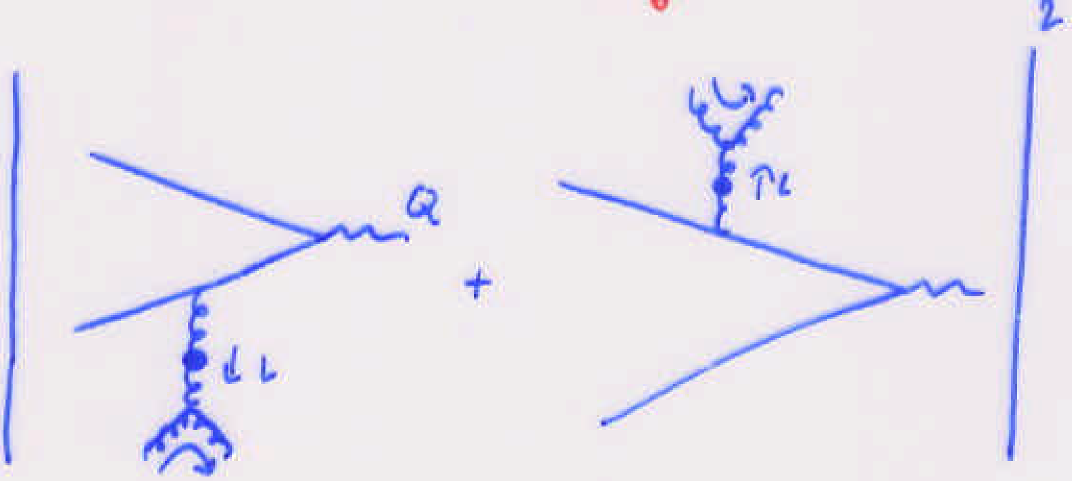
P, R, Q hard =



L soft

Bremsstrahlung

+



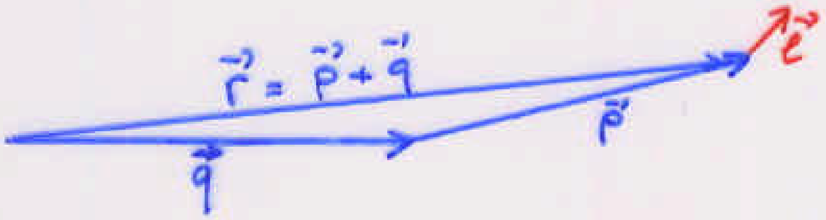
Annihilation with scattering

+ ...

Likewise :

$$(P+L)^2 - m_q^2 \Big|_{(R+L)^2 = m_q^2} \sim 2rq\left(1-\cos\theta' + \theta'\left(\frac{m_q^2}{r^2}\right)\right)$$

$$\sim m_q^2 \quad \text{when } \theta' = (\vec{r}+\vec{\ell}, \vec{q}) = 0$$



So, when

$\vec{p} \,//\, \vec{q} \rightsquigarrow \vec{r} \,//\, \vec{q}$

$\sim \vec{r} + \vec{\ell} \,//\, \vec{q}$

hard soft

and both denominators become small at same time.

Thus

$$\text{Im}\,\Pi(q) \sim \alpha g^2 \int d\cos\theta \, \frac{1}{1-\cos\theta + O\left(\frac{m_q^2}{T^2}\right)} \, \frac{1}{1-\cos\theta + O'\left(\frac{m_q^2}{T^2}\right)}$$

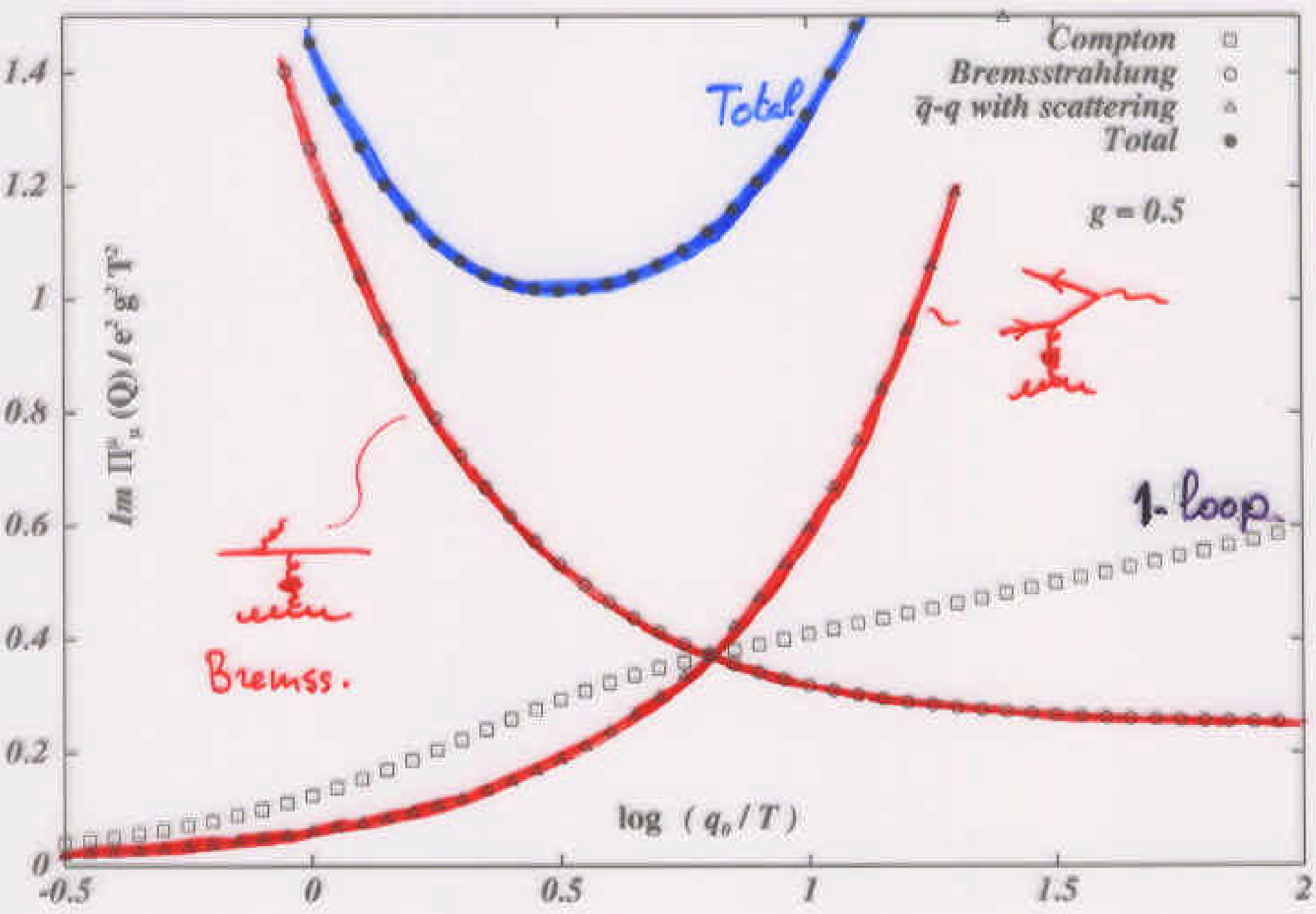
$$\text{Im}\,\Pi(q) \sim \alpha g^2 \frac{T^2}{m_q^2} \sim \alpha g^2 \times \frac{1}{g^2} \sim \alpha \; !$$

The effect of collinear singularities is to cancel g^2 factor.

$\text{Im}\,\Pi(Q)\Big\vert_{\text{bremss.}} \sim \alpha g^2 T^2 \times C$	C large coefficient
$\text{Im}\,\Pi(Q)\Big\vert_{\text{annihil.}} \sim \alpha g^2 T q^0 \times C'$	dominates when $q^0 > T$.

— 2 boucle

— Total : 1 + 2 boucles



Srivastava

LHC

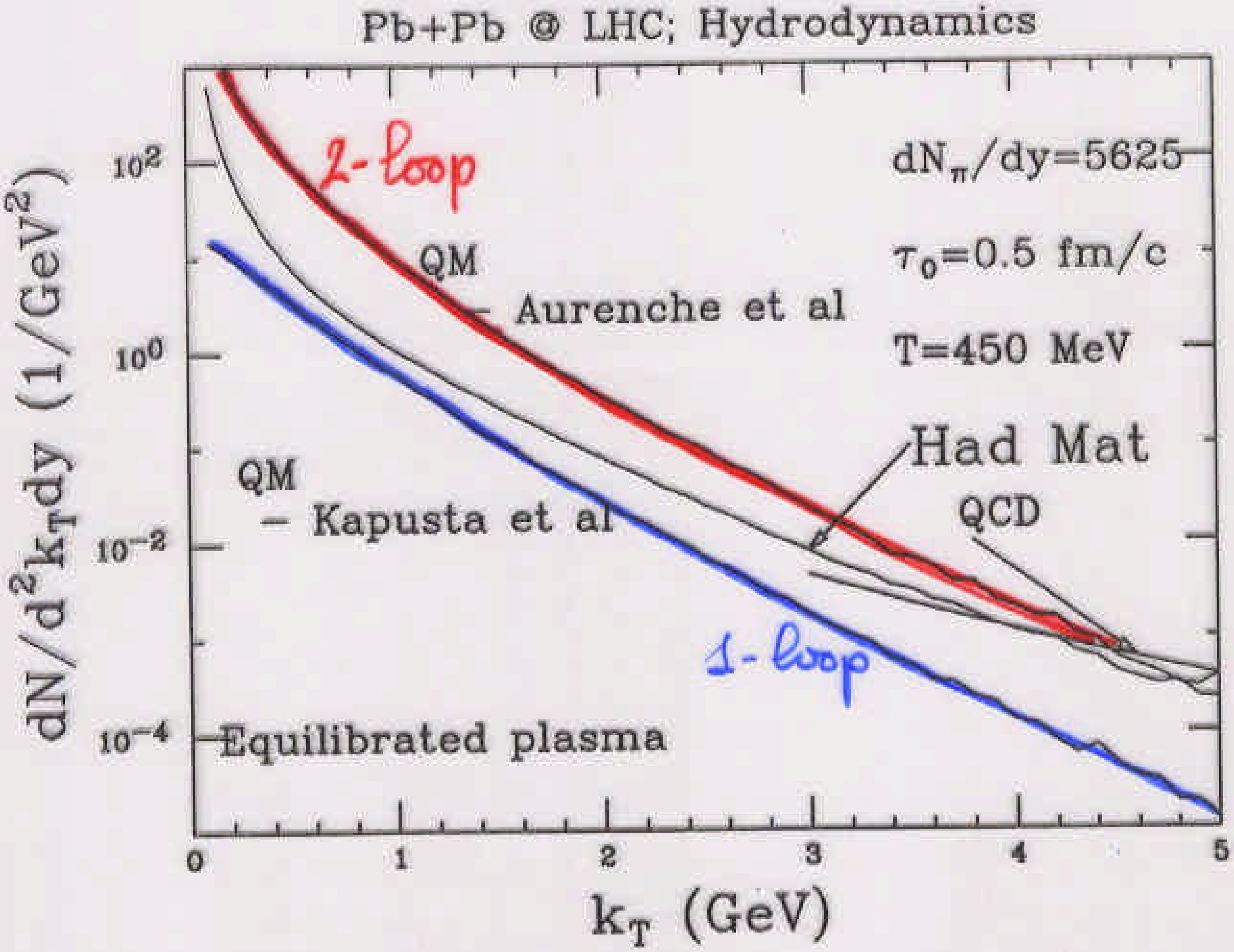


Fig. 4. The same as Fig. 2, but for LHC energies

Srivastava, Sinha

WA98, SPS

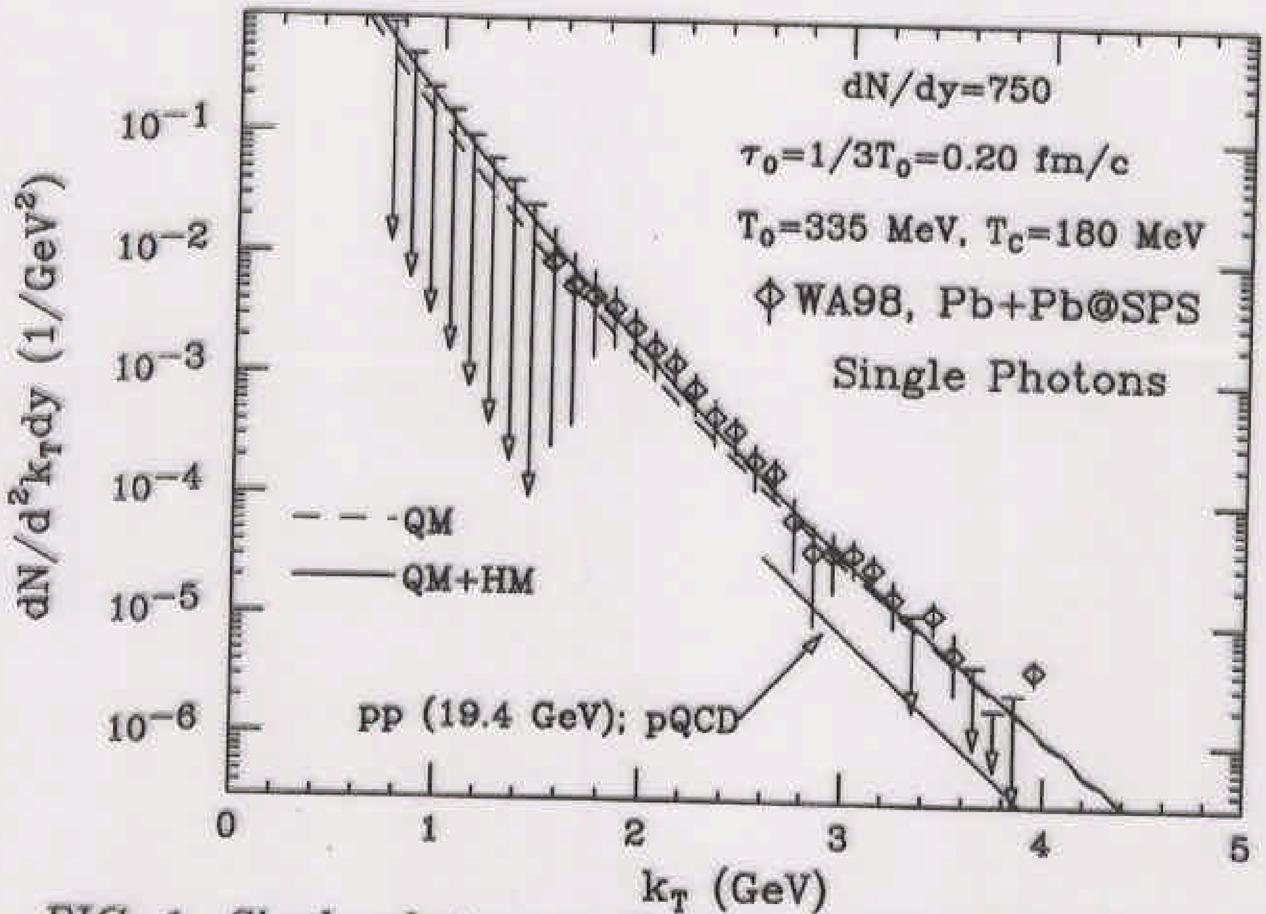


FIG. 1. Single photon production in $Pb + Pb$ collision at the CERN SPS. An equilibrated (chemically and thernally) quark-gluon plasma is assumed to be formed at $\tau_0 = 1/3T_0$

Conclusions :

$$\boxed{\left.\Im m\, \Pi(Q)\right|_{2\text{-loop}} \sim \left.\Im m\, \Pi(Q)\right|_{1\text{-loop}} \quad \text{(up to log.)} \qquad > \qquad q^0 > T}$$

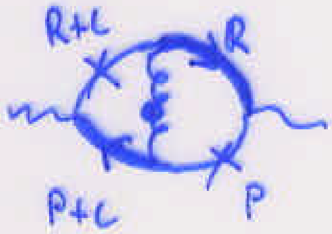
Phenomenology (?, !)

Srivastava, Sinha
Mustafa, Thoma } : 2 loop processes in hydrodynamic code

⇒ predictions for RHIC, LHC.

⇒ predictions for WA98.

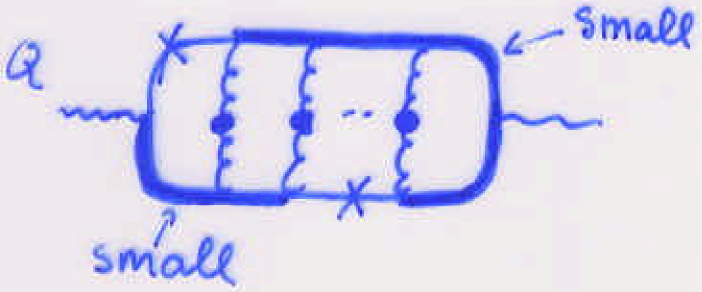
III Is dN^γ calculable in perturbation theory?



$$\sim \oint \frac{1}{R^2 (P+L)^2} \sim \frac{T^2}{m_q^2} \sim \frac{1}{g^2}$$

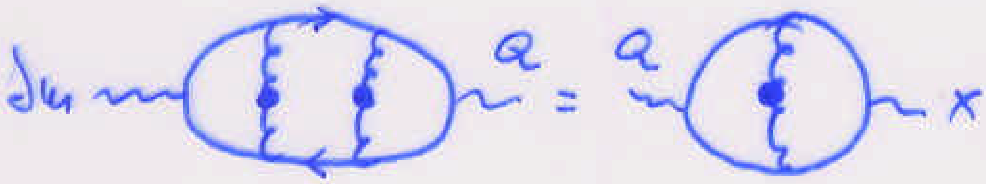
What happens at higher orders?

1) Ladder diagrams.



Accumulation of collinear singularities!

Explicit calculation of 3-loop diagram:



$$\times \; \frac{g^2 T}{\ell_{min}}$$

["KLN" theorem]

ℓ_{min} is the maximum of 2 cut-off:

- collinear cut-off:

$$\boxed{\ell^{(1)}_{min} = \frac{q_0}{p_0 r_0} M^2_{eff}} \qquad \boxed{M^2_{eff} = m_q^2 + \frac{Q^2}{q_0^2} p_0 (p_0 + q_0)}$$

- infrared cut-off, associated to gluon-exchange

$$\ell^{(2)}_{min} = m_D \sim gT \qquad \text{Longitudinal gluon}$$

$$= m_{mag} \sim g^2 T \qquad \text{Transverse gluon}$$

[Recall: Longit. $\sim \dfrac{1}{\ell^2 + m_D^2}$; Transv. $\sim \dfrac{1}{\ell^2 + m^2_{mag}}$]

- For <u>longitudinal gluon</u>: $\quad l_{min} \gtrsim m_D$

so the extra rung brings:

 $\lesssim \dfrac{g^2 T}{m_D} \sim g$, perturbative.

$[m_D \sim gT]$

- For <u>transverse gluon</u>:

$l^{(1)}_{min} = \dfrac{q_0}{p_0 r_0} M^2_{eff} \sim g^2 T + \dfrac{Q^2}{q_0}$ compared to $m_{mag} \simeq g^2 T$

$\Rightarrow$ regulators $\sim g^2 T$ (unless $\dfrac{Q^2}{q_0}$ not too small)

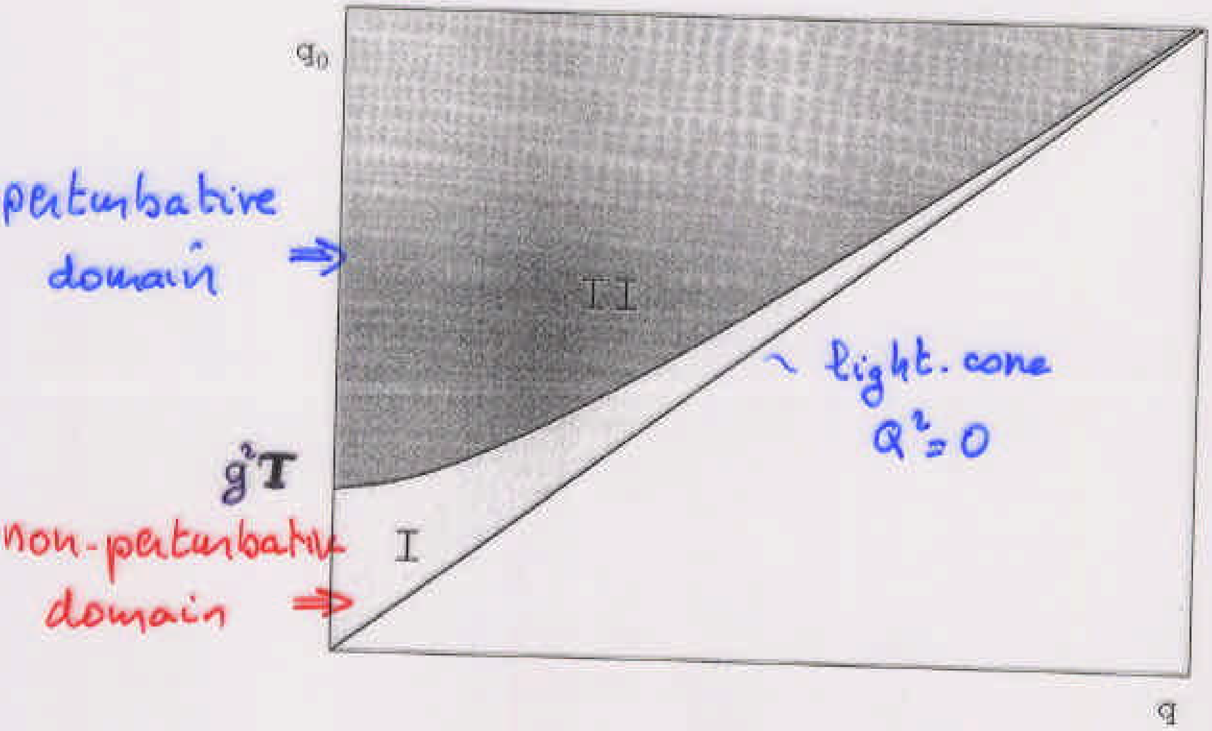
and

 $\sim \dfrac{g^2 T}{m_{mag}} \sim 1$, non perturbative.

$[m_{mag} \sim g^2 T]$

More generally: $\exists$ two domains (figure)

<u>Note:</u> Occurence of factor $\dfrac{g^2 T}{m_{mag}}$ is similar to <u>Linde problem</u> in pressure at higher order. Here problem occurs at lowest order in rate of γ production!



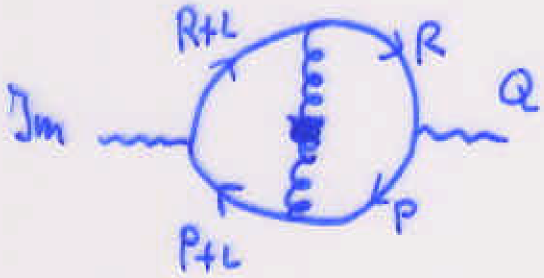
Separation between two domains given by

$$\frac{q_0}{p_0(p_0+q_0)} M^2_{eff} = m_{mag} = g^2 T$$

with

$$M^2_{eff} = m_q^2 + \frac{Q^2}{q_0^2} p_0(p_0+q_0)$$

2) Collisional width.



$$S(R):\ \frac{\not R}{R^2} \xrightarrow[\text{HTL}]{} \frac{\not R}{R^2 - m_q^2}$$

$\sim g^2T^2$

But collisions in a plasma generate a width to prop.:

damping rate : $\Gamma \sim g^2 T \ln(\frac{1}{g}) \sim \mathrm{Im}(\ldots)$

$\ll m_q \sim gT$

and

$$S(R):\ \frac{\not R}{R^2 - m_q^2 + i\Gamma r_0}$$

, Γ neglected in effective theory since $\Gamma \ll m_q$.

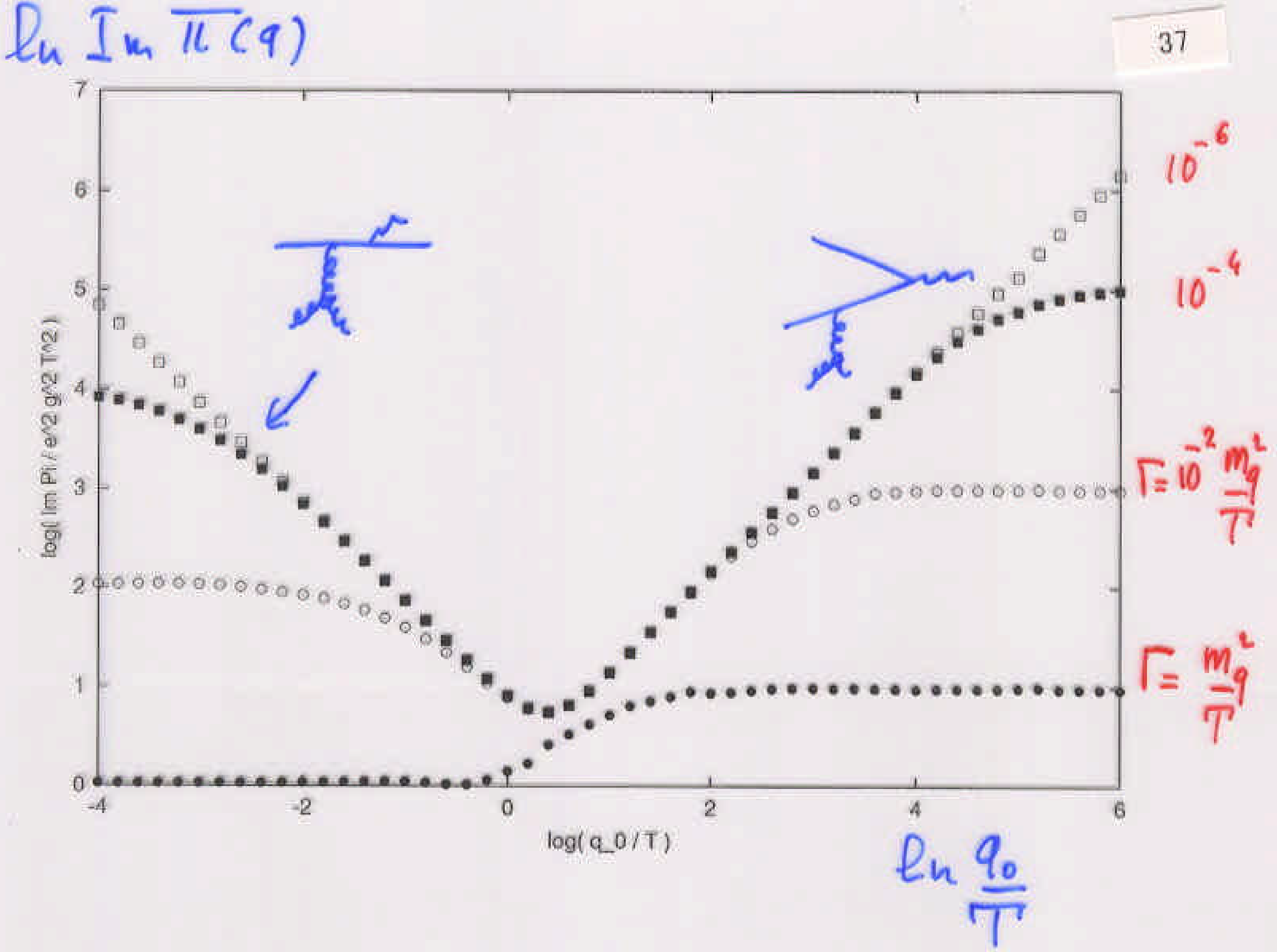
Obviously, Γ can provide a regulator for collinear sing. since pole is shifted away from real axis.

Redo 2-loop calculation with Γ. One finds

$$\boxed{M_{eff}^2 = \underbrace{m_q^2 + \frac{Q^2}{q_0^2} p_0(p_0+q_0)}_{\text{Same as before}} + 4i\Gamma \frac{p_0(p_0+q_0)}{q_0}}$$

The regulator is provided by largest of $\mathrm{Re}\, M_{eff}^2$, $\mathrm{Im}\, M_{eff}^2$.

Shape of photon spectrum is affected. (figure)



Effect of the width on $\mathrm{Im}\,\pi^{\gamma}(q^0, \vec{q})$

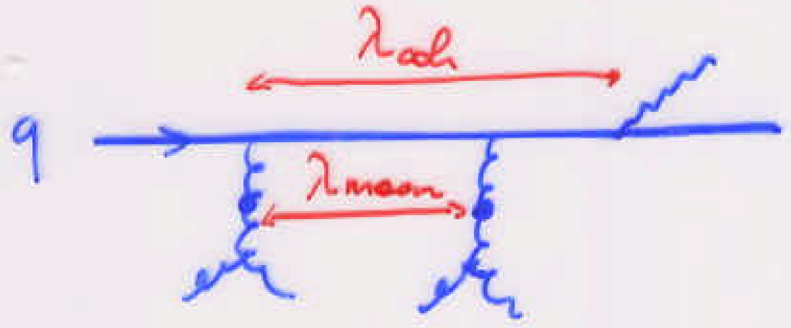
$(Q^2 = 0)$

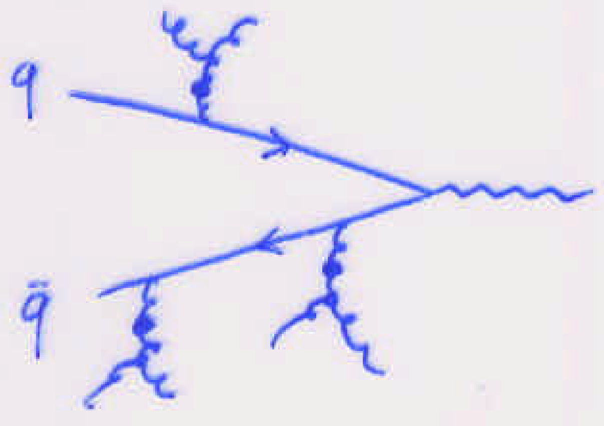
Physical interpretation

$$\boxed{M^2_{eff} = \frac{2p_0(p_0+q_0)}{q_0}\left(\frac{1}{\lambda_{coh}} + \frac{i}{\lambda_{mean}}\right)}$$

λ_{coh} = formation time of the γ in plasma

$\lambda_{mean} = \frac{1}{\Gamma}$ mean scattering length of the quark in plasma





If $\lambda_{coh} > \lambda_{mean}$

rescattering correction important $\Rightarrow$ higher order important

$\Rightarrow$ **LPM effect**

Landau, Pomeranchuk Migdal

The condition $\frac{1}{\lambda_{coh}} < \frac{1}{\lambda_{mean}} \sim \frac{1}{\Gamma} \sim \frac{1}{g^2T}$ $\rightsquigarrow$ non pert.

similar to condition obtained from ladder diagrams

$\lambda_{coh} < \frac{1}{m_{mag}} = \frac{1}{g^2T}$ $\rightsquigarrow$ non pert.

Conclusions

Study of γ, γ^* production in HTL effective theory reveals a rich structure.

- At two-loops new processes occur (bremß., annih. scatt.) which dominate over one-loop processes

- Higher-loop corrections are important when

$$Q^2 = 0 \; , \quad \frac{Q^2}{q_0^2} \ll 1$$

LPM effect with new features (G_T ; energetic γ)

If $\frac{Q^2}{q_0^2} < 1$ but not too small, perturb. theor. holds

- To do : a consistent calculation to all order !