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Complete RG-improvement – Avoiding scale dependence in QCD predictions

CJM and A. Mirjalili, Nucl. Phys. B577 (2000) 209.
S.J. Burby and CJM, hep-ph/0008---

- Perturbative QCD for dimensionless observable $R(Q)$:

$$R(Q) = a\left(1 + \sum_{n>0} r_n a^n\right)$$

$a \equiv \frac{\alpha_s(\mu)}{\pi}$ is the RG-improved coupling.

- State of the art is NNLO ($n=2$) calculation for e.g. QCD vacuum polarization ($R_{e^+e^-}$, R_τ ...)

- For e^+e^- jet observables measured at LEP have NLO calculation ($n=1$) available.

- standard fixed-order NLO RG-improved predictions depend on chosen renormalization scale ($\mu = xQ$). Renormalized coupling $\alpha_s(\mu)$ is dependent on convention used to remove UV divergences. Q-dependence of $R(Q)$ is independent of such conventions. Want to trade unphysical μ-dependence for physical Q-dependence. Keep μ independent of Q!

- μ and $\alpha_s(\mu)$ are irrelevant unphysical quantities. Focus instead on $R(Q)$ (the data) and Λ, the dimensional transmutation parameter of the theory.

(2)

Direct relation between R and Λ

Dimensional analysis (massless quarks) $\Rightarrow$

$$R(Q) = \Phi\left(\frac{\Lambda}{Q}\right)$$

where Λ is a dimensionful constant (related to the dimensional transmutation parameter, e.g. $\Lambda_{\overline{MS}}$)

Try to invert

$$\frac{\Lambda}{Q} = \Phi^{-1}(R(Q)) .$$

Can obtain the structure of the inverse function Φ^{-1} starting from the obvious dimensional analysis statement

$$\frac{dR}{dQ} = \frac{B(R(Q))}{Q}$$ [cf. Stevenson Ann. Phys. 152 (1981) 383]

where $B(R)$ is a dimensionless function of R.

$$\Rightarrow \quad \frac{dR(Q)}{d\ln Q} = B(R(Q))$$

Separable first order ODE. Solve with boundary condition $R(\infty)=0$, Asymptotic Freedom.

$$\ln\frac{Q}{\Lambda_R} = \int_0^{R(Q)} \frac{dx}{B(x)} + K$$

Λ_R is a dimensionful constant dependent on R

K is a universal infinite constant needed to implement $R(\infty) = 0$.

- To define $\mathcal{K}$ need to know power series for $B(x)$ around $x = 0$.
- Using perturbative expansion of R in terms of a, and beta-function equation

$$\frac{da}{d\ln\mu} = \beta(a) = -ba^2\left(1 + ca + \sum_{n>1} c_n a^n\right)$$

$b = (33 - 2N_f)/6$, $c = (153 - 19N_f)/12b$ are universal. $c_2^{\overline{MS}}$, $c_3^{\overline{MS}}$ have been computed.

$$\Rightarrow \quad B(x) = -bx^2(1 + cx + \rho_2 x^2 + \dots)$$

$$\rho_2 = c_2 + r_2 - r_1 c - r_1^2 \qquad \underline{\text{NNLO RS-invariant}}$$

- $$\mathcal{K} = -\int_0^{C} \frac{dx}{K(x)}, \quad K(x) = -bx^2(1 + cx + \Delta(x))$$

$\Delta(x)/x^2$ finite as $x \to 0$.

Different choices for C and $\Delta(x)$ can be absorbed into the definition of Λ_R. $C = \infty$ and $\Delta(x) = 0$ are convenient choices.

$$b\ln\frac{Q}{\Lambda_R} = \underbrace{\int_{R(Q)}^{\infty} \frac{dx}{x^2(1+cx)}}_{F(R) \equiv \frac{1}{R} + c\ln\left[\frac{cR}{1+cR}\right]} + \underbrace{\int_0^{R(Q)} dx\left[\frac{b}{B(x)} + \frac{1}{x^2(1+cx)}\right]}_{G(R) = \rho_2 R + O(R^2)}.$$

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- Exponentiating, the inverse relation $\Phi^{-1}(R)$ is then

$$\boxed{\frac{\Lambda_R}{Q} = \mathcal{F}(R(Q))\,\mathcal{G}(R(Q))}$$

$$\mathcal{F}(R) \equiv e^{-F(R)/b} = e^{-1/bR}\left(1+1/cR\right)^{c/b} \quad \underline{\text{universal}}$$

$$\mathcal{G}(R) \equiv e^{-G(R)/b} \simeq 1 - \frac{\rho_2}{b}R + \cdots$$

- How is Λ_R related to $\Lambda_{\overline{MS}}$ (for instance)?

Have the operational definition

$$\boxed{\Lambda_R = \lim_{Q\to\infty} Q\exp\left[-F(R(Q))/b\right]}$$

If we define $a(Q)$ to be the $\underline{\overline{MS}\text{ coupling}}$ with $\mu = Q$, then

$$\frac{da(Q)}{d\ln Q} = \beta_{\overline{MS}}(a) = -b a^2\left(1 + ca + c_2^{\overline{MS}}a^2 + \cdots\right)$$

Can integrate as above with $B(R) \to \beta_{\overline{MS}}(a)$ and $\Lambda_R \to \tilde{\Lambda}_{\overline{MS}}$, with $c_2' = \infty$, $\Delta(x) = 0$ as before

$$\boxed{\tilde{\Lambda}_{\overline{MS}} = \lim_{Q\to\infty} Q\exp\left[-F(a(Q))/b\right]}$$

Have $F(R(Q)) = F(a(Q)) - r + \cdots$

where $r \equiv r_1^{\overline{MS}}(\mu = Q)$ (independent of Q)

$$\Rightarrow \quad \boxed{\Lambda_R = e^{r/b}\,\tilde{\Lambda}_{\overline{MS}}}$$

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($\tilde{\Lambda}_{\overline{MS}}$ differs from standard definition $\Lambda_{\overline{MS}}$ by $K \to K + c\ln(b/2c)$ so that

$$\tilde{\Lambda}_{\overline{MS}} = \left(\frac{2c}{b}\right)^{-c/b} \Lambda_{\overline{MS}}$$)

So, finally have relation between R and $\Lambda_{\overline{MS}}$.

$$\boxed{\Lambda_{\overline{MS}} = Q\, \mathcal{F}(R(Q))\, \mathcal{G}(R(Q))\, \underline{\underline{e}}^{-r/b} \left(\frac{2c}{b}\right)^{c/b}}$$

- Note all dependence on subtraction convention used to remove UV divergences resides in $e^{-r/b}$ term. Celmaster and Gonsalves: different Λ's can be related by a one-loop calculation, exactly.

- If only have a NLO calculation our state of knowledge of $B(x)$ is $B(x) = -bx^2(1+cx)$ since the NNLO invariant ρ_2 will be unknown $\Rightarrow$ $\mathcal{G} \approx 1$. This is the asymptotic $(Q \to \infty)$ expectation. Deviation of $\mathcal{G}$ from 1 provides a direct measure of how close to asymptotia we are at a given value of Q.

- Best we can do in extracting $\Lambda_{\overline{MS}}$ from data is then (setting $\mathcal{G} = 1$)

$$\boxed{\underbrace{\Lambda_{\overline{MS}}}_{\text{Universal parameter}} = Q\, \mathcal{F}(\underbrace{R(Q)}_{\text{Data}})\, \underbrace{e^{-r/b}}_{\text{NLO correction}} \left(\frac{2c}{b}\right)^{c/b}}$$

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- Have not opted for a particular choice of μ, but have simply integrated up dR/dQ from dimensional analysis, with QCD input for boundary condition and $B(x)$. Inverting the boxed relation, however, gives the NLO result in the Effective Charge (FAC) scheme with $\mu = e^{-r/b} Q$ ($\overline{MS}$), corresponding to $r_1 = 0$. (so $R = a$).

Grunberg

Gardi
Grunberg-
Karliner

$$R(Q) = \frac{-1}{c\left[1 + W(z(Q))\right]}$$

$$z(Q) \equiv -\frac{1}{e}\left(\frac{Q}{\Lambda_R}\right)^{-b/c}$$

$W(z)\exp(W(z)) = z$, Lambert W-function

- By construction solving the ODE directly gives the correct physical Q-dependence of $R(Q)$. What is the special feature of $\mu = e^{-r/b}Q$ in the context of standard RG-improvement?
- Q-dependence is built by UV-logarithms contained in $r_1(\mu)$,

$$\boxed{r_1(\mu) = b \ln \frac{\mu}{\tilde{\Lambda}_{\overline{MS}}} - b \ln \frac{Q}{\Lambda_R}}$$

unphysical μ-log　　　physical UV log

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- Self-consistency of perturbation theory means that r_n is an n^{th}-order polynomial in r_1 with Q-independent but scheme-dependent coefficients. Simplifying to a one-loop beta function $c = c_2 = c_3 = \cdots = 0$

$$a(\mu) = \frac{1}{b \ln \frac{\mu}{\tilde{\Lambda}_{\overline{MS}}}}$$

Given a NLO calculation the <u>RG-predictable</u> part of r_n is r_1^n so geometric series

$$R(Q) = a + r_1 a^2 + r_1^2 a^3 + r_1^3 a^4 + \cdots$$

$$R(Q) = \frac{1}{b \ln \frac{\mu}{\tilde{\Lambda}_{\overline{MS}}}} \left(1 + \sum_{n>0} \left(b \ln \frac{\mu}{\tilde{\Lambda}_{\overline{MS}}} - b \ln \frac{Q}{\Lambda_R}\right)^n \left(\frac{1}{b \ln \frac{\mu}{\tilde{\Lambda}_{\overline{MS}}}}\right)^n\right)$$

Q - dependence

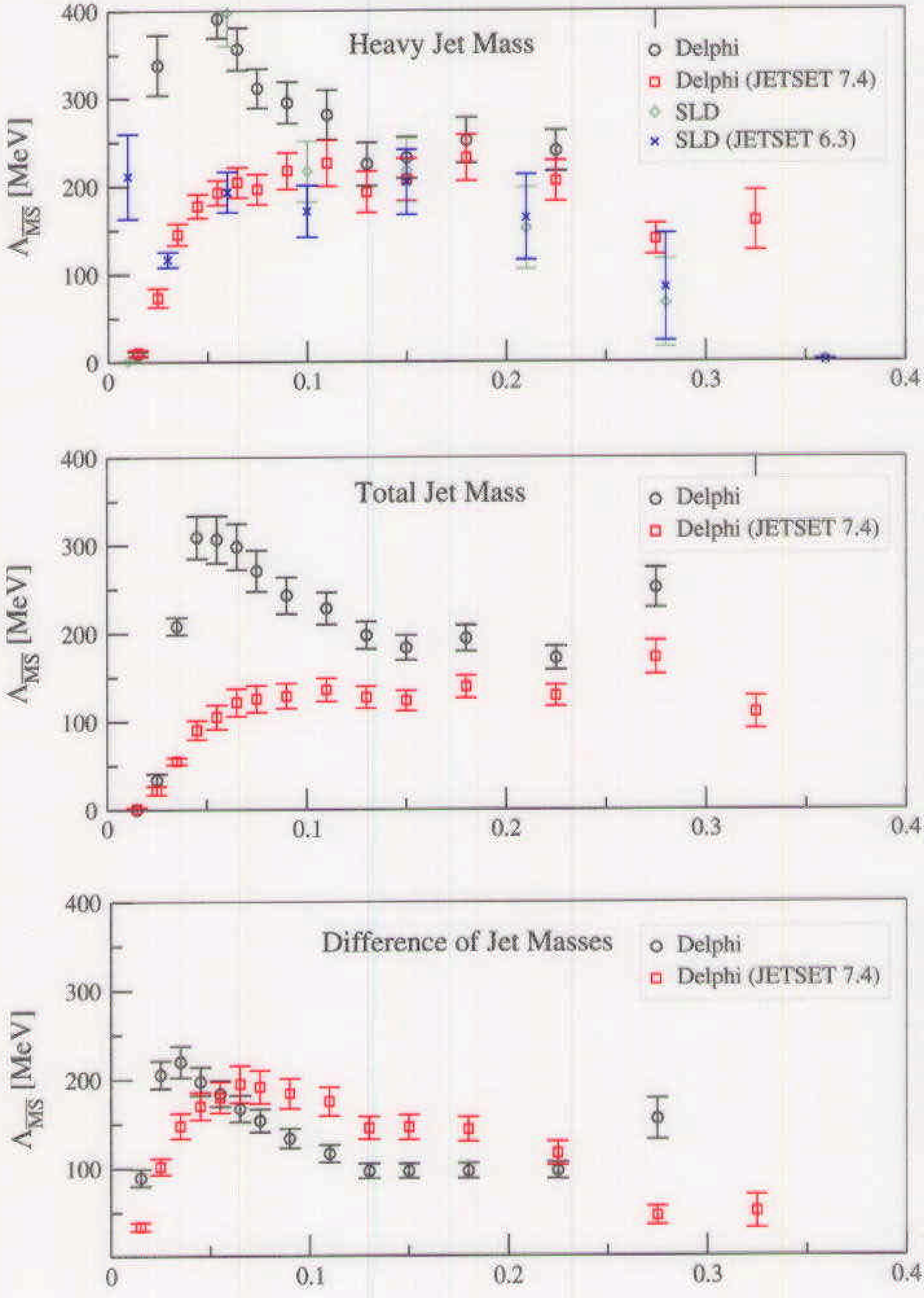
$$= \frac{1}{b \ln \frac{Q}{\Lambda_R}}$$

<u>μ-independent</u> result $a(e^{-r/b} Q)$.

If hold <u>μ independent of Q</u> fixed-order truncation no longer adequate! Asymptotic freedom only emerges if UV logs are resummed to <u>all-orders</u>. <u>Complete RG-improvement</u>.

With standard RG-improvement $\mu = xQ$, $r_1(xQ)$ is <u>Q-independent</u>. Q dependence from scheme-dependent $a(xQ)$.

$\Lambda_{\overline{MS}}$ for e^+e^- Event Shapes



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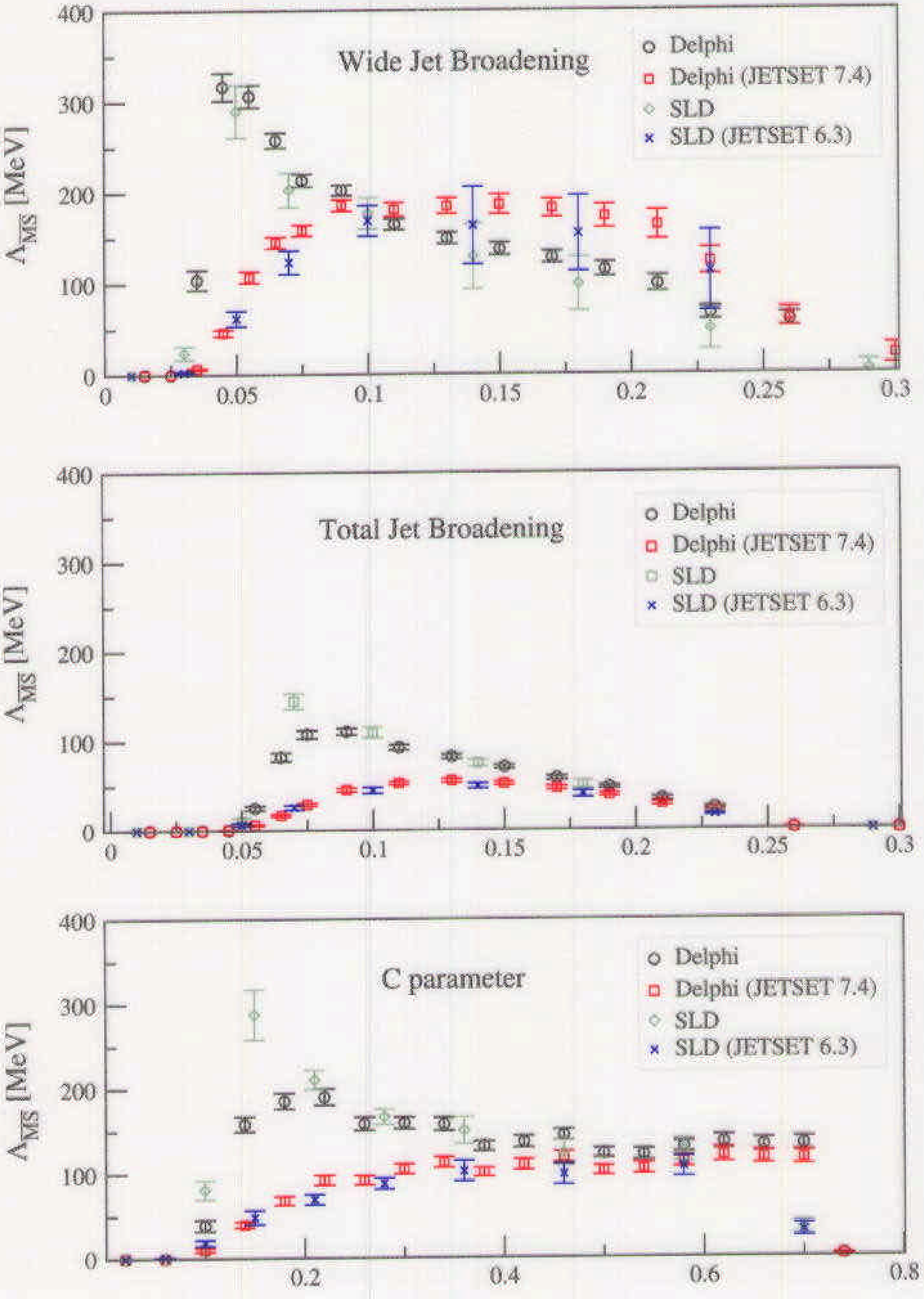

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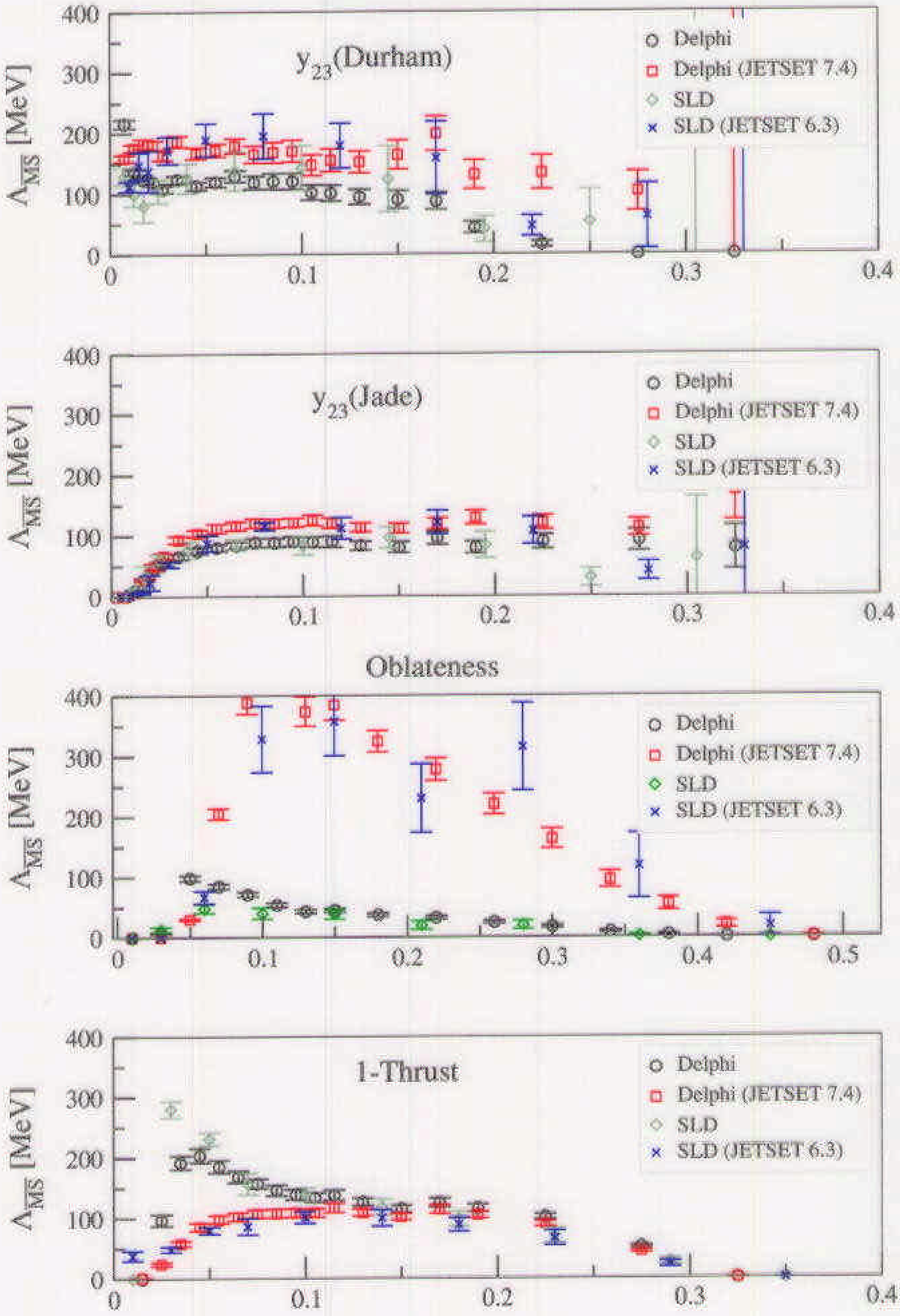

Conclusions

(1) Fundamental parameter is universal Λ (e.g. $\Lambda_{\overline{MS}}$), rather than $\alpha_s(M_Z)$. Different Λ's are related by a one-loop calculation exactly. Celmaster and Gonsalves.

(2) Can relate R (data) to $\Lambda_{\overline{MS}}$ without specifying a particular renormalization scale, μ, by direct integration of the form for $dR/d\ln Q$ implied by dimensional analysis. This automatically resums all UV logs of Q, and is by construction the best one can do in extracting $\Lambda_{\overline{MS}}$ from the data R(Q).

(3) In standard fixed-order RG-improvement with $\mu = xQ$ an infinite subset of UV logs is omitted. The particular choice $\mu = e^{-r/b}Q$ at NLO does resum all UV logs and reproduces the above R(Q) dependence (Corresponds to Effective Charge, FAC approach). Grunberg.

(4) Can replace standard RG-improvement by Complete RG-improvement (CORGI) where μ is independent of Q. One is then forced to resum all UV logs to produce Asymptotic Freedom.