# Recent Progress in NNLO QCD Calculations

Z. Bern, L. Dixon and D. Kosower

Osaka
July 27, 2000

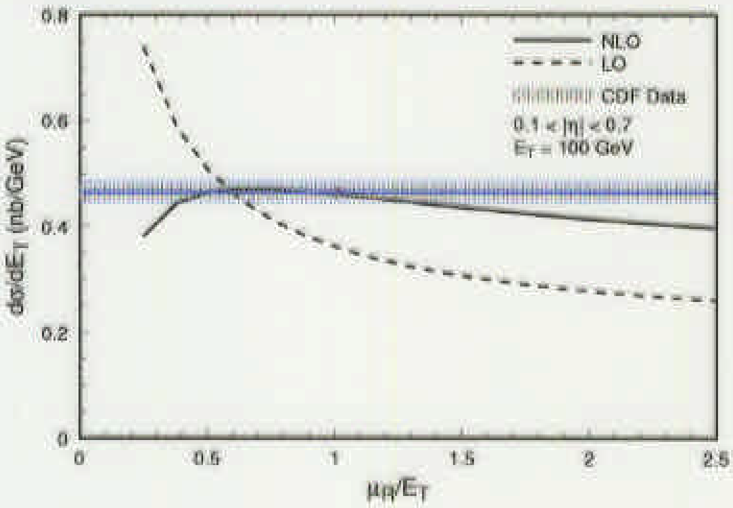
# Higher Order QCD

Higher order contributions can be large.

Basic problem: $\alpha_s$ is relatively large. Also large logs: $\alpha_s \log(y_{IR})$.

Physical quantities should be independent of renormalization scale, $\mu_R$. This can be used to estimate uncertainty due to missing higher order terms.

Example: At LO renormalization scale dependence is large.



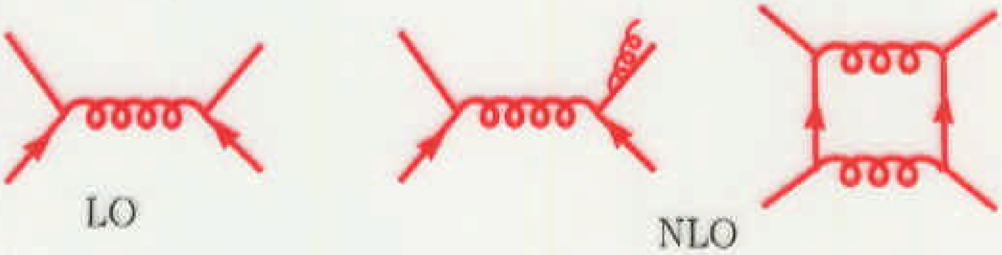

Varying $\mu$ by factor of 2 gives *standard* rough estimate of uncertainty due to missing higher order.

Note: This does not tell you size of higher order corrections; it's only a rough estimate for the uncertainty.

The only way to be sure of the uncertainty is to calculate one order beyond what you expect you need. Generally not easy.

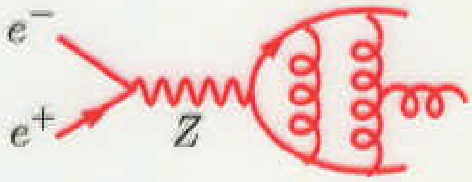
Higher order contributions can give new information not present at LO:

At LO there is no prediction as to the structure of jets – are they fat or skinny?




At NLO we obtain information on jet structure. NNLO would refine this.

An example of an important NNLO perturbative calculation that has not yet been performed is $e^+e^- \to 3$ jets.




It is needed to reduce theoretical uncertainty in $\alpha_s$.

Moreover, at the Tevatron and LHC, QCD enters in essentially all aspects of the physics, whether precision electroweak, new physics searches, etc.

# The difficulties at NNLO

Every step in the construction of a NNLO jet cross-section has major difficulties.

1. Loop integrals.

2. Infrared divergences and phase space integrals.

3. Matrix elements

4. Jet programs

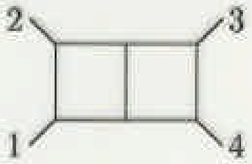
The case of $e^+e^- \to 3$ jets is too difficult at the moment.

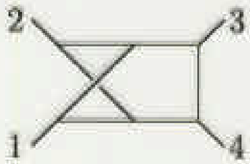
Strategy: Work out $p\bar{p} \to 2$ jets, $pp \to 2$ jets instead. Still very difficult but there has been substantial recent progress.

# Integrals

The integrals are a big problem.

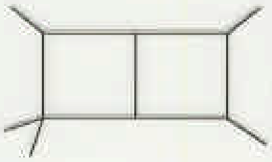
Recent breakthrough by V.A. Smirnov: evaluation of planar massless scalar double box integral. Tausk has also evaluated [redacted] analogous non-planar double box.






Smirnov, Veretin and Anastasiou, Gehrmann, Oleari, Remiddi, Tausk have presented algorithm for evaluating all massless tensor double box integrals.

Example of an integral needed for $e^+e^- \to 3$ jets:

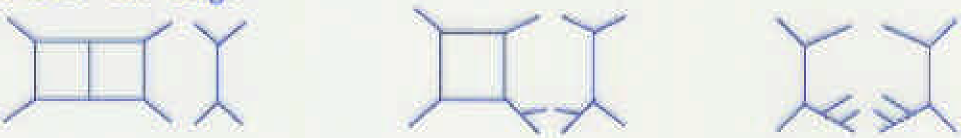


Very recently calculated by Smirnov, but non-planar not yet done.

# Infrared Cancellations

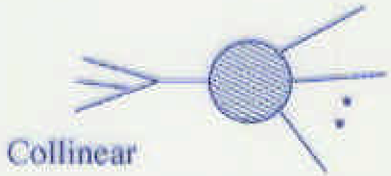
Significant amount of work to combine amplitudes to get jet cross-section.

All same order in $\alpha_s$:



IR divergences cancel amongst diagrams.

Problem solved by Campbell and Glover and by Catani and Grazzini.




Infrared divergences occur in the regions where three particles become collinear or soft.

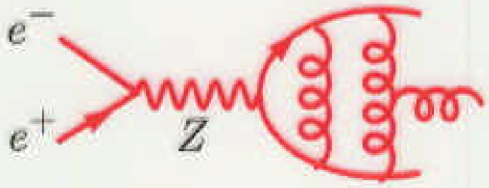
At one-loop there are also non-trivial contributions. Bern, Kilgore, Del Duca, and Schmidt; Kosower and Uwer.


In the presence of IR divergences the Feynman diagrams do not factorize on poles. All such contributions have been determined explicitly (hep-ph/9903516).

Everything would have to be put together in a jet program.

# Two-Loop Matrix Elements

No two- or higher-loop amplitudes have been computed which involve more than 1 kinematic variable.




Another example is NNLO DGLAP splitting functions for parton evolution.

# Quantization via analytic properties.

Claim: We can perturbatively quantize theories by making use of analytic properties, especially as a function of dimension. This leads to an efficient organization of the $S$-matrix.

This is a proven technology – state of the art calculations.

See, e.g., Z.B., L. Dixon, D.A. Kosower, Ann. Rev. Nucl. Part. Sci. 46:109 (1996) [hep-ph/9602280] for refs. and details.

State of the Art One- and Two-Loop Results:

- $Z \to 4$ partons


Z.B., Dixon, Kosower, Weinzierl

- Two-loop calculations in $N = 8$ supergravity – gives divergence properties in disagreement with previous superspace power counting arguments. Also used to show that $D = 11$ supergravity diverges at two loops.

- Infinite sequences of maximally helicity violating one-loop amplitudes, in QCD, gravity and in susy versions of the theories. These are exact expressions!


Z.B.
Chalmers
Dixon
Dunbar
Kosower

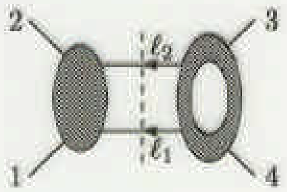
Results for $N = 4$ susy amplitudes are particularly simple.

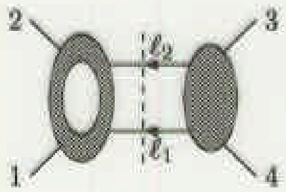
# Generalized Cuts

To performed NNLO QCD calculations it is useful to define a generalized notion of unitarity cuts.

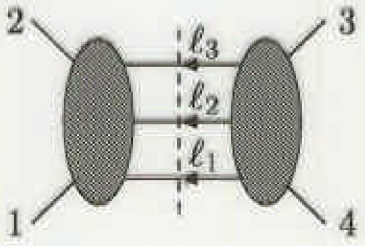
Standard cuts:

Two-particle cuts:

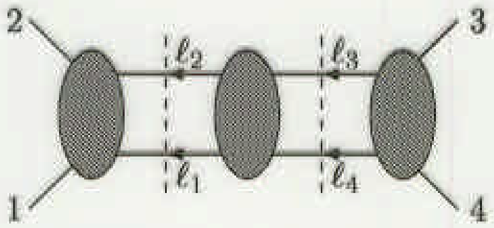



Three-particle cuts:




Generalized double two-particle cut:




This does *not* mean "imaginary part of imaginary part". It should be interpreted as demanding that cut propagators do not cancel.

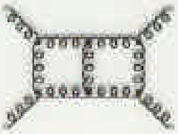
The double two-particle cuts, are very useful since they are relatively easy to calculate. It gives us precise information on the analytic structure of the amplitudes.

# Two-Loop QCD

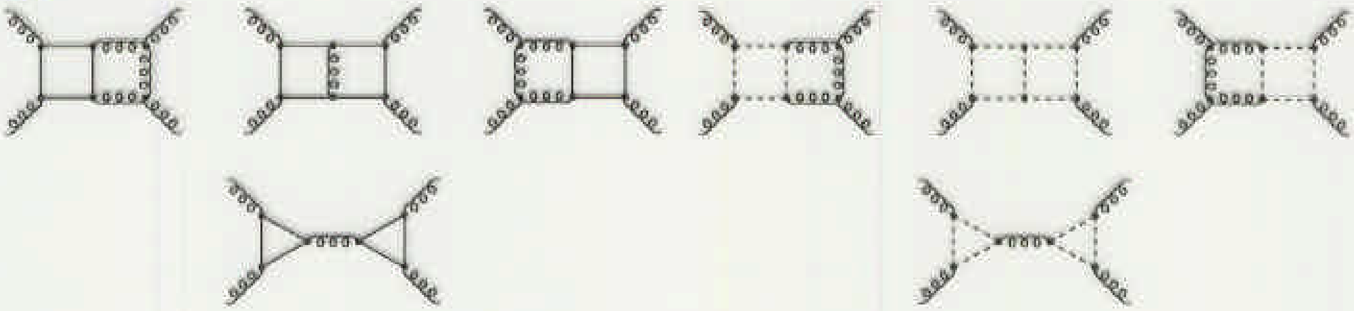
ZB, Dixon and Kosower

Susy is extremely helpful for doing QCD.

Consider case of identical helicities (hep-ph/0001001) corresponding to 931 Feynman diagrams.



Susy can be used to relate gluon diagrams to quark and scalar diagrams.



Scalars are relatively easy to obtain (i.e. can be done by hand using cutting method.)

## Strategy

- Calculate scalar in loop case via cutting method.
- Use susy to relate scalar to quark and gluon loop amplitudes.
- Correct for susy breaking of dimensional regulator.
- Check everything numerically by evaluating cuts at a few points in phase space.
- Face up to integrals and calculate them.

# Two-Loop QCD Results with Scalars




$$|A_4^{\text{2-loop,(a)}}(1^+,2^+,3^+,4^+)|\vert_{2\times 2\text{-cut}}$$
$$= |A_4^{\text{loop}}(1^+,2^+,-L_2^s,-L_1^s) \times A_4^{\text{loop}}(L_1^s,L_2^s,-L_3^s,-L_4^s) \times A_4^{\text{loop}}(L_4^s,L_3^s,3^+,4^-$$
$$= \Big(\frac{s_{12}\mu_1^2\mu_4^2}{(L_1-k_1)^2(L_1-L_4)^2(L_4+k_4)^2} + \frac{1}{2}\frac{\mu_1^2\mu_4^2}{(L_1-k_1)^2(L_4+k_4)^2}\Big),$$

where $L_1 = \ell_1 + \mu$ and $D = 4 - 2\epsilon$.

$$|A_4^{\text{loop}}(1^+,2^+,-L_2^s,-L_1^s)| = \frac{\mu_1^2}{(L_1-k_1)^2},$$
$$|A_4^{\text{loop}}(L_1^s,L_2^s,-L_3^s,-L_4^s)| = \Big(\frac{s_{12}}{(L_1-L_4)^2} + \frac{1}{2}\Big);$$

Just multiply the trees together! (Note: $s_{12} = (k_1+k_2)^2$)

This gives us ansatz for the amplitude which we then check against other cuts.

Final answer:

$$|A_4^{\text{2-loop,(a)}}(1^+,2^+,3^+,4^+)| = [s_{12}\mathcal{I}_4^{\text{P}}[\mu_p^2\mu_q^2](s_{12},s_{23})$$
$$+\frac{1}{2}\mathcal{I}_4^{\text{bow-tie}}[\mu_p^2\mu_q^2](s_{12}) + (s_{12}\leftrightarrow s_{23})],$$

$$\mathcal{I}_4^{\text{P}}[\mathcal{P}(\mu_i,p,q,k_i)](s_{12},s_{23}) \equiv \int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}$$
$$\times \frac{\mathcal{P}(\mu_i,p,q,k_i)}{p^2\,q^2\,(p+q)^2(p-k_1)^2\,(p-k_1-k_2)^2\,(q-k_4)^2\,(q-k_3-k_4)^2}$$

ZB, Dixon and Kosower; ZB, De Freitas, Dixon, Wong

## Two-Loop QCD Results for Gluon and Fermion Loops



Applying susy we convert scalar loop answer to a gluon or fermion loop result.

Complete pure glue answer including all color factors:

$$\mathcal{A}_4^{\text{2-loop}}(1^+,2^+,3^+,4^+) = g^6 [C^{\rm P}_{1234}\, A^{\rm P}_{1234} + C^{\rm P}_{3421}\, A^{\rm P}_{3421} + C^{\rm NP}_{12;34}\, A^{\rm NP}_{12;34} + C^{\rm NP}_{34;21}\, A^{\rm NP}_{34;21} + \ \mathcal{C}(234)]\,,$$

where $C^{\rm P}_{1234}$ and $C^{\rm NP}_{34;21}$ are color factors.

$$\begin{aligned} A^{\rm P}_{1234} = i\,\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\Big\{ & s_{12}\,\mathcal{I}_4^{\rm P}[(D_s-2)(\mu_p^2\,\mu_q^2+\mu_p^2\,\mu_{p+q}^2+\mu_q^2\,\mu_{p+q}^2) \\ & + 16((\mu_p\cdot\mu_q)^2-\mu_p^2\,\mu_q^2)](s_{12},s_{23}) \\ & + 4(D_s-2)\,\mathcal{I}_4^{\text{bow-tie}}[(\mu_p^2+\mu_q^2)\,(\mu_p\cdot\mu_q)](s_{12}) \\ & + \frac{(D_s-2)^2}{s_{12}}\,\mathcal{I}_4^{\text{bow-tie}}[\mu_p^2\,\mu_q^2\,((p+q)^2+s_{12})](s_{12},s_{23})\Big\}\,, \end{aligned}$$

$$\begin{aligned} A^{\rm NP}_{12;34} = i\,\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\, & s_{12}\mathcal{I}_4^{\rm NP}[(D_s-2)(\mu_p^2\,\mu_q^2+\mu_p^2\,\mu_{p+q}^2+\mu_q^2\,\mu_{p+q}^2) \\ & + 16((\mu_p\cdot\mu_q)^2-\mu_p^2\,\mu_q^2)](s_{12},s_{23})\,, \end{aligned}$$


Quark loop contributions similar.

Explicit values of integrals in terms of polylogs given in appendix of hep-ph/0001001.

## Next Steps

1. Other helicities for 2 partons $\rightarrow$ 2 partons.
2. NNLO $e^+e^- \rightarrow 3$ jets
3. Phenomenology!

There is reason to be optimistic that in the next few years many more multi-leg multi-loop calculations will be possible and that there will be substantial refinements in associated theoretical predictions.